Physics

Laboratory

Manual

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By

CLIFFORD N. WALL, PH.D.

Professor of Physics, University of Minnesota

and

RAPHAEL B. LEVINE, PH.D.

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PRENTICE-HALL PHYSICS SERIES

Donald H. Menzel, *Editor*

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PRINTED IN THE UNITED STATES OF AMERICA

Preface

The performance of experiments and the formulation of theories play complementary roles in the development of physics. Only the constant interplay between theory and experiment prevents the subject from falling into a morass of speculation on the one side, or a fruitless taking of data on the other. To neglect either the one or the other of these two aspects of physics is hardly the course of wisdom. For this reason laboratory work in a general physics course should be regarded as an integral part of the subject. It is highly questionable that a sound working knowledge of physics, or even its proper appreciation, can be gained by the student without some direct experience in the laboratory. The precise nature of this laboratory experience is another matter and depends, in large part, upon the objectives of the course.

The authors, in writing this manual, have assumed that direct experience in the laboratory is of value to a student of physics not only because the results illustrate and illuminate general physical principles, but also because this procedure introduces the student to the raw material upon which general physical principles are based.

Laboratory work deals with particulars rather than with universals; e.g., one measures a particular length in a particular manner with a particular instrument. Whence it arises that laboratory manuals, if they are to be helpful, must deal with particulars, and are often criticized for being cook-books. Aside from the fact that there is much to be said for a good cook-book, it is clear that a laboratory manual must be specific where specificity is required if it is to serve any useful purpose for the beginning laboratory student. In this manual the authors have attempted to be explicit enough to enable the student to proceed with the experiment without relying upon other aids. In this way the entire laboratory period is available for laboratory work, and the laboratory instructor is free to help those students who are in real trouble. In the more advanced experiments certain directions are omitted since by this time the student should be familiar with basic laboratory procedures.

A reasonable treatment of errors is highly important in experimental work. The method of error analysis used in this manual goes much further than the usual reliance upon significant figures alone but does not involve the use of distribution functions. This method permits the student to compute an error in the result in terms of estimated errors in the measured quantities which yield this result. Error calculations are called for in most of the experiments. It is hoped that this insistence upon error calculations will make the student "error conscious."

The majority of the experiments described in this manual involve only standard equipment and are traditional in character. A few of these traditional experiments have been extended to show their wide applicability. For example, in the experiment on the gas thermometer the student not only calibrates the thermometer, but also uses it to measure the sublimation temperature of dry ice.

Experiments on the thermocouple, the oscilloscope, and the d-c and a-c amplifiers have been included because of the increasing importance of these devices in all fields of science and engineering.

Several experiments in modern physics have been included. Although these experiments are difficult, they do not lie outside the realm of possibility in a general physics course. Two experiments on thermionics and one on the measurement of e/m require practically the same equipment.

The amount of theory given in the discussion of any experiment depends to a large extent upon its accessibility. If the theory of an experiment is readily available in most general physics textbooks, it is likely to be omitted in this manual. On the other hand, if this theory is not readily accessible, the authors have generally presented it in somewhat more detail than is the usual case in a laboratory manual.

The body of the manual is divided into five parts: Mechanics; Heat; Electricity and Magnetism; Wave Motion and Sound; and Light. In each part the experiments are numbered consecutively; gaps in the numbering system between parts permit the introduction of additional experiments, should the need arise, without revision of the numbering system.

A selected list of reference books is given in Appendix I. Many of these books have been sources of valuable material used in the preparation of this manual. The authors wish to acknowledge their indebtedness to the writers of these books. Particular mention should be made of Hoag and Korff's *Electron and Nuclear Physics*, which furnished much of the material for the experiments on modern physics; Cork's *Heat*, for its material on thermocouples; Robertson's *Introduction to Physical Optics*, for its excellent treatment of the telescope and microscope; Millikan and Mills' *Electricity, Sound, and Light*, for its thorough treatment of wave motion; and the *Smithsonian Physical Tables*, from which many of the tables in this manual were drawn.

The authors also wish to thank their colleagues in the Physics Department of the University of Minnesota, who have contributed many valuable suggestions and criticisms.

C.N.W.
R.B.L.

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Introduction

The purpose of laboratory work in a general physics course is twofold. First it serves to exemplify and illuminate the physical principles studied in the classroom. But in addition to that, and of equal importance, laboratory work gives the student a working knowledge of some of the methods and the instruments which are used quite generally in many laboratories to solve problems of the most varied types, both physical and biological.

In laboratory work it is usually necessary to transform a set of mental operations into a corresponding set of physical operations which can be performed in the laboratory. The correspondence is seldom, if ever, perfect. As a result, errors creep into the set of physical operations and must be taken into account. One not only wishes to know the magnitude of a physical quantity determined in the laboratory. One also desires to know with what accuracy the quantity has been determined, *e.g.*, the value of the quantity may be accurate to within 1 part in 1000 (0.1%), 1 part in 100 (1.0%), or 1 part in 10 (10%). Whatever the error may be, it is generally necessary to know, at least approximately, what it is. Hence one of the important things to be learned in the laboratory is how to make reasonable estimates of the errors involved in any physical measurements and how to handle the propagation of these errors, *i.e.*, to know how these errors affect the final result in an experiment. For this reason the following section on errors is included in the introduction.

A. ERRORS

1. *Indeterminate or Accidental Errors.* These errors are inherent in practically all measurements. They arise because of uncontrollable conditions affecting the observer, the measuring device, and the quantity to be measured. On the basis of probability it is assumed that these errors are as likely to be positive as negative, and more likely to be small than large. Hence their effect may be minimized by taking a number of measurements of a given physical quantity and using the arithmetic average or mean of these values in any computation.

The precision with which a physical quantity has been measured depends upon the *spread* of the set of measurements about their mean value. If the separate measurements are widely dispersed about the mean, the precision is small; whereas if the dispersion of the separate measurements is small, the precision is large. There are many methods of estimating the spread of a set of measurements about the mean value. In general one usually determines the deviations of the measured values from the mean value and then uses some function of these deviations to represent the spread, and hence the precision of the set of measurements. Unfortunately many of these methods involve considerable computation and are based upon assumptions which are seldom satisfied in the general physics laboratory. A simple but effective measure of the spread of indeterminate errors in the measurement of a quantity is the *average deviation from the mean*. Thus if a given quantity is measured several times and the mean value of these measurements is computed, then the indeterminate error may be represented by the average difference between the mean value and the measured values without regard to sign. For example, suppose that four measurements of a length yield, in centimeters, 7.65, 7.61, 7.66, 7.68. The mean value is 7.65. The deviations without regard to sign are 0.00, 0.04, 0.01, 0.03. The average deviation from the mean is 0.02. If this average deviation is first

subtracted from, and then added to, the mean value, an interval is defined (7.63 to 7.67) which generally brackets about half of the measured values. In this example the measurements 7.65 and 7.66 lie within the interval but 7.61 and 7.68 lie outside the interval. In many cases it is desirable to extend the interval so as to include practically all of the measured values. This can generally be done by using twice the average deviation as a measure of the indeterminate errors. The interval in the above example then becomes 7.65 ± 0.04 and includes all of the measured values.

Any measured value whose deviation from the mean is more than four times the average deviation should be viewed with suspicion. It should probably be discarded and a new measurement taken.

In some experiments it is impossible to repeat a measurement. In this case it is impossible to use the method described above to estimate the error; but some estimate of the error should be made nevertheless. Often it is satisfactory to take this error as some fraction (frequently one-half) of the smallest division of the measuring device.

Frequently a set of measurements has to be taken in an experiment under a *prescribed* condition. It may be difficult to judge exactly when this condition is satisfied. In this case it may be necessary to vary the physical quantities which produce this condition and to note how much each may be varied without appreciably altering the prescribed condition. This variation may be used to determine the error.

Finally, the measuring devices (meter stick, voltmeter, etc.) are not perfectly accurate even if one could make an exact reading with them. The manufacturer usually guarantees a certain specified accuracy in the measuring device; for example, a particular type of voltmeter may be guaranteed to give readings which are accurate to at least $\pm 1\%$ of full-scale reading. If this instrument is used to make a voltage determination, a possible error of at least $\pm 1\%$ may be introduced. In a sense such an error as this is not strictly indeterminate for it could be determined and eliminated by calibrating the voltmeter. Such procedure, however, is the exception rather than the rule in the general physics laboratory. Consequently such instrumental errors are usually regarded as indeterminate in the sense that they could be reduced by using several instruments of the same type to make a measurement rather than just one instrument. In taking an average, these instrumental errors would tend to balance out.

2. *Determinate or Constant Errors.* Determinate errors occur in an experiment because of faulty methods, defective measuring apparatus, or incomplete working equations. They are definite in sign and magnitude and cannot be reduced by taking the average of a number of measurements because the same error is included in each measurement.

These errors are often more important than the indeterminate errors. They may be reduced by modifying the method, calibrating the measuring apparatus, or changing the working equations. These procedures are generally described as "corrections" which are to be made in performing an experiment. For example, the reading of a micrometer caliper may not be zero when the jaws are in contact. This so-called zero error must be corrected for in using the instrument; otherwise all measurements made with the caliper will be in error by a determinate amount, the zero error.

3. *The Error Interval.* The determination of a physical quantity that is directly measurable, such as the length or diameter of a cylinder, involves the use of a set of equipment and a set of operations. By means of these one expects to be able to assign a numerical value to the quantity being measured. As has been pointed out, there is usually an element of uncertainty about the numerical value. One attempts to indicate the amount of this uncertainty by means of an error interval. This interval encloses or brackets the value assigned to the physical quantity and gives an indication of the accuracy of the measurement. The size of the interval is estimated by the observer on the basis of the type of equipment and operations used in making the measurements. Experience plays a large part in making a suitable estimate of the error interval. Beginners in laboratory work are generally inclined to underestimate the size of this interval.

It is practically impossible to establish general rules for estimating the size of the error interval, since the interval depends primarily upon the specific set of equipment and operations actually used in making the measurement. Perhaps the most that can be said about the error interval in any direct measurement is that it should be chosen large enough for the observer to be fairly confident that a more accurate measurement would fall within the interval, and small enough to reflect the accuracy of the measurement actually

made. Beyond this it is difficult to go without entering into statistical arguments that lie outside the realm of this discussion.

The results of a direct measurement of a quantity may be expressed in the form $X \pm \Delta X$. X represents the assigned value of the quantity (usually the mean of several measured values). ΔX represents the half-width of the corresponding error interval.

It is often convenient to express the error ΔX as a fraction or as a percentage of X. Thus $\Delta X/X$ is the fractional error in X and $100\ \Delta X/X$, the percentage error. It is assumed that $\Delta X/X \ll 1$. Hence the number of figures used in denoting an error is usually one, or at most, two.

4. *Propagation of Errors.* The experimental determination of some physical quantity such as density or volume is seldom obtained by direct measurement. Usually the quantity to be determined is related in some known manner to one or more measurable quantities. The procedure, of course, is to measure the latter quantities and thence to compute the former by means of the known relation. For example, the volume of a cylinder may be computed if its length and diameter are given. These latter may be measured directly. Now each of the measured quantities has an error interval associated with it. These error intervals determine the error interval of the computed quantity. It is important to know how to make this determination of the propagation of errors.

Suppose that the quantity R to be determined is related to two (for simplicity) measurable quantities X and Y by means of the equation

$$R = f(X, Y), \tag{1}$$

where f is a known function of X and Y. If X and Y are measured, then R may be computed by means of Eq. (1). But the values of X and Y are uncertain and these uncertainties are represented by the errors $\pm\Delta X$ and $\pm\Delta Y$. In order to obtain the uncertainty or error in R which we represent by the symbol $\pm\Delta R$, we vary X and Y by amounts $\pm\Delta X$ and $\pm\Delta Y$ and compute the corresponding variation $\pm\Delta R$ in R. On this basis the meaning of ΔR is just this: If the "true" value of the first measured quantity lies somewhere within the interval $X \pm \Delta X$, and the "true" value of the second measured quantity lies somewhere within the interval $Y \pm \Delta Y$, then the "true" value of the computed quantity will be somewhere within the interval $R \pm \Delta R$. Note that the converse of this is not necessarily true.

There are two different methods by which ΔR may be computed. One method is simply to compute the values of the function in Eq. (1) for the four sets of arguments:

$$X + \Delta X,\ Y + \Delta Y; \qquad X + \Delta X,\ Y - \Delta Y; \qquad X - \Delta X,\ Y + \Delta Y; \qquad \text{and} \qquad X - \Delta X,\ Y - \Delta Y.$$

The difference between the largest and smallest values of the function for these arguments will be $2\Delta R$. This method, while always legitimate, is often tedious and cumbersome. The computations must be made with extreme care since $2\Delta R$ is the difference between two numbers that are almost equal. Any mistakes in computation will very likely be fatal to the success of this method.

The second method, the one customarily used, is more direct in the sense that it gives ΔR at once in terms of ΔX and ΔY. It is based upon the use of differential calculus applied to equation (1) and leads to a few simple rules for computing ΔR in terms of ΔX and ΔY. The basis for these rules is given in the next paragraph. Students who are not familiar with differential calculus may skip the following paragraph.

The determination of ΔR for given values of ΔX and ΔY is most directly effected by taking the differential of Eq. (1). This gives

$$dR = \frac{\partial f}{\partial X}\,dX + \frac{\partial f}{\partial Y}\,dY. \tag{2}$$

The differentials in Eq. (2) may be replaced by the errors ΔR, ΔX, and ΔY, provided these errors are small enough to justify neglecting terms of higher order than the first. With this limitation in mind, we may write

$$\Delta R = \frac{\partial f}{\partial X}\,\Delta X + \frac{\partial f}{\partial Y}\,\Delta Y. \tag{3}$$

Equation (3) is the determinate-error equation to be associated with Eq. (1). If ΔX and ΔY are indeterminate errors, *i.e.*, their signs are $\pm$, then the largest value of ΔR is wanted. This will be given by the equation

$$\left|\Delta R\right| = \left|\frac{\partial f}{\partial X}\,\Delta X\right| + \left|\frac{\partial f}{\partial Y}\,\Delta Y\right|. \tag{4}$$

Equation (4) is the indeterminate-error equation to be associated with Eq. (1). Frequently it is more convenient to take the differential of the natural logarithm of Eq. (1). This is especially true if the function $f(X, Y)$ is a product or quotient of X and Y. In this case Eq. (3) is replaced by

$$\frac{\Delta R}{R} = \frac{\left(\frac{\partial f}{\partial X}\right)}{f}\,\Delta X + \frac{\left(\frac{\partial f}{\partial Y}\right)}{f}\,\Delta Y. \tag{5}$$

With these equations available, the student may work out the rules for the propagation of errors given in the following paragraphs.

Three simple rules suffice to handle most problems in the propagation of errors. These may be stated as follows:

(a) If the result R is the sum or difference of two measured quantities X and Y, the indeterminate error in R is the *sum* of the errors in X and Y.

EXAMPLE:

Mass of bulb with air: 66.928 ± 0.001 gm
Mass of bulb empty: 66.682 ± 0.001 gm
Mass of air: 0.246 ± 0.002 gm

Note that, although the mass of the bulb is reliable to about 1 part in 67,000, the mass of the air is reliable to only 1 part in 123 or 0.8%. Also notice that the errors are added even though the masses are subtracted.

(b) If the result R is the product or quotient of two measured quantities X and Y, the *percentage error* in R is the *sum* of the *percentage errors* in X and Y.

EXAMPLE:

Mass of object: $M = 345.1 \pm 0.1$ gm
Volume of object: $V = 41.55 \pm 0.05$ cm^3
Density of object: $D = \frac{M}{V} = \frac{345.1}{41.55} = 8.306$ gm/cm^3
Percentage error in M: $\frac{\Delta M}{M} \times 100 = 0.03\%$
Percentage error in V: $\frac{\Delta V}{V} \times 100 = 0.12\%$
Percentage error in D: $\frac{\Delta D}{D} \times 100 = 0.15\%$
Error in D: $\Delta D = 0.012$ gm/cm^3
Density of object: $D = 8.31 \pm 0.01$ gm/cm^3

Note that in this case the error in the result affects the third place in the density. Hence only three figures in D need be retained.

(c) If the result R is some power n of the measured quantity X, then the *percentage error* in R is *n times the percentage error in X.*

EXAMPLE:

Diameter of a sphere: $d = 7.65 \pm 0.03$ cm
Volume of sphere: $V = \frac{1}{6}\pi d^3 = 234$ cm^3

Percentage error in d: $\frac{\Delta d}{d} \times 100 = 0.4\%$

Percentage error in V: $\frac{\Delta V}{V} \times 100 = 3 \times 0.4 = 1.2\%$

Error in V: $\Delta V = 3 \text{ cm}^3$
Volume of sphere: $V = 234 \pm 3 \text{ cm}^3$

5. *Propagation of Determinate Errors.* The rules for the propagation of determinate errors are based upon the same analysis as those for indeterminate errors. However, in this case, the errors have a *definite* sign which must be taken into consideration in combining errors.

6. *General Example.* Suppose we wish to compute the density D of a metal cylinder from measurements of its mass m, its length l, and its diameter d. At the same time we wish to compute the error in D due to errors in the measured quantities m, l, and d. We know that the density (mass per unit volume) is given by the equation

$$D = \frac{4m}{\pi d^2 l}. \tag{6}$$

In order to obtain the corresponding error equation we take the differential of the logarithm of Eq. (6) and get

$$\frac{\Delta D}{D} = \frac{\Delta m}{m} - 2\frac{\Delta d}{d} - \frac{\Delta l}{l}. \tag{7}$$

This Eq. (7) shows us exactly how the errors Δm, Δd, and Δl combine to give the error ΔD. If the errors are determinate (have a definite sign), then Eq. (7) is used as it stands. In this case it is perfectly possible that the errors on the right side of Eq. (7) might balance out leaving $\Delta D = 0$.

If, however, the errors are indeterminate ($\pm$), then it is clear that the signs in Eq. (7) should be chosen in such a manner as to give the largest value of ΔD. This may be achieved by simply *adding* the various error terms on the right side of Eq. (7) regardless of signs. Hence for indeterminate errors, the error equation may be written

$$\frac{\Delta D}{D} = \frac{\Delta m}{m} + 2\frac{\Delta d}{d} + \frac{\Delta l}{l}. \tag{8}$$

This is the general procedure for obtaining the indeterminate-error equation from the determinate-error equation. The determinate-error equation in turn can always be derived from the working equation by the method which we have used above.

It is important to note that the error in a result is always a linear function of the errors in the measured quantities. Hence errors are not multiplied or divided, they are only added or subtracted.

7. *Significant Figures.* In recording data and results it is customary to keep only those figures which are trustworthy and have some significance. These figures, called significant figures, are always determined by the amount of error in the value expressed by these figures. To illustrate the idea of significant figures consider the example given in 4(a). The mass of the bulb with air is given as 66.928 ± 0.001 gm. There are five significant figures in this value, the last one, 8, being in doubt by one unit as indicated by the amount of error, ± 0.001. On the other hand the mass of the air written as 0.246 ± 0.002 gm has only three significant figures, the last one, 6, being in doubt by two units. The first 0 in this value is not counted as a significant figure because it is put in simply to emphasize the position of the decimal point. In example 4(b) the mass of the object has four significant figures, its volume also has four, but its density only has three significant figures because the error in the density affects the third digit and makes it doubtful. Notice that the density was computed to be 8.306 but because of the error was "rounded off" to be 8.31. In casting off nonsignificant figures, if the value of the rejected figures is one-half or greater than one-half unit in the last place retained, increase the last digit retained by 1; if it is less than half, leave this digit unchanged.

It is clear that the amount of error in any measured or computed quantity determines the number of significant figures in the value of that quantity. Hence in recording the value of any quantity all figures up

to and including the first figure affected by the error should be retained. For example, if the error in the value of a quantity is 1 part in 100 (1%), it is fairly evident that the number of significant figures in that value will never be more than three although it may sometimes be only two. Consider a 1% error in the values 5.024, 1.135, and 9.807. These errors (to one place) are respectively 0.05, 0.01, and 0.1. Thus the values with their errors may be written 5.02 ± 0.05, 1.14 ± 0.01, and 9.8 ± 0.1. Notice that the first two values have three significant figures although the last one has only two figures.

In computing with logarithms it is advisable to use a five-place table when the errors are approximately 0.01%, a four-place table for errors of about 0.1%, and a slide rule for errors of about 1%. Where angles are involved, errors of 0.01, 0.1, and 1% call for angles expressed to the nearest 1, 6, and 30 min, respectively.

In writing a very small or a very large number it is customary to express it as a power of 10. The number is written as the product of two factors. The first factor contains as many digits as there are significant figures, the decimal point usually appearing to the right of the first digit. The second factor is a power of 10. Thus the speed of light is written 2.99796×10^{10} cm/sec and implies that this speed has been determined to six significant figures.

B. GRAPHICAL METHODS

Frequently the relation between two varying quantities may be clearly shown by means of a graph or curve. The independent variable is usually plotted along the X axis (abscissa) and the dependent variable along the Y axis (ordinate).

The choice of scales is arbitrary and should be made on the basis of convenience and completeness of representation. In general each scale unit should be chosen in such a manner that one-tenth of the smallest division on the coordinate paper represents a unit of the last significant figure of the measurement.

It is not necessary that the intersection of the two axes correspond to the zero point of each scale. The scale values assigned to this intersection point should be such that the curve occupies as much of the coordinate paper as possible while still satisfying the provisions of the previous paragraph.

The experimental values obtained are represented on the coordinate paper by means of sharp dots with small circles drawn about them.

A *smooth* curve drawn through these points as nearly as possible, so that very few points are far from the curve and so that there are as many points on one side of it as on the other, will graphically represent the observations. The exact form of the curve is a matter of judgment. An error in an observation may be indicated by the erratic location of a point.

A title for the curve should always accompany the curve. Also the coordinates along each axis should be labeled with a statement of the quantity plotted and the units in which it is expressed.

A typical graph is shown in Fig. 43-9.

C. METRIC SYSTEM

In general all measurements in the laboratory are to be made in the metric system of units unless distinctly required otherwise. The fundamental units of mass, length, and time in this system are the gram, the centimeter, and the second.

Upon these three fundamental units are founded the many derived units in physics, such as the dyne, erg, joule, watt, etc. There are also related units such as the calorie, degree centigrade, ampere, etc. These units are generally described in connection with the experiment in which they occur.

A table of metric and English equivalents may be found in your general physics textbook.

D. GENERAL INSTRUCTIONS

1. *Laboratory Materials.* In addition to the laboratory manual you will need a notebook, data pad with a sheet of carbon paper, and write-up paper.

2. *Rules of Conduct:*
   (a) Much of the apparatus is delicate and must be handled with care. The student should *never* try to operate the apparatus until the signal to proceed has been given.

(b) Breakage of any part of the apparatus always means an interruption in the experiment and a handicap to the group working with it. The student should examine the apparatus carefully before using it and should take every possible precaution to prevent injury to it.
(c) Should any apparatus be found in an unworkable condition or should any breakdown occur during the experiment, it should be reported at once to the instructor so that repairs can be made as soon as possible.
(d) Calculations and scratch work are to be done in the laboratory notebook provided by the student, not on the table tops.
(e) Each student in a group is individually responsible for the condition in which its location is left at the end of the period.
(f) Apparatus is not to be moved from one location to another without the instructor's permission.

3. *Laboratory Procedure.* At the beginning of the laboratory period, the instructor may briefly discuss the nature and purpose of the experiment to be performed, the apparatus to be used, and the procedure to be followed.

After this discussion the students, generally working in groups of two, perform the experiment. All *data* and *notes* concerning the experiment should be entered directly into the student's notebook. The student should make a definite effort to record the data of an experiment in organized form, paying particular attention to proper significant figures and units. When the experiment has been completed, each student should have a complete record of the experiment in his notebook. The importance of having this original record complete in the notebook can hardly be overemphasized, since this material will form the basis of any laboratory report required. It may also be used in laboratory quizzes.

Each student makes a copy (may be a carbon copy) of the original data he has recorded, including a list of the apparatus and apparatus numbers. The data sheet should be used for this purpose. This sheet is given to the instructor.

The remainder of the laboratory period is used by the student in carrying out the calculations called for in the experiment. These calculations should always be made in the laboratory notebook in conjunction with the original data. Errors in calculations are generally inexcusable and may be largely avoided by using a systematic method of computation. Near the end of the period, at a signal from the instructor, the students clean up their location, put away their equipment, and check out.

Experiment 1.

Density of a Solid

Object: To determine the density of a solid in the form of a metal cylinder.

Apparatus: Metal cylinder, vernier caliper, micrometer caliper, trip balance.

Theory: The density D of a substance is its mass per unit volume. In this experiment the density of the metal cylinder is determined in terms of its mass M, its length l, and its diameter d by means of the equation,

$$D = \frac{4M}{\pi d^2 l}. \tag{1}$$

The determinate error equation corresponding to Eq. (1) is

$$\frac{\Delta D}{D} = \frac{\Delta M}{M} - \frac{\Delta l}{l} - 2\frac{\Delta d}{d}. \tag{2}$$

For the method of obtaining this equation and its use see the Introduction, A-6.

Method: Make five determinations of the mass (gm) of the cylinder by "weighing" it on a trip balance. Shift the position of the cylinder on the pan of the balance after each weighing. The scale on the balance reads directly to 0.1 gm. It is not necessary to attempt a reading more accurate than one-half this smallest division.

Measure the length (cm) of the cylinder five times with a vernier caliper to the nearest 0.01 cm. See Appendix II-A for the use of the vernier caliper. Place the cylinder at different positions along the jaws of the vernier caliper.

Measure the diameter (cm) of the cylinder ten times with a micrometer caliper to the nearest 0.0001 cm. This will require estimating to tenths of the smallest division. See Appendix II-B for the use of the micrometer caliper.

Record these data in tabulated form. Compute the average values of M, l and d; the deviations, and the mean deviations. Compute the density of the cylinder using these average values. Compute the indeterminate error in the density using as the errors in M, l and d either the mean deviation or one-half the smallest division of the measuring device, whichever is the larger.

Record: (Sample).

Cylinder #40 (brass)
Vernier caliper #5 (zero error negligible)
Micrometer caliper #11, zero error +0.0003 cm.

Trial	*M*, gm	dev.	*l*, cm	dev.	*d*, cm (uncorrected)	dev.
1	13.70	0	8.99	0	0.4767	4
2	13.85	15	8.98	1	0.4765	2
3	13.60	10	8.99	0	0.4769	6
4	13.60	10	9.00	1	0.4761	2
5	13.75	5	8.99	0	0.4758	5
6					0.4763	0
7					0.4762	1
8					0.4762	1
9					0.4763	0
10					0.4762	1
Average	13.70 ±.08		8.990 ±.004		0.4763 ±.0002	

$M = 13.70 \pm .08$ gm., $l = 8.990 \pm .005$ cm., $d = 0.4760 \pm .0005$ cm (corrected).

Note that the error in M is taken as the mean deviation because it is larger than one-half the smallest division on the balance but that the errors in l and d are taken as one-half the smallest division because they are larger than the mean deviations.

$$D = \frac{4M}{\pi d^2 l} = \frac{4(13.70\text{ gm})}{(3.1416)(0.4760\text{ cm})^2(8.990\text{ cm})}$$
$$= 8.564\text{ gm/cm}^3.$$
$$\frac{\Delta D}{D} = \frac{\Delta M}{M} + \frac{\Delta l}{l} + 2\,\frac{\Delta d}{d}.$$
$$= 0.006 + 0.0006 + 2(0.001)$$
$$= 0.009.$$
$$\Delta D = \pm 0.08\text{ gm/cm}^3.$$

Logs

0.6021	0.4971
1.1367	$\bar{1}.6776$
1.7388	$\bar{1}.6776$
0.8061	0.9538
0.9327	0.8061

Final result: $D = 8.56 \pm 0.08$ gm/cm^3.

QUESTIONS

1. If the zero error in the micrometer caliper had been neglected in the sample data given in this experiment, what *constant* error would have been introduced into D? Is this error significant?

2. Determine ΔD from the sample data by calculating first the maximum value of D, and then its minimum value. This work must be done accurately if a satisfactory value of ΔD is to be obtained. Why?

3. What error in D would have been introduced by using $\pi = 22/7$ instead of 3.1416 in the sample given in this experiment? Is this error significant? What type of error is it—determinate or indeterminate?

Experiment 2.

Equilibrium of Forces

Object: To test the conditions of equilibrium for a set of concurrent coplanar forces.

Apparatus: Force table, weight holders, and weights. The force table consists of a circular metal table top mounted on a vertical rod held in a tripod support with leveling screws. The rim of the circular top has a 360° scale engraved on it along which it is possible to clamp a number of pulleys. The forces are produced by weights attached to suspension cords which pass over the pulleys and are fastened to a small ring held in the center of the table by means of a pin. When the forces along the cords acting upon the small ring are balanced, the ring remains in the center of the table without being held there by the pin.

Theory: A set of forces acting upon a particle will hold that particle in equilibrium provided the *vector sum* of those forces is equal to zero, *i.e.*, provided the vectors representing those forces form a closed polygon when placed end to end. This is equivalent to saying that the algebraic sum of the components of those forces along any straight line must be zero.

A special case of considerable importance is that of three balanced forces F_1, F_2, F_3, as shown in Fig. 2-1a. The angles opposite the forces F_1, F_2, F_3 are designated as ϕ_1, ϕ_2, ϕ_3. These forces, if in equilibrium, must form a closed triangle as shown in Fig. 2-1b.

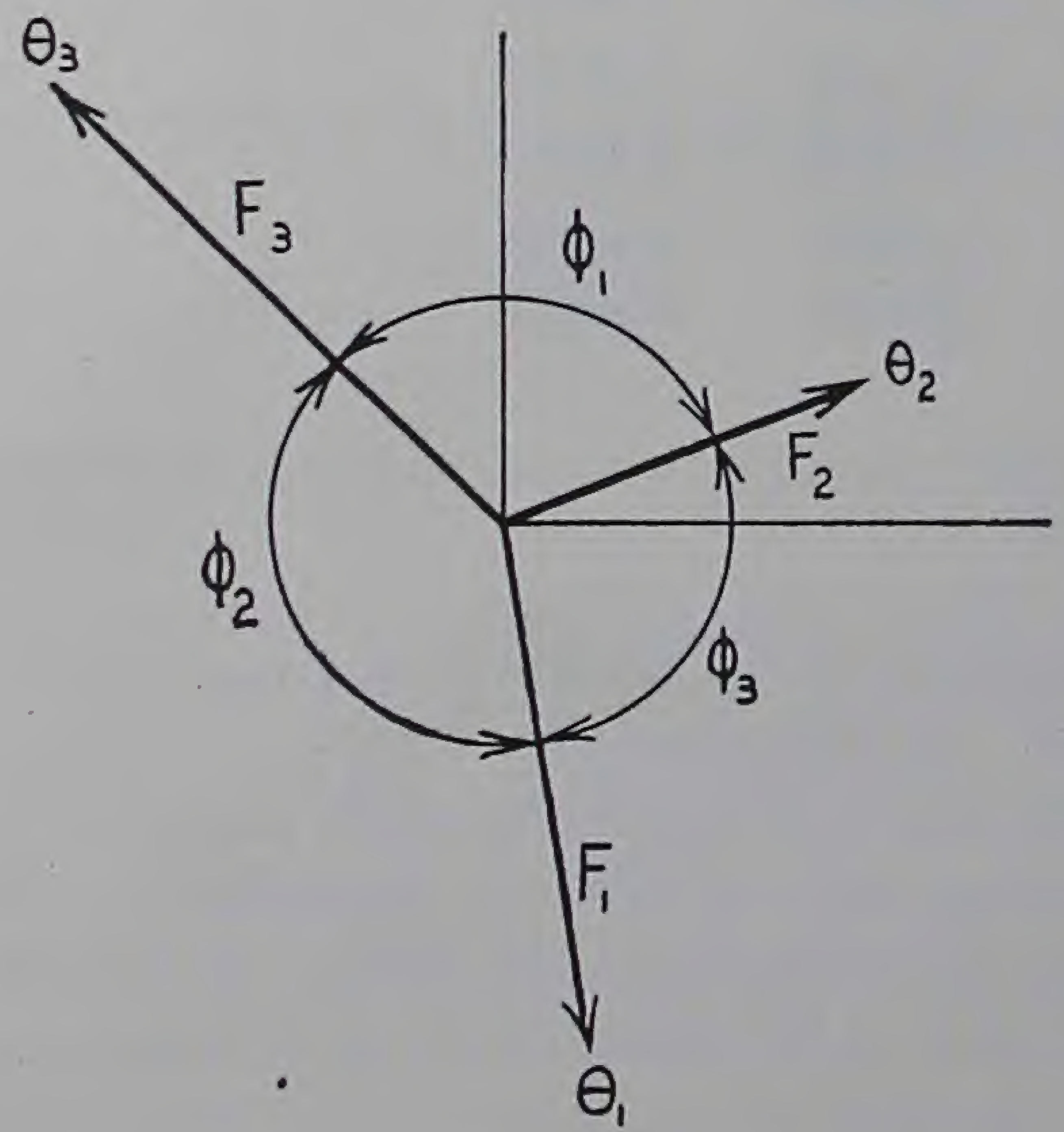


Fig. 2-1a.

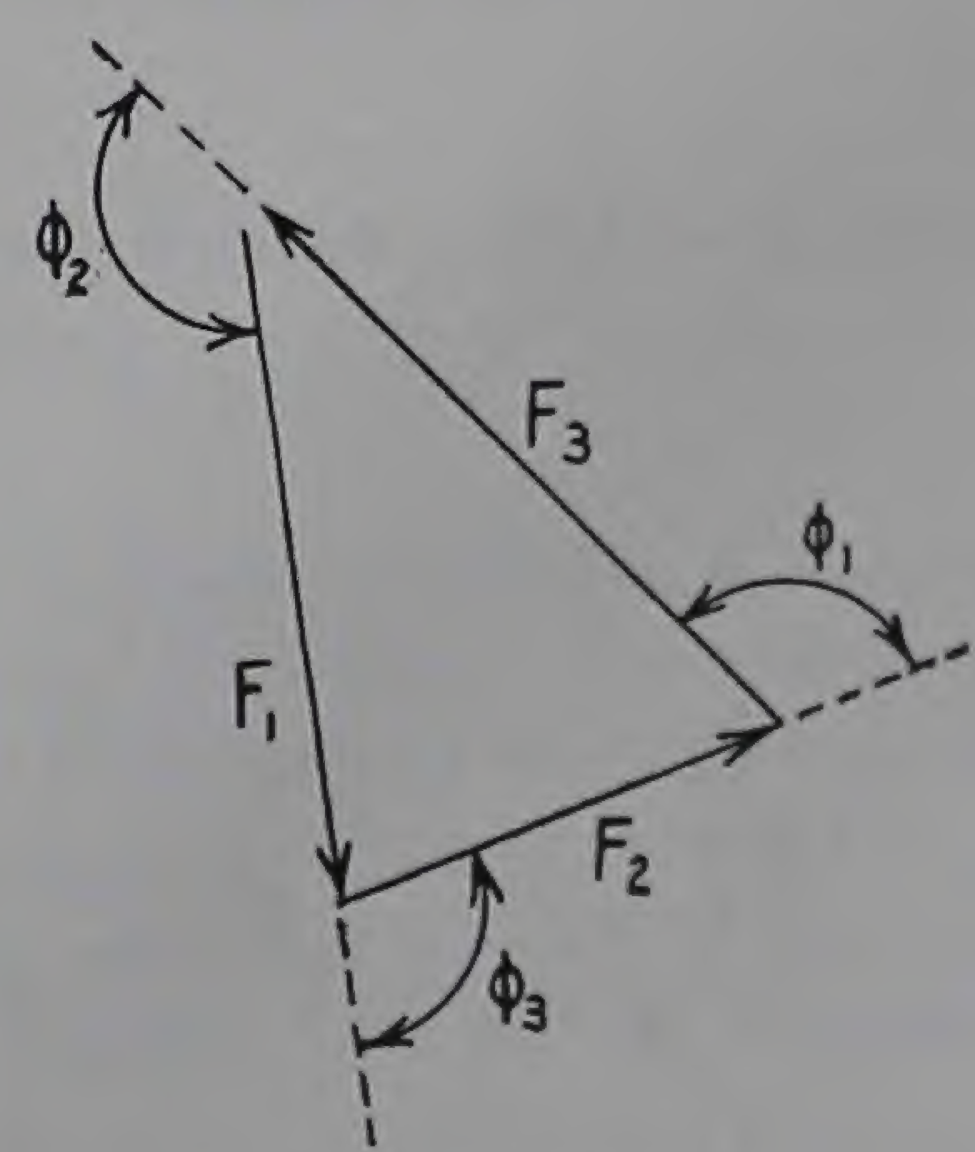


Fig. 2-1b.

By the sine law,

$$\frac{F_1}{\sin \phi_1} = \frac{F_2}{\sin \phi_2} = \frac{F_3}{\sin \phi_3}, \tag{1}$$

since $\sin (180° - \phi_1) = \sin \phi_1$, etc. This relationship is known as Lami's theorem.

Another form of the equilibrium condition may be obtained by resolving each of the forces into its rectangular components X and Y. Then the algebraic sum of the X components of all the forces must equal zero. Also the algebraic sum of the Y components must equal zero. Thus let θ_1, θ_2, and θ_3 be the direction angles, respectively, of the three forces F_1, F_2, F_3 with respect to the X axis. Then the X components of the three forces are given by the equations

$$\left\{\begin{aligned} X_1 &= F_1 \cos \theta_1 \\ X_2 &= F_2 \cos \theta_2 \\ X_3 &= F_3 \cos \theta_3 \end{aligned}\right\}. \tag{2}$$

Correspondingly the Y components are

$$\left\{\begin{aligned} Y_1 &= F_1 \sin \theta_1 \\ Y_2 &= F_2 \sin \theta_2 \\ Y_3 &= F_3 \sin \theta_3 \end{aligned}\right\}. \tag{3}$$

For equilibrium we must have

$$\left\{\begin{aligned} X_1 + X_2 + X_3 &= 0 \\ Y_1 + Y_2 + Y_3 &= 0 \end{aligned}\right\}. \tag{4}$$

The distinct advantage of this latter form of the equilibrium condition over that of Lami's theorem is that it is applicable to any number of balanced forces, whereas Lami's theorem is restricted to three balanced forces.

Method: In this experiment each student should take an independent set of data and compute results based on these data alone.

Part I. Three balanced concurrent forces. Level the force table. Set three pulleys at the angular positions $\theta = 0°$, $110°$, and $260°$ on the force table. Load the weight holders until equilibrium is achieved. The individual loads should not be less than 200 gm or greater than 600 gm. At equilibrium the ring will remain in the center of the force table even though the pin is removed.

Friction in the pulleys often causes trouble. It may be partially avoided by raising the ring vertically upward about 1 cm and then releasing it. It will oscillate up and down for a short time but should come to rest at the center of the table for good equilibrium conditions. The oscillations tend to relax any particular "set" in the pulleys.

The errors in this experiment may be estimated in the following manner. After equilibrium has been achieved with the ring at the center of the table, determine the amount by which each force *in turn* must be increased in order to shift the equilibrium point about 2 mm away from the center of the table in the direction of the increased force. Use this amount as the error in that force. Neglect the errors in the angular settings of the pulleys, for these errors are at least partially accounted for by the above procedure.

Record the magnitude of the three forces, their errors, and their angular positions. Compute the angles ϕ_1, ϕ_2, ϕ_3, and the ratios $F_1/\sin \phi_1$, $F_2/\sin \phi_2$, $F_3/\sin \phi_3$ as well as the errors in these ratios. Compute the X and Y components of the three forces as well as the errors in these components.

Check the validity of Lami's theorem by examining the constancy of the ratio $F/\sin \phi$. Also check the validity of the equilibrium equations by adding the X components and the Y components of the three forces. These sums will not be exactly zero but they should be less than the sum of errors in each case.

Draw the force triangle formed by these forces on a sheet of graph paper using a protractor and a ruler. Choose a scale such that the force triangle covers most of the graph paper. In general the triangle will not be entirely closed because of the errors in the experiment.

Part II. Three unbalanced concurrent forces A, B, C held in equilibrium by two forces P, Q at right angles to each other (the rectangular components of the anti-resultant of forces A, B, C).

Set five pulleys at the angles 0°, 90°, 160°, 240°, and 300°. Apply loads of 200 to 400 gm at each of the latter three angles, as the forces A, B, C. Vary P and Q until equilibrium is obtained. Make an estimate

of the errors in P and in Q by the method already described. Compute the values of P and Q by using the rectangular components of A, B, C. Compare these with the observed values of P and Q.

Draw to scale the force polygon represented by the observed forces A, B, C, P, Q on a sheet of graph paper observing the same rules as given in Part I of this experiment.

***Record:** Part I.* (*Sample.*)
App. No. 24

DATA | COMPUTATIONS

	F, gm	θ	$\sin\theta$	$\cos\theta$	X, gm	Y, gm	ϕ	$\sin\phi$	$\frac{F}{\sin\phi}$
1	300 ± 5	355°	-0.0872	0.996	299 ± 5	-26.2 ± 0.5	140°	0.643	467 ± 8
2	270 ± 5	70°	0.940	0.342	92 ± 2	254 ± 5	145°	0.574	470 ± 9
3	450 ± 5	210°	-0.500	-0.866	-390 ± 4	-225 ± 3	75°	0.966	467 ± 5
				Sum	1 ± 11	3 ± 8			

In the above table it should be noted that although the sums of the X components and the Y components are not exactly zero, they are less than the sums of the errors in both cases. Also, in the last column, $F/\sin\phi$ is not exactly the same for all three forces, but it is a constant within the limits of error of the experiment.

Part II.

DATA | COMPUTATIONS

	F, gm	θ	$\cos\theta$	X, gm	$\sin\theta$	Y, gm
A		160°				
B		240°				
C		300°				
P		0°	$-P$ (calc) = ______			
Q		90°			$-Q$ (calc) = ______	

QUESTIONS

1. Suppose that the weight holders in this experiment all have the same weight. May their weights be neglected? Explain.

2. Suppose that the force table is not level but is tipped slightly around the X axis ($\theta = 0$), *i.e.*, the X axis is level but the Y axis makes a small angle with the horizontal. Which force, P or Q, in Part II of this experiment will be most affected; which least affected? Assume that the ring has no weight.

Experiment 3.

Coefficient of Friction

Object: To determine the coefficient of friction for various pairs of surfaces.

Apparatus: An inclined plane which may be adjusted to any angle with respect to the horizontal, and provided with a pulley at the upper end; various blocks and surfaces; string; weight pan and weights. The inclined plane is provided with three scales: one indicates its angle with the horizontal, and the others indicate length and corresponding height, respectively, of the inclined portion. See Fig. 3-1.

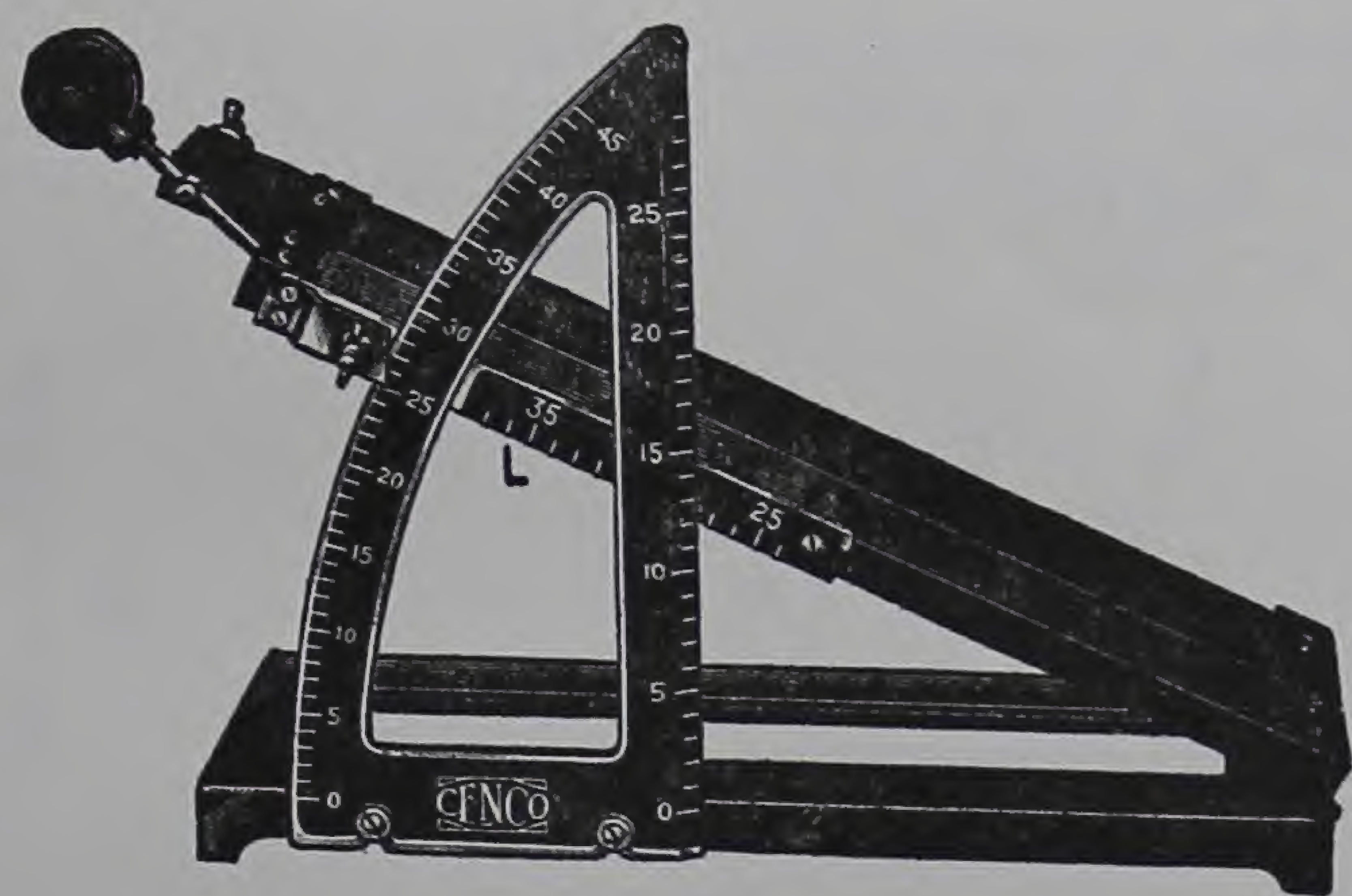


Fig. 3-1.

Theory: To cause one surface to move over another surface against which it is pressed, it is necessary to overcome the frictional force which is in a direction tangential to the two surfaces and opposite to the direction of motion. The force required to overcome friction is found to be substantially independent of everything except the normal force (the force pressing the two surfaces together) and the materials of which the surfaces are composed. Thus for any two surfaces, the ratio of the force of friction and the normal force is a constant:

$$\frac{F}{N} = \mu, \tag{1}$$

where F = force of friction,
N = normal force between the two surfaces, and
μ = constant, the *coefficient of friction*.

Distinction may be made between two different types of friction: *kinetic* (sliding) *friction* and *static friction*. The former type is that which exists between two surfaces which are moving at a constant velocity with respect to each other, and is practically independent of that velocity. It is also independent of the amount of area in contact, provided that the area of one surface does not become so small that it begins to penetrate the other surface. The latter type of friction is that which exists between two surfaces which are

stationary with respect to each other. It is much more difficult to measure accurately than is kinetic friction; in this experiment we shall be interested mainly in its relative magnitude, as compared with the kinetic type. The following discussion refers primarily to kinetic friction. Its coefficient depends only on the materials of the two surfaces; it is generally less between two different surfaces than between two identical surfaces.

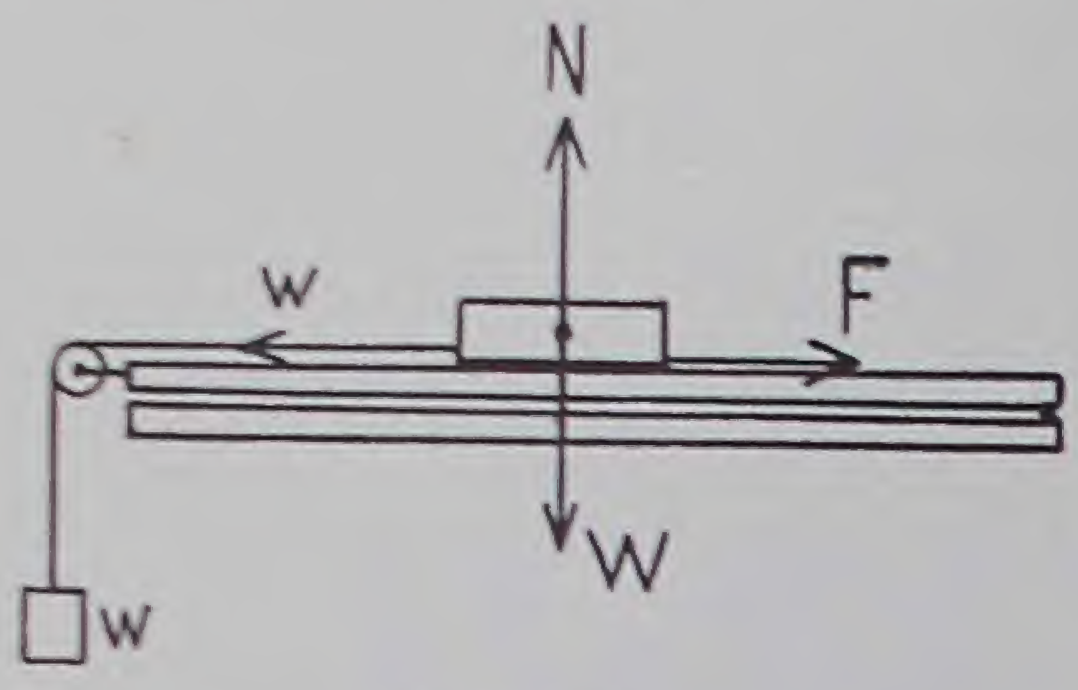


Fig. 3-2.

When a body moves along a horizontal surface at a constant velocity, as shown in Fig. 3-2, the normal force on the body is equal and opposite to its weight, and the force of friction on the body is equal and opposite to the force pulling it along. Here

$$\mu = \frac{F}{N} = \frac{-w}{-W} = \frac{w}{W}. \tag{2}$$

When a body slides down an inclined plane at a constant velocity under the action of gravity, as shown in Fig. 3-3, the three forces acting on the body—its weight, the normal force, and the force of friction—are in equilibrium, and may then be drawn to form a closed triangle as shown. This triangle and the physical triangle formed by the inclined plane itself are similar, since both are right triangles, and the sides of angle θ in one triangle are perpendicular to the sides of θ in the other triangle. Using the proportionality of similar sides in similar triangles it is clear that

$$\mu = \frac{F}{N} = \frac{h}{b}. \tag{3}$$

In the case of static friction, the inclined plane is tilted to just the angle that causes the body to *start* sliding. At the instant of starting, the three forces shown in Fig. 3-3 are in equilibrium, where F is now the force of static friction. It is clear that Eq. (3) again holds with the appropriate different values of the quantities substituted.

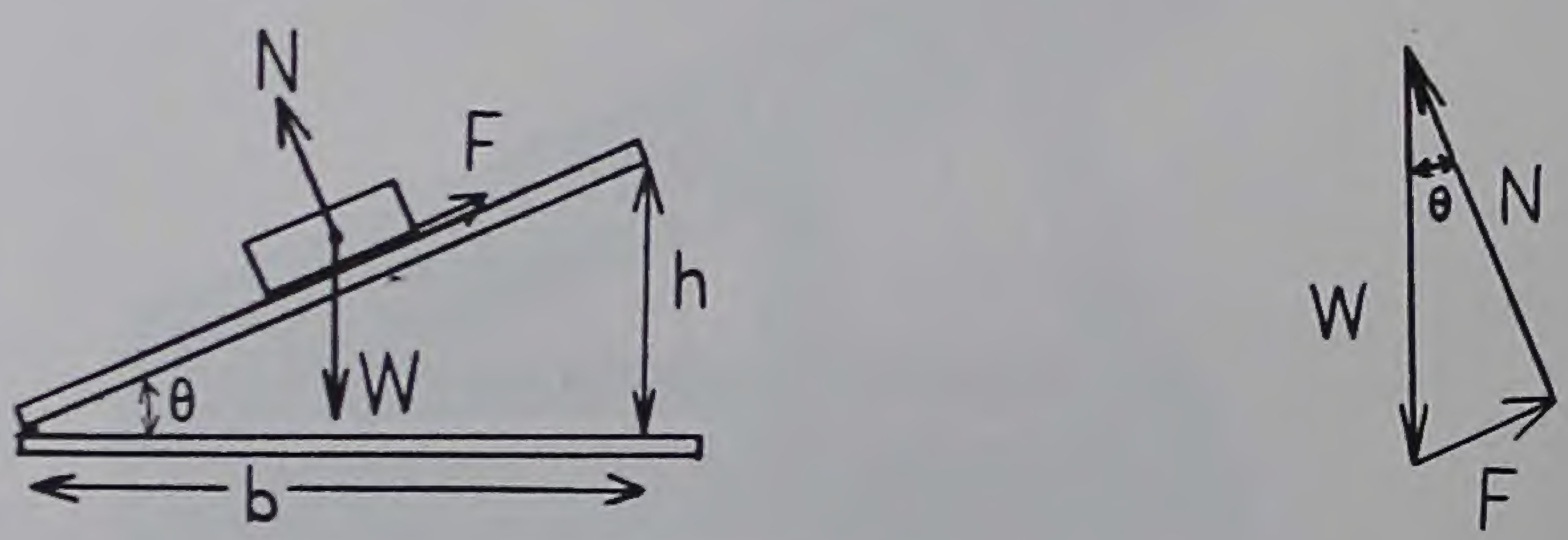


Fig. 3-3.

***Method:* *Part I*.** With the inclined plane horizontal, place the wooden block on it and load it with about 1 kg. Find the weight w which will cause the loaded block to move slowly and uniformly along the plane, giving it a gentle push to start it. See Fig. 3-2. Determine the amount by which w may be changed without causing appreciable acceleration or deceleration.

Repeat with loads up to 4 kg, increasing by 1 kg each time. In each case give an initial push which establishes as nearly as possible the same velocity as in the first case. Weigh the block itself, and use Eq. (2) to find the coefficient of kinetic friction in each case. Compare these results with the appropriate values given in Table J, Appendix III.

Part II. Record the value of the base b of the inclined plane triangle by noting the length of scale L when the plane is horizontal. Remove the weight pan and adjust the angle of the plane to the point where the metal block just slides down the plane without acceleration, after having been given a slight push. Record the angle θ and the height h and repeat three times, recording the angles and heights. Using the mean value of the heights, and the value of b obtained earlier, apply Eq. (3) and find the coefficient of sliding friction. By use of Table A, Appendix III, find the value of tan θ. Note that this is essentially h/b, and is therefore an alternative method of finding μ.

Repeat, using two other pairs of surfaces.

Compare these results with the corresponding values given in Table J, Appendix III.

Part III. Adjust the angle of the plane to the point where the block just *begins* to slide (*i.e.*, just breaks away from rest), and record the angle and the height h at which this happens. Repeat three times, recording the angles and heights. Using the mean value of the heights, and the value of b obtained earlier, apply Eq. (3) and find the coefficient of static friction.

Repeat, using the same pairs of surfaces as in Part II.

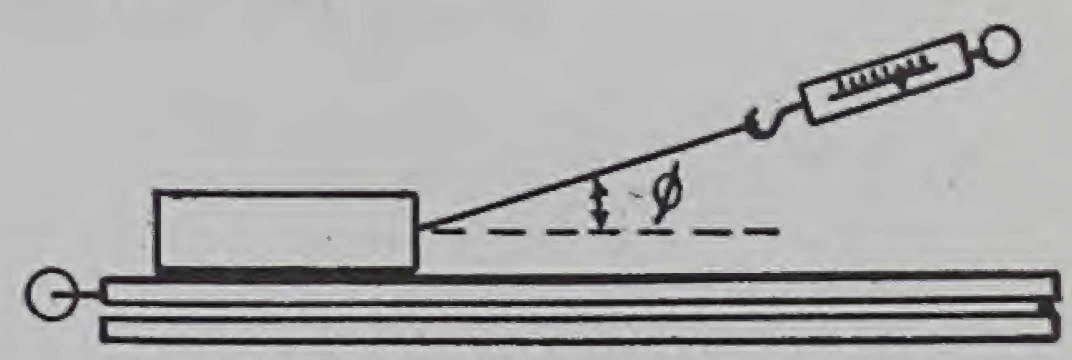


Fig. 3-4.

Part IV. With the plane horizontal, attach the spring balance to the metal block and, varying the angle the connecting string makes with the horizontal (see Fig. 3-4), determine whether there is any angle ϕ at which the force required to move the block slowly and uniformly along the surface is *a minimum.* If so, measure this angle, and compare it with the angle obtained for the same surfaces in Part II.

Record: Record your data and results in tabular form.

QUESTIONS

1. Why must the bodies move with uniform velocity, *i.e.*, with no acceleration, in Parts I, II, and IV of this experiment?

2. Comparing the data of Parts II and III, explain why a motorist stuck in a snowy ditch has more chance of getting out if he does *not* spin his wheels. Roughly, using your own data for the different surfaces, how much better are his chances of getting out by not spinning his wheels as compared to spinning? How much increase in traction does he get by spinning them fast as compared to slowly?

3. Show that the tangent of the angle indicated in Fig. 3-4 is just equal to the coefficient of kinetic friction.

Experiment 4.

Falling Body

Object: To show that the acceleration of a freely falling body is constant; to determine this acceleration: the acceleration due to gravity.

Apparatus: Falling-body apparatus, waxed tape, level, steel rule.

The falling-body apparatus consists essentially of an electromagnet free to fall vertically between guide rods. The electromagnet when connected to a 110-volt 60-cycle power source drives a vibratory-tracing element at 120 vibrations per second. When allowed to fall under these conditions the stylus of the element traces a wavy line (time trace) on a waxed paper held near the stylus. See Fig. 4-1 for a sketch of the electromagnet and vibratory-tracing element.

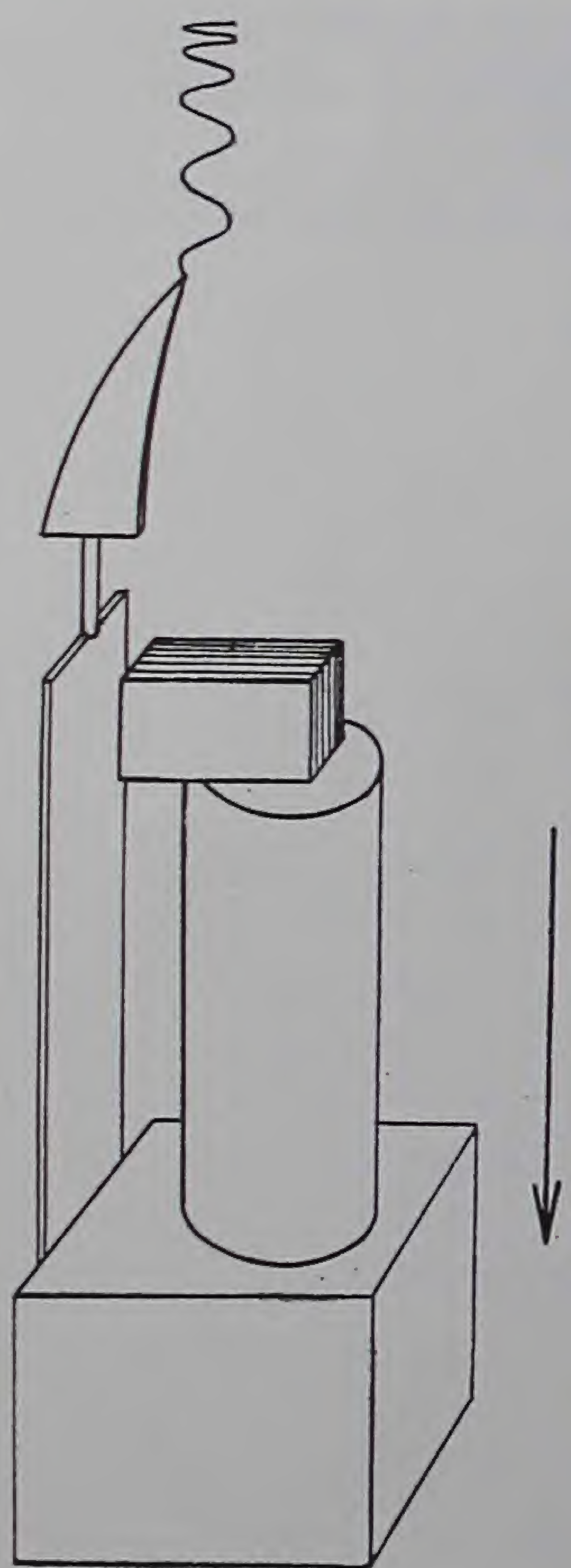


Fig. 4-1.

Theory: Velocity is the time rate of displacement, and for straight-line motion is equal to the distance covered divided by the time of transit. If the velocity is not constant during a finite interval of time, the quantity obtained by such a division will be the *average velocity* during the interval, and this may differ greatly from the *actual* velocity at any particular instant. To obtain such an *instantaneous* velocity, the ratio of distance to time must be found during a time interval small enough so that the velocity does not change significantly during the interval. Thus it is readily seen that the exact value of an instantaneous velocity is obtained by taking the limiting value of the ratio of distance to time as the time interval becomes infinitesimal.

In such a case, where the velocity is changing from instant to instant, the motion is said to be *accelerated*. Acceleration is defined as the rate of change of velocity, and is, of course, equal to the amount of change of velocity divided by the length of time in which the change takes place. If the velocity is changing at a uniform rate, the acceleration is constant. It should be noted that when the acceleration is constant, the average velocity during any interval of time is equal to the instantaneous velocity at the center of that interval.

This experiment will study a particular case of uniformly accelerated motion, that of a freely falling body, whose acceleration is that due to gravity. A continuous record of the position of the body as a function of time will be made, and from this its velocities at various times and its acceleration may be determined.

Method: With the carpenter's level held against one of the guide rods, adjust the foot screws until the apparatus is perfectly vertical. Place the level against the other guide rod parallel to its position against the first, and adjust the screws to the best average vertical position of the two rods. Turn the level 90° about the guide rod, and level the apparatus in this plane, again adjusting for the best average vertical position of the two guide rods. *Great care must be taken* in performing this operation since in an apparatus

out of plumb, the falling body would not only drag, thus giving erroneous results, but would damage the air cup on the bottom of the body and the plug onto which it falls.

Inspect the apparatus, noting that the stylus is driven by an electromagnet which connects to 110-volt 60-cps current. Since the magnet attracts the stylus twice during each cycle of the electric current, the frequency of stylus vibration is therefore 120 cps. The magnet and stylus pivot about a vertical axis which permits the tension of the stylus against the recording tape to be adjusted. In use, the tension is set at the lightest value which will give a clear trace the entire length of the fall. When raising the body to the top, in order to avoid catching the stylus and bending it, the set screw which releases the magnet pivot should be loosened, the magnet turned with the stylus away from the paper, and the screw tightened. The body should be grasped by the heavy framework which rides along the guide rods and *not* by the light frame above the coil.

Insert a strip of tape in the tape frame, which can be removed for the purpose, keeping the white side toward the stylus. Replace the frame, sliding it over to one side so that room will be left for successive readings. With the falling body in its catch at the top, turn on the magnet switch and release the catch. As the body strikes the plug at the bottom, open the switch. Bring the body again to the top, remembering the precautions in the preceding paragraph, move the tape frame about a centimeter over, and take another run. In this manner, obtain four or five traces about a centimeter apart.

Remove the tape from the frame and stretch it on the table. Select the two best traces, and, neglecting the first centimeter or two of the first of these, scratch a fine line across the exact center of a crest. Count off six more cycles, and mark with a fine line, and so on, until the entire trace has been marked every six cycles. (See Fig. 4-2.) Mark the scratches 0, 1, 2, 3, etc. Repeat with the second of the two best traces, making the zero mark about two centimeters lower than the original zero mark. Time will be counted from the instant the stylus was at the zero position; distances will also be measured from this point. Lay the steel meter rule on the tape and measure *carefully* (to a tenth of a millimeter) the distance *from the zero mark* to each scratch. Record these distances in the third column of the record. Repeat for the second trace.

If each reading in this third column is subtracted from the following reading, the differences represent the distances fallen during the individual 6-cycle time intervals. For instance, during the third time interval in the sample record, the body fell $12.80 - 6.08$ or 6.72 cm. Knowing the distance fallen and the length of time interval—in this case one-twentieth of a second—*the average velocity during this interval* is seen to be 134.4 cm/sec. Since this is equal to the instantaneous velocity at the center of the interval, the velocity is entered midway between the second and third interval marks.

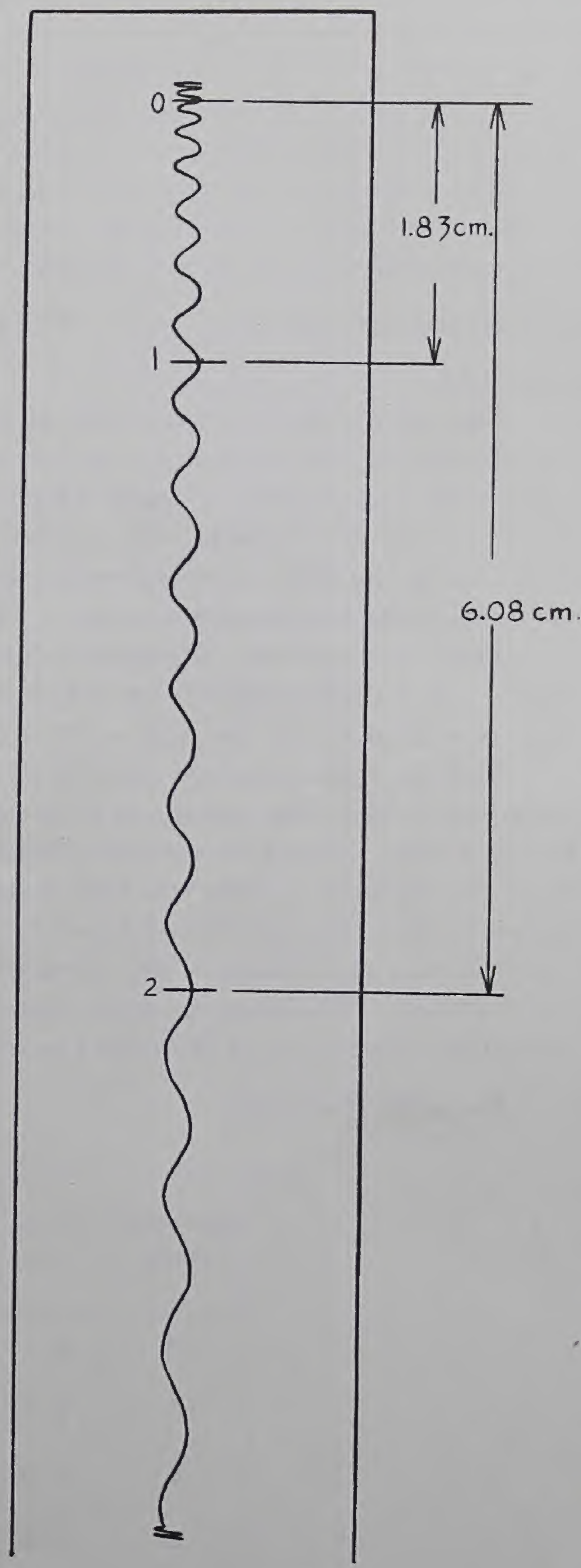


Fig. 4-2.

In the sample record, the difference between the velocity at the center of the second interval and that at the center of the first interval is $85.0 - 36.6$ or 48.4 cm/sec. This change takes place in an amount of time equal to one interval, that is in one-twentieth of a second, so the average acceleration during this interval is 968 cm/sec/sec. Similarly, the average acceleration during the time from the center of the second interval to the center of the third is 988 cm/sec^2, and so on.

Estimate the errors in reading the positions of the interval marks. Assume that the errors in the time intervals are negligible. From the above errors calculate the errors in the velocities and in the accelerations. Note how the errors mount up as one goes from distances to velocities (first differences) and then to accelerations (second differences). This is shown in the sample record and is characteristic of all difference methods. The use of significant figures alone may not exhibit this fact. For this reason one cannot expect to obtain a very good value of g, the acceleration, in this experiment. However the values of the accelerations in the fifth column of the sample record are constant within the limits of their errors. This is one of the important points of the experiment—to show that a freely falling body has constant acceleration.

The average value of g over the complete space-time interval in this experiment is simply the total change in velocity divided by the total time interval during which this change takes place. In terms of the data and results given in the sample record, this is $(280.4 - 36.6) \div 0.25$ or 975 cm/sec². Also the error in this average value is $\frac{\pm 1.6}{243.8} \times 975 = \pm 6$ cm/sec². Compute the average value of g and its error from your data.

The sample record shows that the same average value of 975 cm/sec² can be obtained by simply taking the average of the values of g given in column five. The reason for this is that these values of g are not completely independent of each other; for example, the first value of g in the column depends upon data at the 0, 1, 2 interval marks, the second upon data at the 1, 2, 3 marks, etc. As a result the average value of the g's in the fifth column depends only upon the first and last velocity values in column four and is independent of the intermediate values. This is another characteristic of difference methods.

There are methods of treating data such as these which make use of all the data in getting an average value. But these methods are necessarily more complicated than the one given here and the gain in accuracy is seldom worth the added effort.

Plot the instantaneous velocity as a function of the time for one best run, remembering that the average velocities in the table are the instantaneous values at the centers of the intervals, and draw the best possible straight line. (See Introduction, Section B.) For the same run plot the total distances against the times. Find the value of g from the first graph and compare with the value obtained earlier. Is this more or less accurate than the calculated result? Why? Also, determine the area under the v versus t graph out to $t = 0.25$ sec and compare this area with the value of the total distance given by the s versus t graph at $t = 0.25$ sec. We began to count time somewhat after the body had started falling; thus the body will have an initial velocity, v_o, at the time $t = 0$. Find the value of v_o from the first graph.

Record: (Sample).

Trial I

Interval mark	Time, sec	Total distance, cm	Average velocity, cm/sec	Acceleration, cm/sec²
0	0	0 ± 0.02		
			36.6 ± 0.8	
1	0.05	1.83 ± 0.02		968 ± 32
			85.0 ± 0.8	
2	0.10	6.08 ± 0.02		988 ± 32
			134.4 ± 0.8	
3	0.15	12.80 ± 0.02		960 ± 32
			182.4 ± 0.8	
4	0.20	21.92 ± 0.02		972 ± 32
			231.0 ± 0.8	
5	0.25	33.47 ± 0.02		988 ± 32
			280.4 ± 0.8	
6 etc.	0.30	47.49 ± 0.02		

App. No.______________

$$\text{Average } g = \frac{280.4 - 36.6}{0.25} = 975 \text{ cm/sec}^2$$

$$\Delta g = \pm 6 \text{ cm/sec}^2$$

QUESTIONS

1. Show that, in rectilinear motion of constant acceleration, the average velocity during any interval of time is equal to the instantaneous velocity at the center of that interval.

2. Suppose that in this experiment the apparatus was not level but that the guide rods were perfectly smooth (no drag). Would this produce any error in the value of g? Explain.

3. An error of 1% in the time-interval measurements in this experiment would produce what percentage error in the value of g?

Experiment 5.

Velocity of a Projectile

Object: To determine the initial velocity of a projectile (1) by measurements of range and fall, (2) by means of a ballistic pendulum.

Apparatus: Blackwood ballistic pendulum, trip balance, metric steel scale, plumb bob, steel tape, carbon paper, and a wooden box.

The Blackwood pendulum is a combination of a ballistic pendulum and a spring gun for propelling the projectile. The pendulum (Fig. 5-1) consists of a massive cylindrical bob C, hollowed out to receive the projectile and suspended by a strong rod K that is pivoted at its upper end at the top of a heavy support rod.

The projectile is a brass ball B which, when shot into the pendulum bob, is held there by the spring S in such position that its center of gravity lies in the axis of the suspension rod K. A brass indicator I is attached to the pendulum bob C in such a way that its tip indicates the height of the center of gravity of the loaded pendulum.

When the projectile is shot into the pendulum, it swings upward and is caught at its highest point by means of the pawl P which engages a tooth in the curved rack R.

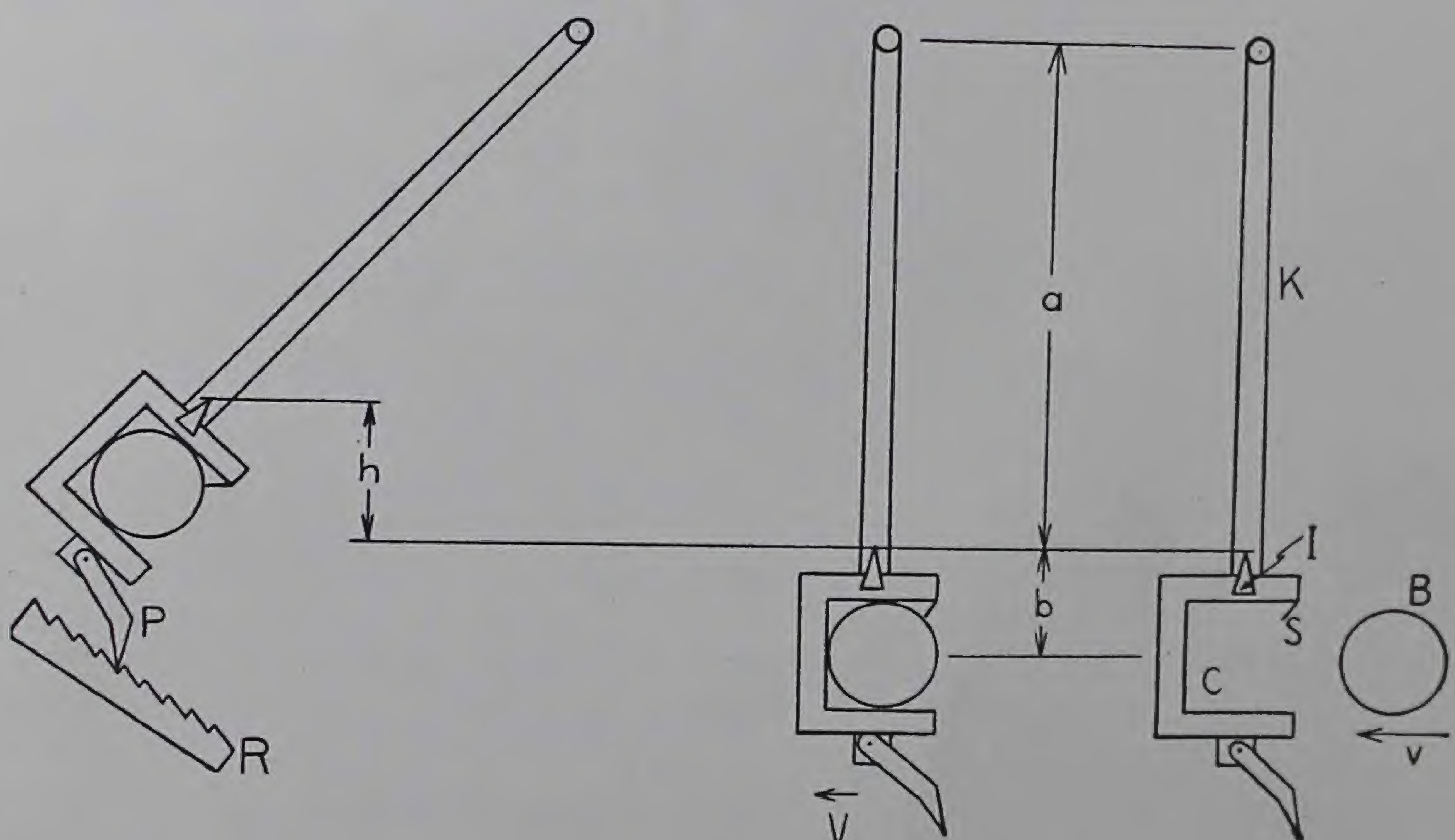


Fig. 5-1.

Theory: *Part I.* The initial velocity of a projectile shot *horizontally* from a gun and allowed to fall freely toward the earth may be determined in terms of the acceleration of gravity, the horizontal range of

the projectile, and its vertical fall. In time t the projectile will fall vertically through a distance y given by the equation

$$y = \tfrac{1}{2}gt^2. \tag{1}$$

In the same time its horizontal displacement x will be given by

$$x = vt, \tag{2}$$

where v is the initial horizontal velocity. See Fig. 5-2. If we eliminate t between these two equations and solve for v, we get

$$v = x\sqrt{\frac{g}{2y}}. \tag{3}$$

By means of this equation we may compute v in terms of measured values of x and y and the value of g.

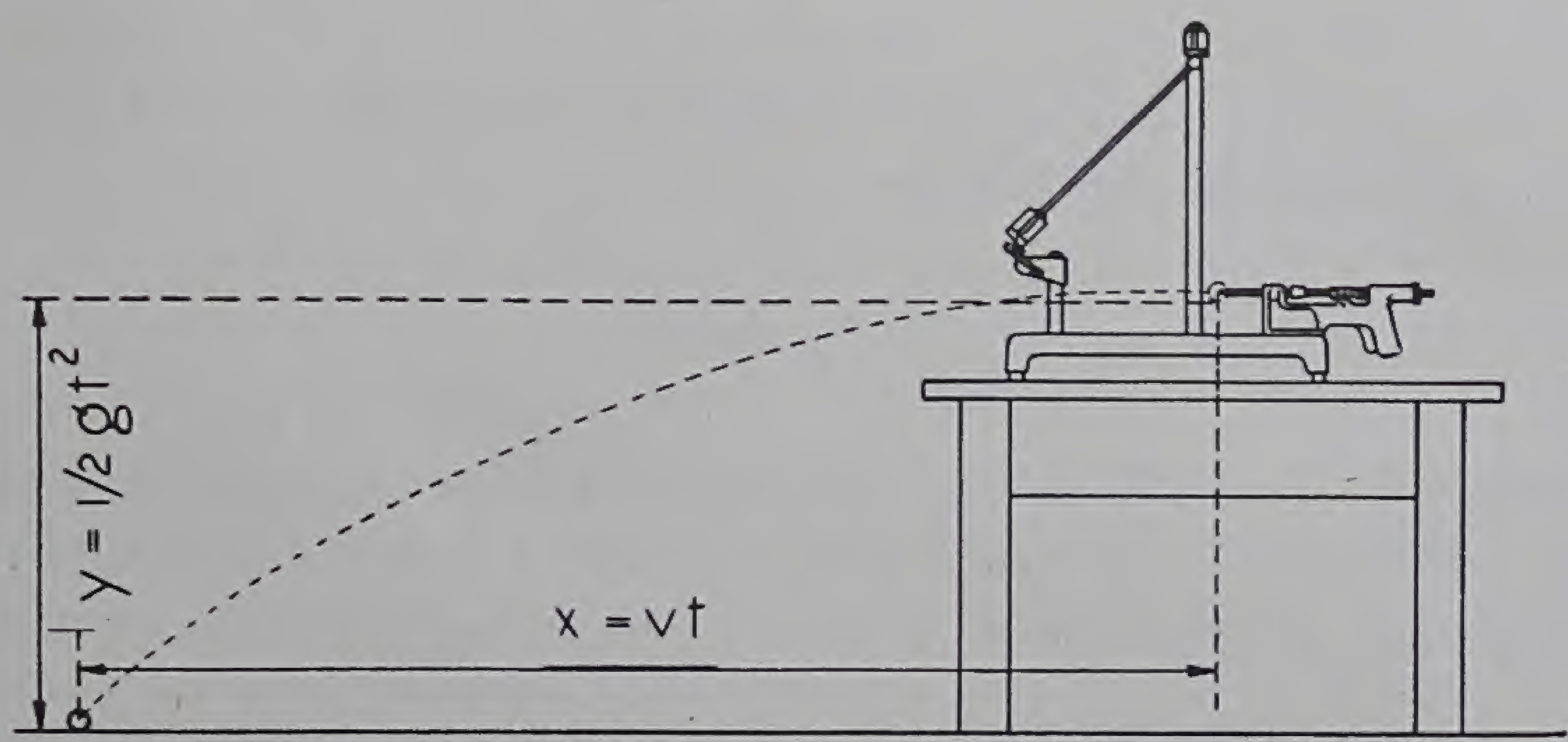


Fig. 5-2.

The determinate-error equation corresponding to Eq. (3) will be

$$\frac{\Delta v}{v} = \frac{\Delta x}{x} - \frac{1}{2}\frac{\Delta y}{y} \tag{4}$$

where the sign rules concerning determinate and indeterminate errors should be strictly observed.

Part II. Another method of determining the initial velocity of a projectile fired horizontally is by means of the ballistic pendulum.

Suppose the projectile of mass m and velocity v is fired into the pendulum initially hanging at rest in a vertical position. As a result of this collision the pendulum bob with the projectile trapped inside it is given a velocity V. Since momentum is conserved even for an inelastic collision such as this, the following relation must be satisfied:

$$mv = (m + M)V. \tag{5}$$

The mass M in this equation is the *effective* mass of the pendulum rather than its real mass M_o since the mass of the pendulum is actually distributed throughout the pendulum rather than being concentrated entirely in the bob of the pendulum. Only in this latter case would M and M_o be the same because only in this case would the entire mass of the pendulum have the same velocity V. Fortunately in the apparatus used in this experiment most of the mass of the pendulum is concentrated in the bob so that the difference between M and M_o is small. The relation between M and M_o will be given later.

The velocity V given to the pendulum bob by the impact of the projectile causes it to swing up along a circular arc until the center of gravity of the loaded pendulum rises to such a vertical height h that its initial kinetic energy is entirely converted into potential energy, *i.e.*,

$$\tfrac{1}{2}(M + m)V^2 = (M_o + m)gh. \tag{6}$$

Here again it is necessary to make the distinction between M and M_o.

By eliminating V between Eqs. (5) and (6) and then solving for v, we get

$$v = \frac{1}{m}\sqrt{2gh(M_o + m)(M + m)}. \tag{7}$$

The relation between M and M_o is developed in the following section (fine print) and is shown to be

$$M + m = (M_o + m)\frac{a}{a + b}, \tag{8}$$

where a and b are the dimensions indicated in Fig. 5-1. Since b is small compared to a, the correction factor $a/(a + b)$ is only slightly less than unity.

In Eq. (7) we may substitute the value of $M + m$ from Eq. (8), obtaining

$$v = \frac{M_o + m}{m}\sqrt{2gh\left(\frac{a}{a + b}\right)}. \tag{9}$$

By the binomial theorem the quantity $\sqrt{a/(a + b)}$ is approximately equal to $1 - \frac{1}{2}b/a$, provided b is small compared to a. Hence Eq. (9) may be written as

$$v = \frac{M_o + m}{m}\sqrt{2gh}\left(1 - \frac{b}{2a}\right). \tag{10}$$

The factor in the parentheses represents the correction due to the fact that not all of the mass of the pendulum is concentrated in the bob. If it were, this term would reduce to unity. Actually this term has a value for the apparatus in this experiment of about 0.96 or 0.95 representing an error of 4 or 5% if the term is not included.

The determinate-error equation for this part of the experiment may be obtained in the usual manner by taking the logarithmic derivative of Eq. (10). The factor in parentheses is treated as a constant since small errors in M_o, m, b, and a will not appreciably change this factor. Hence we get

$$\frac{\Delta v}{v} = \frac{M_o}{M_o + m}\left(\frac{\Delta M_o}{M_o} - \frac{\Delta m}{m}\right) + \frac{1}{2}\frac{\Delta h}{h}. \tag{10a}$$

The relation between the effective mass $M + m$ of the loaded pendulum and its real mass $M_o + m$ may be obtained from the formula for the kinetic energy of rotation of the loaded pendulum. As a result of the collision between projectile and pendulum, the loaded pendulum is given an initial kinetic energy of rotation of amount $\frac{1}{2}I\omega^2$, where I is the moment of inertia of the loaded pendulum about its axis of suspension, and ω is the angular velocity of the pendulum just after the collision. Now I in this formula may be replaced by $(M_o + m)K^2$, where K is the radius of gyration of the loaded pendulum about its axis of suspension. Also ω may be replaced by $V/(a + b)$, where a is the distance between the axis of suspension and the center of gravity of the loaded pendulum, and where b is the distance between the center of gravity and the center of the bob. See Fig. 5-1. Thus

$$\frac{1}{2}I\omega^2 = \frac{1}{2}\frac{(M_o + m)K^2}{(a + b)^2}V^2. \tag{11}$$

The right side of Eq. (11) may be written in the form

$$\tfrac{1}{2}(M + m)V^2,$$

provided that

$$M + m = (M_o + m)\frac{K^2}{(a + b)^2}. \tag{12}$$

Equation (12) gives the effective mass of the loaded pendulum in terms of its real mass and the constants K, a, and b. The constants a and b may be measured directly. The radius of gyration K could be determined by observing the period of oscillation of the loaded pendulum and applying the theory of the physical pendulum. This procedure is hardly necessary, however, because the pendulum is manufactured in such a way that its center of oscillation (or percussion) is approximately at the center of the bob at the distance $a + b$ from the axis of suspension. This means that this pendulum will oscillate like a simple pendulum of length $a + b$. Hence from the theory of the physical pendulum $K^2/a = a + b$. It follows then that the effective mass of the loaded pendulum is given by Eq. (8).

Method: *Part I. Initial Velocity from Measurements of Range and Fall.* Make sure that the apparatus is level and clamped in position on the table. In this part of the experiment the pendulum is not used and should be swing up onto the rack so that it will not interfere with the free flight of the ball.

Cock the gun by placing the ball on the end of the firing rod and pushing it back, compressing the spring until the trigger is engaged. Fire the gun and note the place where the ball strikes the floor. Place the wooden box with a carbon-paper recorder at this position. Shoot the ball five more times from the gun. Measure the horizontal range x of the ball along the floor from the point immediately below the projection point of the ball (use plumb bob) to the mean position of the points at which the ball strikes the bottom of the box on the floor. Estimate the average deviation of these points from the mean position and use this as the error in x.

At the same time measure the vertical fall y of the ball, making allowance for the box thickness and estimating the error in y.

By use of Eqs. (3) and (4) compute the initial velocity of the ball and the error in this velocity.

Part II. Initial Velocity by Use of Ballistic Pendulum. Release the pendulum from the rack and allow it to hang freely *without swinging*. *Without changing the spring tension* in the gun, load the gun and fire the ball into the pendulum bob. This will cause the pendulum with the ball inside it to swing up along the rack where it will be caught at its uppermost position. A scale along the outer edge of the rack provides a means for noting and recording the position of the pendulum on the rack. To remove the ball from the pendulum, push it out with the finger or with a rubber-tipped pencil, meanwhile holding up the spring catch.

Repeat this process four more times, recording each time the rest position of the pendulum on the rack. Determine the mean of these positions and set the pendulum at this position. Measure the height h_1 of the index point of the center of gravity above the base of the apparatus. Then release the pendulum, allowing it to hang in its lowermost position, and measure the height h_o of the index point above the base. The difference between these readings gives the vertical height h through which the center of gravity of the loaded pendulum is raised as a result of the collision.

Loosen the thumbscrew holding the axis of rotation of the pendulum and carefully remove the pendulum. Weigh and record the masses of the pendulum and the ball. At the same time measure and record the lengths a and b of the pendulum. These distances need not be measured with extreme accuracy since they only appear in the correction term in Eq. (8).

From these data calculate the initial velocity v of the ball using Eq. (8). Also compute the *indeterminate* error in v. Compare the value of v obtained in Part II with that obtained in Part I.

Change the spring tension in the gun and repeat both Parts I and II.

Record:

App. No.______________

Part I.

$g =$ Thickness of box =

Trial	x, cm	Height above floor, cm	y, cm	v_I, cm/sec
a				
b				

Part II.

$M_o =$ $a =$

$m =$ $b =$

Trial	h_o, cm	h_1, cm	h, cm	v_{II}, cm/sec
a				
b				

QUESTIONS

1. In Part I of this experiment it is assumed that the floor is level. Suppose that this were not true but that the floor tipped down through a *small* angle θ in the direction of flight. Show that the true value of x would still correspond very closely to the value measured along the floor, but that the value of y as measured would have to be increased by $x\theta$ approximately. HINT: For small θ, $\cos\theta \doteq 1$; $\sin\theta \doteq \theta$.

2. Show by use of the binomial theorem that $\sqrt{a/(a+b)}$ reduces to $1 - (b/2a)$ for $b \ll a$.

Experiment 6.

Centripetal Force

Object: To determine the centripetal force on a body rotating at constant speed.

Apparatus: Centripetal-force apparatus, motor-driven rotator, weight holder and weights, extender, template, divider, steel rule, timer.

The centripetal-force apparatus (Cenco) as shown in Fig. 6-1 consists of a cylindrical bob B which slides freely on guide rods. A spring S of adjustable tension exerts the centripetal force upon the bob as the apparatus rotates. This entire apparatus may be mounted in a motor-driven rotator and whirled. By means of a variable-speed friction drive the speed can be adjusted until the mass has moved from its normal position to a predetermined position at the end of the guide rods, as shown in Fig. 6-1. There it actuates an indicator P the point of which is near the axis of rotation and is therefore visible at any speed.

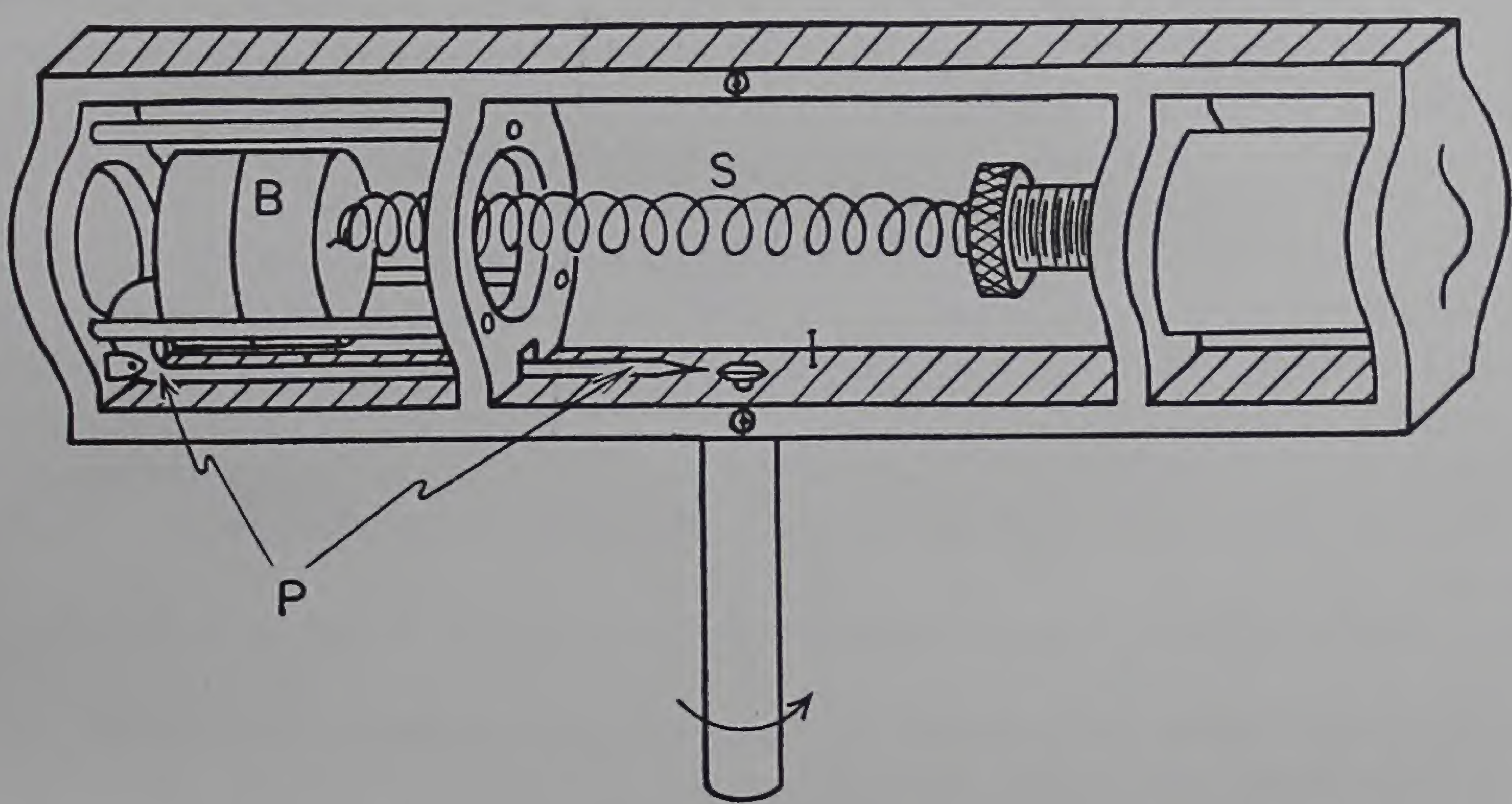


Fig. 6-1.

The rotator (not shown) consists essentially of a large disk, driven by the motor, on which rolls a friction disk mounted on a spindle. The speed of the friction disk is controlled by a screw which varies the position of the friction disk. A revolution counter is mounted on the spindle of the friction disk.

Theory: The centripetal force F acting upon a body of mass m moving with constant speed v in a circle of radius r is given by the equation

$$F = m\frac{v^2}{r}. \tag{1}$$

But $v = 2\pi rn$, where n = number of revolutions per second. Hence Eq. (1) may be written in the form

$$F = m4\pi^2rn^2. \tag{2}$$

The corresponding error equation is

$$\frac{\Delta F}{F} = \frac{\Delta m}{m} + \frac{\Delta r}{r} + 2\frac{\Delta n}{n}. \tag{3}$$

By measuring m, r, and n, one may calculate the centripetal force F by means of Eq. (2). If this force is applied to the body by means of a spring, as it is in this experiment, then the force F may also be determined directly by loading the spring until it stretches the same amount as it does in the rotation experiment.

Method: Remove the centripetal-force apparatus from the rotator and examine it carefully to see how it works. Notice how the spring tension may be changed and also how the indicator lever is actuated. Try to pull the cylindrical bob out to the end of the apparatus with your fingers. Note that a large force is required to do this.

Examine the rotator. Set the friction drive wheel near the center of the driving disk and turn on the motor. Notice that the spindle of the friction drive wheel turns very slowly if at all. Try increasing the speed of the spindle by moving the friction disk away from the center of the driving disk. Engage and disengage the revolution counter. Stop the motor.

Reset the friction disk at the center of the driving disk, reclamp the apparatus on the rotator, and start the motor. Slowly increase the speed until the pointer rises and is even with the fixed index I. Unfortunately, it is impossible to keep the speed constant enough to hold the pointer in this position for any length of time without a continual readjustment of the position of the friction disk. One method of partially avoiding this difficulty is to run the apparatus at a speed *slightly* larger than the critical speed and then to apply a small amount of friction on the rotating spindle sufficient to reduce the speed to its critical value. The side of a pencil held against the spindle frequently works well in this respect.

Determine the value of n (revolutions per second) by counting the number of revolutions with the revolution counter over an interval of 2 min. To do this record the initial reading of the counter, control the speed of the apparatus, engage the counter at the zero instant, disengage the counter at the end of 2 min, and record the final reading of the counter. To prevent the counter from spinning after disengagement touch the counter gear lightly with the finger while it is being disengaged. Make two additional runs of 2 min each under the same conditions. Calculate the average n for the three runs. Use the mean deviation as Δn.

Remove the centripetal-force apparatus from the rotator and hang it from the support stand with bob down. Attach a weight holder to the bob and add weights until the bob reaches the same distance from the axis as it had when revolving. The total weight on the spring expressed in dynes (this includes the weight of the bob itself) is equal to the centripetal force F. The accuracy of the weights may be taken as $\pm 0.1\%$.

With the spring extended as in the previous paragraph, measure the radius r of rotation, *i.e.*, the distance from the axis of rotation to the center line on the bob. If available, use the special extender and template for this measurement.

Calculate the centripetal force F by substituting the values of n, r, and m in Eq. (2). Compute the error in F.

Determine the per cent difference between the observed and calculated values of F.

Change the spring tension and repeat the experiment.

Record:

App. No.__________	
Mass of bob:	$m =$
Radius of rotation:	$r =$
Time interval:	$t =$ 120 sec
Case I.	
No. of rev:	$N =$ 1st ( ) − ( ) = ( )
	2d
	3rd
	Ave
No. rev. per sec:	$n =$
Centripetal force (calc):	$F =$ ( ) dyne
Centripetal force (obs):	Bob ( ) gmf
	Wt holder ( )
	Weights ( )
	Total ( ) gmf
	F (obs) = ( ) dyne

QUESTIONS

1. Show by dimensional argument that Eq. (2) gives the force in dynes provided m is in grams, r in centimeters, and n in revolutions per second.

2. Is one justified in using $\frac{22}{7}$ as the value of π in this experiment? Explain.

3. If, in Case I of this experiment, the apparatus were rotated with twice the angular speed needed for balance, with what force would the bob press against the end of the frame?

4. If this experiment were performed on Mars ($g_{\text{Mars}} = 0.4g_{\text{Earth}}$), which of your measured quantities would be different and by how much?

Experiment 7.

The Simple Pendulum

Object: To investigate the relation between the period of a simple pendulum and its length; also to determine the acceleration of gravity.

Apparatus: Simple pendulum, timing device, meter stick, vernier caliper. The simple pendulum consists of a small brass sphere B suspended from a rigid support S by means of a fine steel wire as shown in Fig. 7-1. The wire is fastened at its upper end to a small threaded rod fastened to a knife-edge holder K. The knife-edge rests in a groove on the support. A hole through the support under the middle of the knife-edge permits the pendulum to swing freely in a vertical plane about a line coincident with the knife-edge. The position of the nut N has been adjusted so that the effect of the supporting system on the period of the pendulum is negligible.

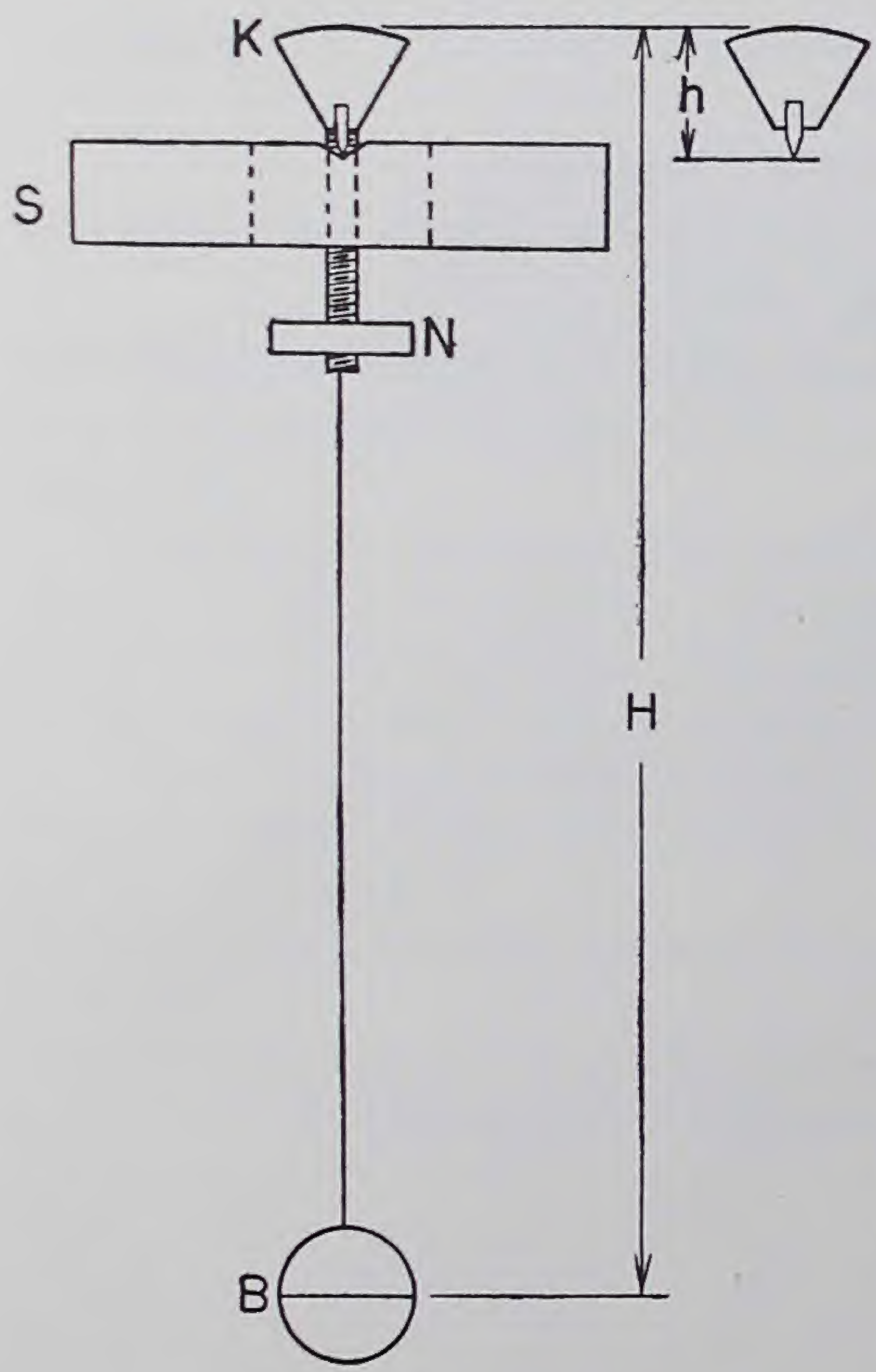


Fig. 7-1.

Theory: The vibration of a pendulum such as that shown in Fig. 7-1 is, for small amplitudes, an example of simple harmonic motion, the period of which is given approximately by the equation

$$T = 2\pi\sqrt{\frac{L}{g}}, \tag{1}$$

where T = period, *i.e.*, the time for a complete to-and-fro vibration, L = length from the point of suspension to the center of the bob, and g = acceleration of gravity.

Strictly speaking, Eq. (1) is only valid for infinitely small amplitudes and for a pendulum with all of its mass concentrated at the end of its suspension. However, the errors introduced by not being able to satisfy these conditions in the laboratory are very small provided that the amplitude of vibration does not exceed 2° and provided that the radius of the bob is small compared to the length of the pendulum.

Under these conditions Eq. (1) states that T is independent of the amplitude of vibration and is directly proportional to $\sqrt{L}$.

The vibration of a simple pendulum also provides a simple and accurate means of determining g, for both T and L can be determined in the laboratory, and hence g may be calculated by use of the equation

$$g = 4\pi^2\frac{L}{T^2}. \tag{2}$$

The determinate-error equation is

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} - 2\frac{\Delta T}{T}. \tag{3}$$

Method: Examine the suspension device for the pendulum, see that the knife-edge is properly placed in the V-shaped groove, and set the pendulum swinging in a small arc in a vertical plane perpendicular to the line of the knife-edge. It should swing freely and smoothly. The amplitude of the swing should not be more than about 4% of the length of the pendulum.

Determine the number of complete vibrations of the pendulum in a time interval long enough so that the total number of vibrations in this interval is at least 100. Estimate the number of vibrations to the nearest quarter of a vibration. Make a second count under the same conditions. These counts should not differ by more than a fraction of a vibration. If they do, an error in counting has almost certainly been made, and a third count should be made. The counting should be started *after* the pendulum is swinging. The instantaneous position and direction of the pendulum bob at the zero count, *i.e.*, when the time is zero, may be marked on the paper attached to the wall in back of the pendulum. This mark then serves as a reference point for counting vibrations.

After two satisfactory counts have been made stop the pendulum, allowing it to hang vertically at rest. Measure the distance H from the top of the knife-edge holder to the center of the bob. Then measure the height h of the knife-edge holder itself with a vernier caliper. The difference between H and h will be the length of the pendulum L. Measure the diameter of the bob.

Make two additional runs of this experiment, using different lengths of suspension wire in each case.

Record all these data in tabulated form, including estimated errors. Calculate the value of g for each case by means of Eq. (2) and the error in g by Eq. (3). Compare these values with the accepted value for your geographical location. See Table P, Appendix III, to determine this value of g.

Record:

App. No.________________

	No. vib	Time, sec	H, cm	h, cm	T, sec	L, cm	g, cm/sec^2
1							
2							
3							

QUESTIONS

1. If the timing device in this experiment were in error by $\pm 0.1\%$, what approximate error in centimeters per second squared would be introduced into the value of g?

2. If an error of ± 1 vibration is made in counting the number of vibrations (assume to be approximately 100), what error would this introduce into the value of g?

3. It may be shown that practically no error in the period is introduced by the supporting system KN (Fig. 7-1) provided its natural period of oscillation is the same as that for the complete pendulum. What *physical* argument could be used to justify the above statement?

4. Show that the effective length of the laboratory pendulum is $L\,[1 + 0.1\,(d^2/L^2)]$ where d is the diameter of the bob. Calculate the error introduced in this experiment by neglecting the diameter of the bob.

Experiment 8.

The Spiral Spring

Object: (1) To determine the force constant of a spiral spring; (2) to determine its period of vibration with several different loads; (3) to compare the observed period with the calculated period.

Apparatus: Spiral spring and support, weight holder and weights, simple cathetometer for measuring displacements, timing device. The spiral spring in this experiment is a steel spring capable of supporting loads of several kilograms. It is suspended by a hook attached to a rigid framework of heavy metal rods. The cathetometer is a vertical metal rod in a tripod base with an engraved metric scale upon it. A vernier scale with a pointer is attached to a sleeve that can be moved along the main scale. Hence vertical displacements may be determined more accurately with this device than with an ordinary meter stick.

Theory: When a load is gradually applied to the free end of a spring suspended from a fixed support, the spring usually stretches until the tension in the spring just balances the weight of the load. Some springs, however, possess an initial strain and tension even without any apparent load. In this case it will be found that the coils of the spring are pressed tightly together. Thus the spring through this action furnishes its own load. If now a gradually increasing external load is applied to such a spring, the internal load is gradually relaxed without appreciable stretch of the spring until the coils of the spring are just pulled apart. Thereafter there is only an external load on the spring, and the spring stretches in a normal manner. That is, within limits, the added load on the spring is directly proportional to the stretch of the spring and the spring obeys *Hooke's law*.

Under these conditions the loaded spring, if set into vibration, will undergo harmonic motion with a period given by the equation

$$T = 2\pi\sqrt{\frac{M}{K}}, \tag{1}$$

where T = period of the motion, M = the effective mass of the vibrating system, and K = the spring constant, *i.e.*, the ratio between the added force and the corresponding stretch of the spring. The effective mass M of the spring and its load is the mass of the load M_o plus one-third the mass of the spring. Thus Eq. (1) may be written

$$T = 2\pi\sqrt{\frac{M_o + (m/3)}{K}}. \tag{2}$$

The contribution of the mass of the spring to the effective mass of the vibrating system may be shown as follows. Consider the kinetic energy of a spring and its load undergoing harmonic motion. At the instant under consideration let the load M_o be moving up with velocity v_o as shown in Fig. 8-1. At this same instant an element of mass of the spring dm will also be moving up but with a velocity v which is smaller than v_o. It is fairly evident that the ratio between v and v_o is just the ratio between y and y_o. Hence, $v = v_o(y/y_o)$. The kinetic energy of the spring alone will be $\int_0^{y_o} \frac{1}{2}v^2\, dm$. But dm may be written as $\frac{m}{y_o}\, dy$, where m is the mass of the spring. Hence the integral equals

$$\frac{1}{2}\left(\frac{m}{3}\right) v_o^2.$$

The total kinetic energy of the system will then be

$$\frac{1}{2}\left(M_o + \frac{m}{3}\right) v_o^2,$$

and the effective mass of the system is therefore $M_o + \frac{m}{3}$.

The determinate-error equation may be written approximately as

$$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta M_o}{M_o} - \frac{1}{2}\frac{\Delta K}{K}. \tag{3}$$

It is assumed that $m/3$ is small compared to M_o, hence an error in m will not appreciably alter the value of ΔT.

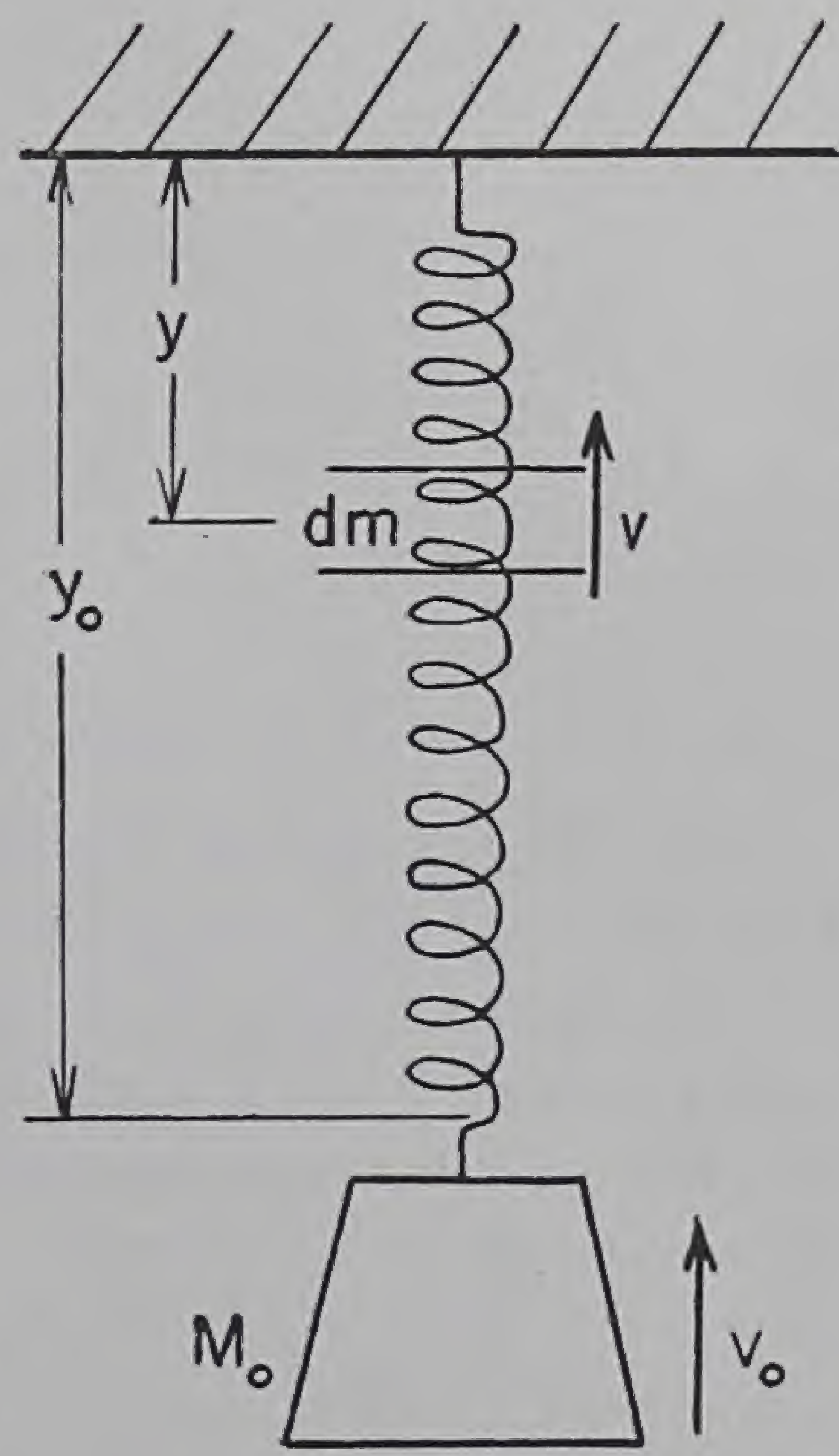


Fig. 8-1.

***Method:** Part I. Determination of Force Constant.* Place a sufficiently large initial load on the spring to pull the coils of the spring apart. A 2-kg load will generally suffice. Measure the spring stretch from this reference point, *i.e.*, determine the vertical position with the cathetometer of some mark on the weight holder. Add 1 kg to the load and again determine the position of the same mark. Continue this process until the initial load has been *increased* by 5 kg. Then reduce the load in steps of 1 kg taking position readings at each step until the load reaches its initial value.

Record these data in tabulated form. Plot the *added* load in grams (ordinate) against the stretch of spring in centimeters (abscissa). Note that the points lie on a straight line, indicating that the spring obeys Hooke's law. Draw the best possible straight line through the plotted points (see Introduction, Section B) and determine the slope of this line by choosing two points on the line, one near the origin with coordinates x_1 cm and y_1 gmf and the other near the upper end of the line with coordinates x_2 cm and y_2 gmf. The slope will be $\frac{y_2 - y_1}{x_2 - x_1}$ gmf/cm and the force constant will be this slope multiplied by 980.

It is rather difficult to compute the error in K. Try to estimate how much y_1, y_2 could be changed and still give a straight line that would fit the observed points. Also take into account the fact that the weights used in this experiment may be in error by $\pm 0.1\%$ and that the cathetometer readings are only good to 0.01 cm.

Part II. Determination of the Period of Vibration. Place a load of about 3500 gm on the spring, set the system into vertical vibration with an amplitude of about 5 cm, and determine the period of vibration by counting the number of complete vibrations in a given time interval. Choose a time interval such that the number of vibrations will be at least 100. Estimate the total number to a quarter of a vibration. Repeat the count over the same time interval. These counts should not differ by more than a fraction of a vibration.

Make two additional runs of this experiment using loads of about 5000 and 7000 gm.

Record in tabulated form the data from these runs and determine the period of the system in each run along with the error in the period.

Finally, calculate the periods for the three different loads using Eq. (2) and their errors using Eq. (3). Remember that M_o is the mass of the *total load* on the end of the spring, including the mass of the weight holder.

***Record:** Part I. Force Constant of Spring.*

Load, gm, Initial +	Readings ↓ cm ↑		Ave stretch, cm
0	31.75	31.77	0
1000	25.43	25.47	6.31
2000	....		....

From graph: K = ______________; ΔK = ______________.

Part II. Period of Vibration.

Mass of spring: = (   ) gm
Mass of weight holder: = (   ) gm

	M_o, gm	No. vib 1st 2nd	Time, sec	T (obs), sec	T (theo), sec	% diff
1						
2						
3						

QUESTIONS

1. What constant fractional error would be introduced into the calculated period of the system in this experiment if the mass of the spring were neglected? Would this error be significant in this experiment?

2. It frequently happens in this experiment that the vibrating spring system, after a time, begins to swing to and fro in a vertical plane like a pendulum. This is an example of resonance, *i.e.*, in both motions the system has about the same period. What change in the system could be made to prevent this resonance phenomenon?

Experiment 9.

Work and Power

Object: To determine the power output which a person can develop under certain conditions.

Apparatus: Prony brake, stop watch, scales.

Theory: Power is the rate of doing work. Its units are those of energy per second. Two special units are the horsepower, which is equal to 550 foot-pounds per second, and the watt, which is equal to 1 joule per second. A device used to measure the power output of a rotating machine is called the *Prony brake*. In its simplest form it consists of a friction band passed around a drum or wheel which is being rotated by the machine. Tension is kept on each end of the friction band by a spring balance. Rotating the wheel at a known rate, and knowing the tensions on the band and the diameter of the wheel, the power output can be calculated as follows:

When the wheel is rotating steadily as shown in Fig. 9-1a and the brake band and balances have assumed their equilibrium position, the new effect is the same as though at any instant the two cords were fastened to the rim and pulling with the forces F_a and F_b. See Fig. 9-1b. (Note that these forces must be tangential ones since the force of friction can only be exerted in the direction of motion.) From this it may be seen that the resultant force tending to prevent turning of the wheel is $F_b - F_a$, acting at the rim. The work done against this force equals the product of the force and the distance traveled by the rim of the wheel. The average power exerted during the run is therefore

$$P = \frac{2\pi r N(F_b - F_a)}{t}, \tag{1}$$

where $2\pi r$ = the circumference (the distance traveled in one turn), N = the number of turns in the run, t = the time of the run.

Method: Adjust the turnbuckle until both balances read about 8 lb. Move the wheel *slightly* to make sure it is in its equilibrium position. Record the readings of the balances with the wheel at rest. Rotate the wheel at as fast a *steady rate* as possible for 30 sec. Count and record the number of revolutions made in this time, and the readings F_a and F_b while turning at the steady rate. Repeat for each partner.

Repeat the preceding paragraph of instructions for an equilibrium force of about 12 lb on each balance.

Again repeat, this time for 20 lb on each balance, and run for 10 sec. Measure the outside diameter of the wheel and the depth of the groove. To eliminate zero errors in the spring balance, find the difference between the readings at rest and in motion of *each* balance and add the absolute values of these differences. This sum is the net force acting on the rim $(F_b - F_a)$.

Run rapidly up a flight of stairs, from a running start, having your partner time the run with a stop-watch. Repeat for each student. Measure the heights of ten steps and record the average. Record the number of stairs and your weight.

Calculate the work and the power developed in each part of the experiment. Express the work in foot-pounds and the power in horsepower and in watts. Note that a person's power output depends mainly on the muscles involved.

Since this experiment is largely qualitative in nature it will not be necessary to make error calculations.

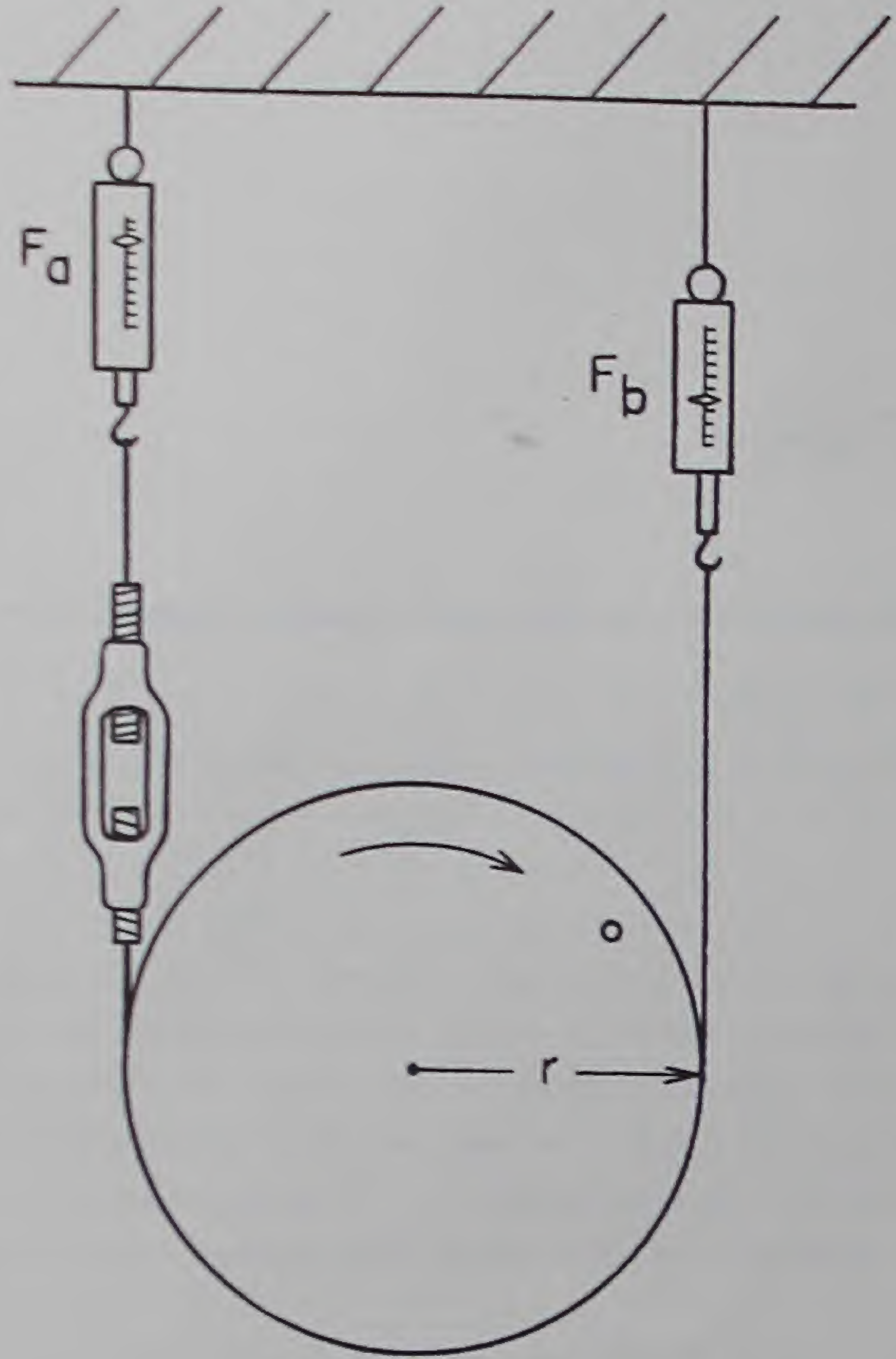


Fig. 9-1a.

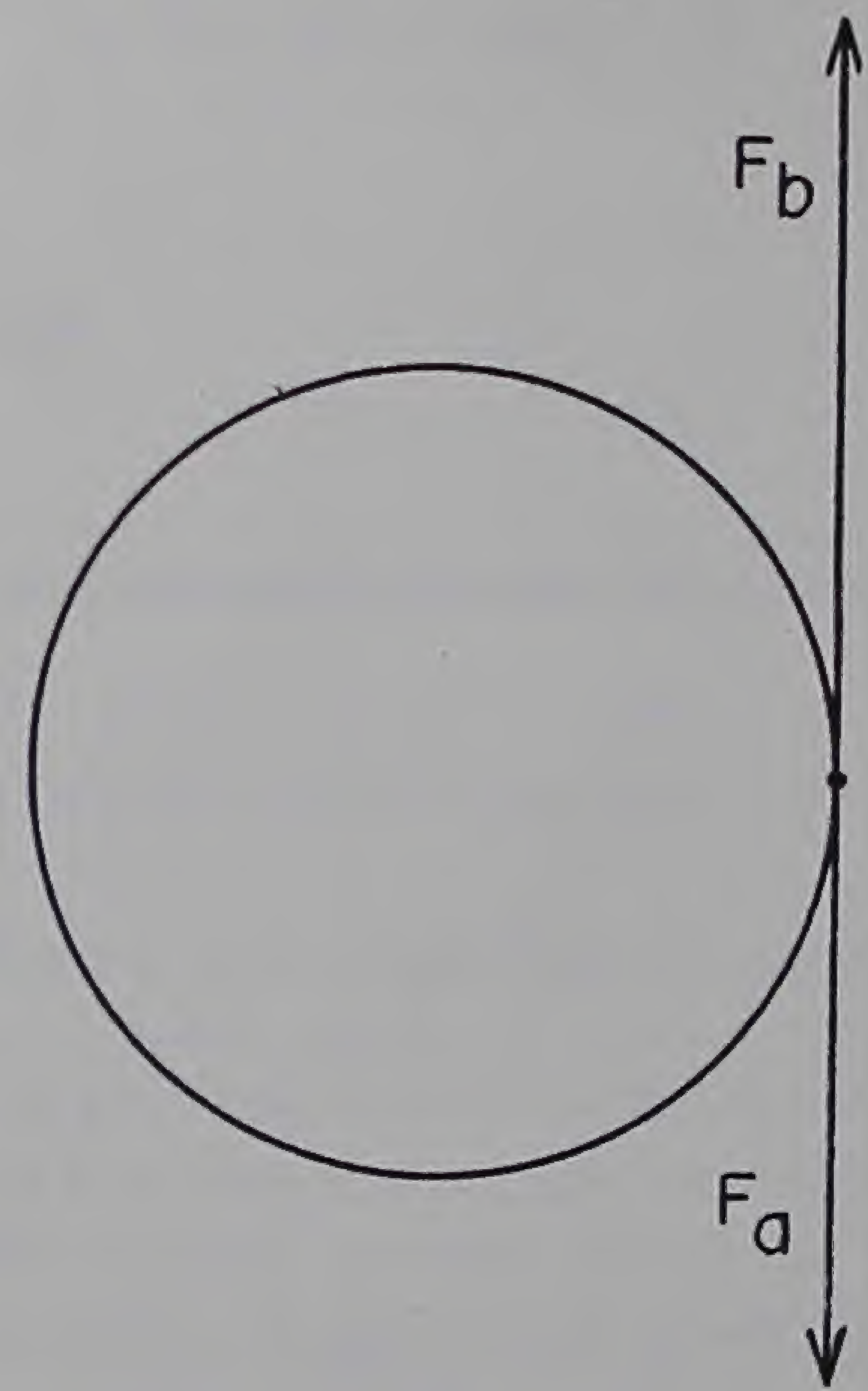


Fig. 9-1b.

Record:

Trial	Student	Balance *A*			Balance *B*			Net force	No. of turns	No. of seconds	Work, ft-lb	Pwr, hp	Pwr, watts
		Rest	Turning	Diff	Rest	Turning	Diff						
I	1	8								30			
	2	8								30			
II	1	12								30			
	2	12								30			
III	1	20								10			
	2	20								10			
IV	1												
	2												

Diam wheel, over-all________ } Radius to inside of groove________
Depth of groove________ }

Height (ave of 10) of a riser________ } Height climbed________
Number of steps________ }

Weight, student 1________

Weight, student 2________

QUESTIONS

1. Transform the work done into ergs, joules, kilogram-meters.

Experiment 10.

Simple Machines

Object: To study some simple machines and to determine their mechanical advantages and efficiencies.

Apparatus: Pulleys and stand, wheel and axle, block and tackle, differential pulley (chain hoist), weights, outside caliper, meter stick, gear train.

Theory: A machine is a device for applying energy to do work in the way most suitable for the purpose at hand. It is used to transform or to transfer energy; it may receive (and deliver) energy in different forms: mechanical (both kinetic and potential), electrical, heat, etc. In the simple machines, the energies are mechanical only. The agent that supplies the energy exerts a single force, and the machine is opposed in doing its useful work by a single resisting force, the load.

The *mechanical advantage* of such a machine is defined as the ratio of the force exerted *by* the machine on the load to the force applied *to* the machine. In simple machines the mechanical advantage may be greater than, equal to, or less than unity, depending on the character of the machine.

Although the force exerted by the machine may be much greater than the applied force (as in the case of jacking up an automobile), the useful work done by the machine is in every instance less than or at most equal to the work supplied to it. The smaller force is applied through a much larger distance than is the larger force. The *efficiency* of the machine is the measure of how much of the work supplied is delivered to the load, and is defined as the *ratio of the delivered to the supplied work.* In an ideal machine none of the supplied energy would be wasted in overcoming friction or in other losses, and the ideal efficiency would be 100%. Few actual machines approach this value. In the ideal machine

$$F_{in}\, S_{in} = F_{out}\, S_{out}, \tag{1}$$

where F_{in} = input force,
S_{in} = distance through which the input force acts,
F_{out} = output force, and
S_{out} = distance through which the output force acts.

It is readily seen that the ratio S_{in}/S_{out} is equal to the mechanical advantage. Since this is true only in an ideal machine, this ratio is called the *ideal or theoretical mechanical advantage.* It is usually possible to obtain the value of this quantity by inspection of the geometry of the machine (for instance the gear-tooth ratio, or the number of ropes in pulley systems). This quantity is also often called the *velocity ratio.* It may be shown that the efficiency is equal to the ratio of the actual to the ideal mechanical advantage.

Method: *Part I. Pulleys.*

A. Use first a simple pulley fastened to a support, with a cord passing over the pulley carrying a weight pan at each end. Place equal weights on each pan, successively 500 and 1500 gm. In each case place on one side the amount of extra weight necessary to keep the pans moving slowly and *uniformly* when lightly touched to start them in motion.

B. Next, rig a traveling block system, with the weight pan suspended from a floating pulley. One end of the cord supporting the floating pulley is tied to the support, and the other passes over a fixed pulley as in

Part A. Use as loads successively 100, 500, 1000, 1500, and 2000 gm, and balance each by the force necessary to keep the load rising slowly and uniformly after touching lightly to start the motion.

C. There will be found in the laboratory a differential pulley (chain hoist) and a block and tackle system, each capable of lifting a considerable weight. Examine these two systems carefully. See Fig. 10-1. Apply a load of about 75 lb to each and, using an appropriate spring balance on the input, determine the mechanical advantage of each of these mechanisms. Measure values of S_{in} and S_{out} to determine the velocity ratios. Count the number of chain indentations on each of the two upper pulleys of the differential pulley.

D. For each of the above pulley systems determine the mechanical advantage, the theoretical mechanical advantage, and the efficiency for each load.

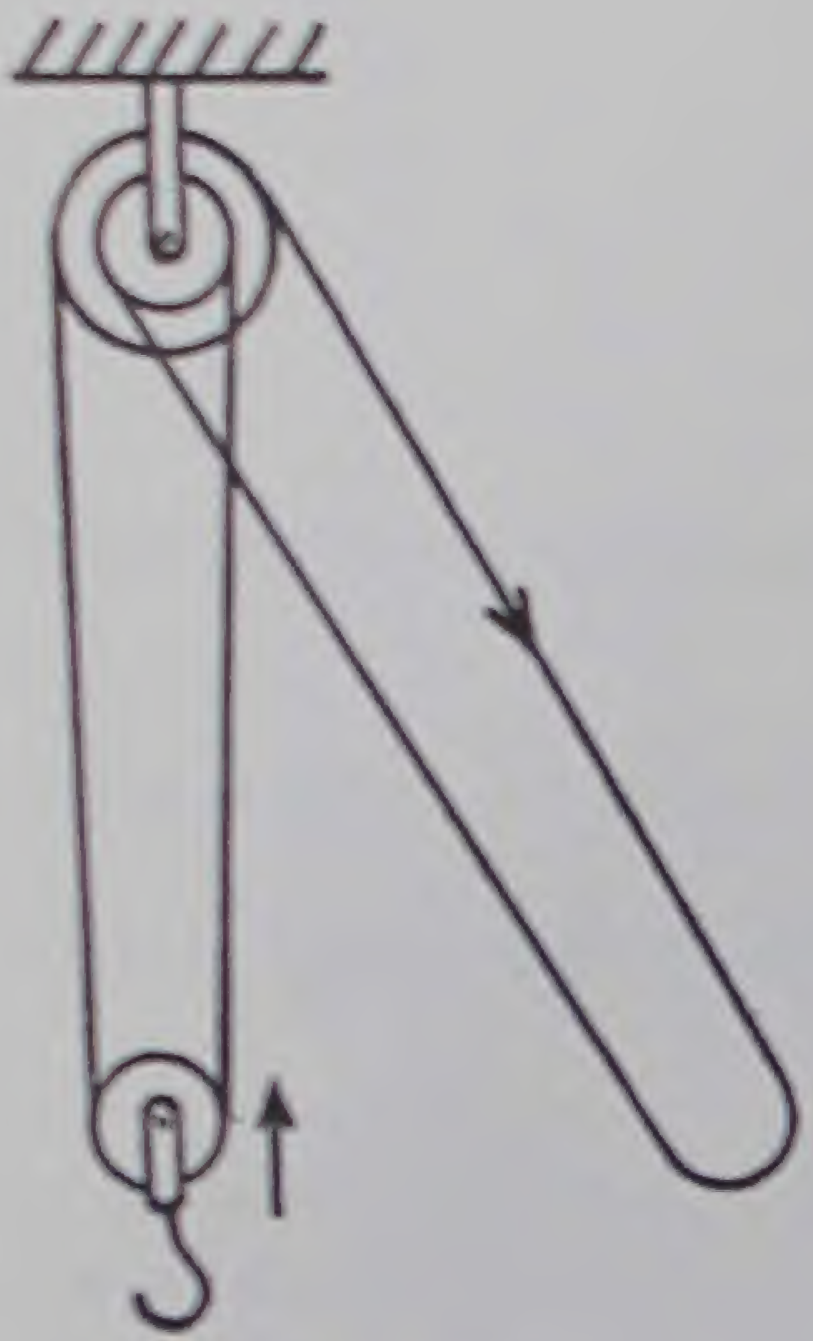


Fig. 10-1.

Part II. Wheel and Axle. The wheel and axle supplied in the laboratory has several different diameters. The smaller diameters will be used to support the loads and the larger diameters will be used in applying the input forces. Measure all diameters using the outside calipers.

A. Use loads of 500, 1000, 1500, and 2000 gm on the smallest diameter, and determine the corresponding forces on the largest diameter using, as in the case of the pulleys, just enough weight to keep the load rising slowly and uniformly.

B. Repeat, supporting the load on the next-to-the-smallest diameter, and balancing with forces on the next-to-the-largest diameter.

C. For each of the above cases determine the mechanical advantage, the theoretical mechanical advantage, and the efficiency.

Part III. Gear Train. Select one of the collection of gear trains found in the laboratory and record its identification mark. Determine its mechanical advantage and its theoretical advantage (by counting teeth). Compute the efficiency.

Record: Record all data and results in neat tabular form. Plot the following set of curves for Part I, B (all on one sheet):

a. loads as abscissas and applied forces as ordinates;
b. loads as abscissas and theoretical applied forces as ordinates (calculate the latter using the theoretical mechanical advantage);
c. loads as abscissas and mechanical advantages as ordinates;
d. loads as abscissas and efficiencies as ordinates.

QUESTIONS

1. Why is it necessary to apply enough force to keep the load rising smoothly instead of just enough to balance the load? Why is it necessary to adjust for *uniform* motion?

2. Compare curves *a* and *b*. What is the reason for the difference in slopes?

3. What can be concluded from the behavior of curve *c*?

4. What can be concluded from the behavior of curve *d*?

5. Show that the theoretical mechanical advantage of a differential pulley is

$$\text{T.M.A.} = \frac{2r_o}{r_o - r_i}, \qquad \text{(in } raising \text{ the load)} \tag{2}$$

where r_o = radius of the larger part of the pulley, and
r_i = radius of the smaller part of the pulley.

What would be the corresponding expression for the differential pulley used in the laboratory in terms of the numbers of chain indentations?

6. Show that

$$\text{efficiency} = \frac{\text{actual mechanical advantage}}{\text{ideal mechanical advantage}}. \tag{3}$$

Experiment 11.

The Torsion Pendulum

Object: To determine (1) the moment of inertia of a torsion pendulum, (2) its constant of torsion, (3) the modulus of rigidity of the support rod of the torsion pendulum.

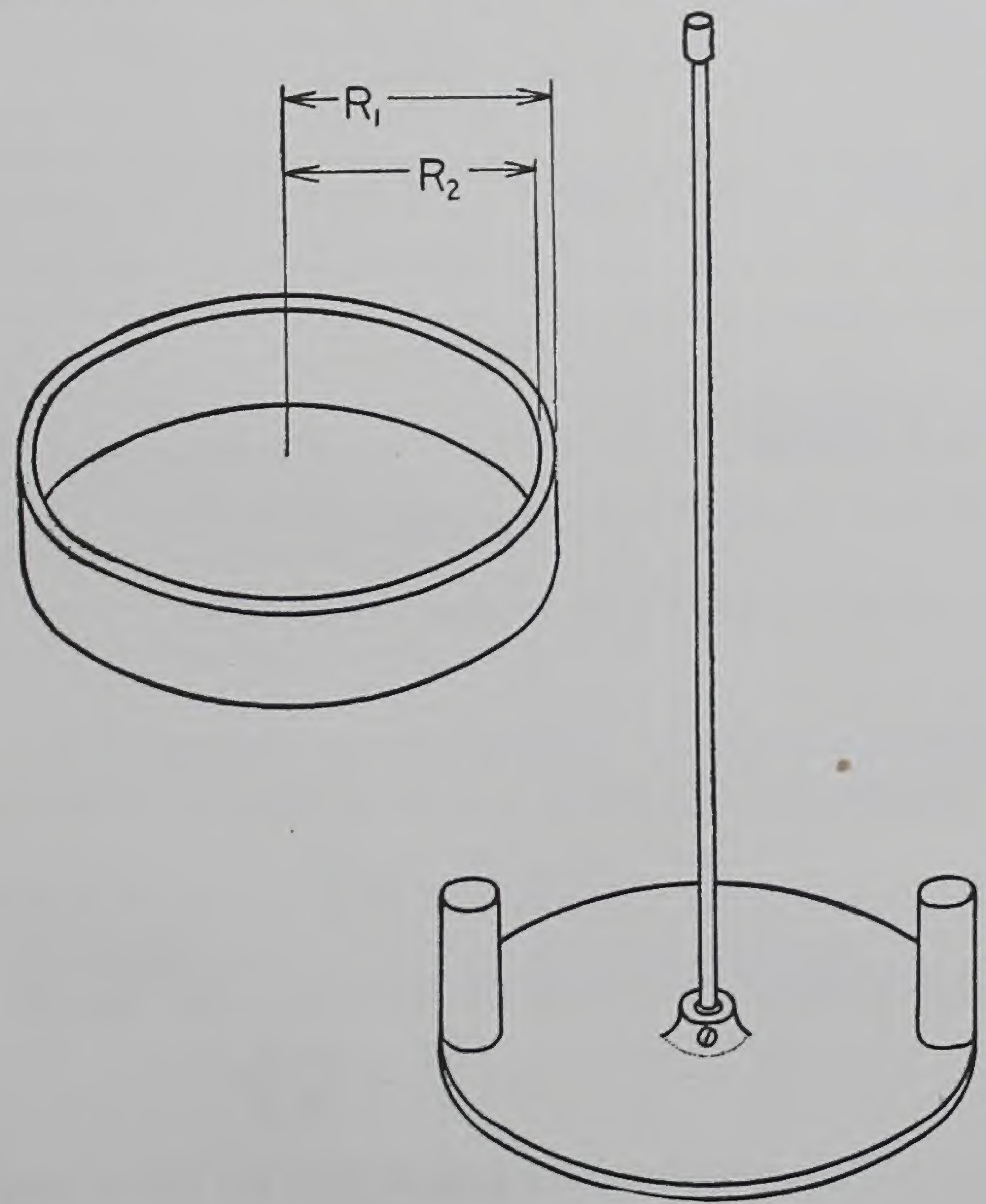


Fig. 11-1.

Apparatus: Torsion pendulum, heavy ring, two identical solid cylinders, caliper, micrometer caliper, steel scale, timer. The torsion pendulum consists of a solid disk suspended at its center by a metal rod. This rod is clamped in a rigid support at the top. See Fig. 11-1. Either the ring or the cylinders may be placed on the disk in order to change the moment of inertia of the pendulum.

Theory: If a body suspended by a rod or wire such as in Fig. 11-1 is given a small twist about the axis of suspension, it will oscillate with angular harmonic motion the period of which is given by the equation

$$T = 2\pi \sqrt{\frac{I}{k}}, \tag{1}$$

where T = period of oscillation,
I = moment of inertia of the system about the axis of rotation, and
k = torsion constant of the suspension, *i.e.*, the constant ratio between the restoring torque and the angular displacement.

The torsion constant of the suspending rod is a function of the dimensions of the rod and the modulus of rigidity of its material. This modulus n is given by the equation,

$$n = \frac{32lk}{\pi d^4}, \tag{2}$$

where n = modulus of rigidity,
l = length of rod,
k = torsion constant, and
d = diameter of the rod.

In this experiment we wish to determine both I and k for the torsion pendulum. By observing the period of oscillation T of the pendulum we can determine I/k by Eq. (1), but not I and k separately. A method of solving this problem is to *add* to the system a body of known moment of inertia I_o and then to observe the new period of oscillation T_o. This gives the equation

$$T_o = 2\pi\sqrt{\frac{I + I_o}{k}}. \tag{3}$$

By eliminating k between Eqs. (1) and (3) and solving for I, we get

$$I = qI_o \tag{4}$$

where

$$q = \frac{T^2}{T_o^2 - T^2}.$$

In a similar manner we may eliminate I and solve for k. We get

$$k = 4\pi^2 p \frac{I_o}{T_o^2} \tag{5}$$

where

$$p = \frac{T_o^2}{T_o^2 - T^2}.$$

The error equations in this experiment are somewhat more complicated than those in preceding experiments. The simplest one is that for Eq. (2), which may be written for determinate errors as

$$\frac{\Delta n}{n} = \frac{\Delta l}{l} + \frac{\Delta k}{k} - 4\frac{\Delta d}{d}. \tag{6}$$

For Eq. (4) the corresponding determinate-error equation is

$$\frac{\Delta I}{I} = \frac{\Delta I_o}{I_o} + 2p\left(\frac{\Delta T}{T} - \frac{\Delta T_o}{T_o}\right). \tag{7}$$

Notice that the fractional error in I may become very large if T_o is not much larger than T, *i.e.*, if p is large. In order to prevent this, I_o should be made as large as possible.

Finally, the determinate-error equation for Eq. (5) is

$$\frac{\Delta k}{k} = \frac{\Delta I_o}{I_o} + 2q\frac{\Delta T}{T} - 2p\frac{\Delta T_o}{T_o}. \tag{8}$$

Here again we see the advantage of using a large value of I_o for this makes both p and q small, thus keeping the fractional error in k small.

The "known" moment of inertia in this case, I_o, is that of a massive ring that has the same external diameter as the torsion disk. Actually its moment of inertia is not given, but it may easily be computed by means of the formula

$$I_o = \tfrac{1}{2}M_o(R_1^2 + R_2^2), \tag{9}$$

where I_o = moment of inertia of the ring about its central axis,
M_o = mass of the ring,
R_1 = external radius of the ring, and
R_2 = internal radius of the ring.

The determinate-error equation for Eq. (9) is approximately

$$\frac{\Delta I_o}{I_o} = \frac{\Delta M_o}{M_o} + \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} \tag{10}$$

since R_1 and R_2 are roughly equal to each other.

A second "known" moment of inertia is the combination of two solid metal cylinders each of mass m_o and radius r_o standing upright at the edge of the torsion disk and on exactly opposite sides of the supporting rod. The moment of inertia of this combination may be computed by means of the theorem (Lagrange) that the moment of inertia of any body about any axis is its moment of inertia about a parallel axis through its center of gravity plus m_oh^2 where m_o is the mass of the body and h is the perpendicular distance between the two axes.

To sum up, we may determine I and k for a torsion pendulum by finding its normal period T, and its new period T_o when a known moment of inertia I_o has been added.

Method: 1. Set the torsion pendulum (without the ring) into oscillation with an amplitude of about 10° to 20° after placing a chalk mark on its front edge. Determine its period of oscillation by counting the number of oscillations (at least 100) in a certain time interval. Estimate this count to the nearest quarter of an oscillation. Repeat this count two more times. The various counts should not vary by more than a half oscillation. Compute the period T and its error.

2. Place the ring on the disk, centering it carefully. Again set the system into oscillation and determine the new period T_o in the same manner as before.

3. Compute the moment of inertia of the ring I_o by measuring R_1 and R_2. The mass of the ring is stamped on its surface. Estimate the error in I_o.

4. By means of Eqs. (4) and (5) compute I and k for the torsion pendulum. Also determine the errors in I and in k by means of Eqs. (7) and (8).

5. Measure the diameter d and length l of the suspension rod. Take at least five measurements of d at various points along the rod. Compute the coefficient of rigidity n by means of Eq. (2); also compute the error in n by use of Eq. (6). Compare the value of n obtained with that given in Table M, Appendix III.

6. If time permits, repeat 2, 3, and 4, using the cylinders instead of the ring. Be sure that the two cylinders are placed on the disk in alignment with the index lines and with their surfaces just tangent to the circumference of the disk. Do not make error calculations for this part of the experiment.

Record:

App. No.__________

Mass of ring: M_o =

External diameter: $2R_1$ =

Internal diameter: $2R_2$ =

Length of rod: l =

Material of rod:

Diameter of rod: d =
1__________
2__________
3__________
4__________
5__________
Ave __________

	Period T		Period T_o	
Trial	No. osc	Time	No. osc	Time
1 2 3				
Ave		$T =$		$T_o =$

$I_o =$ $\Delta I_o =$

$q =$ $\Delta T =$

$p =$ $\Delta T_o =$

$I =$ $\Delta I =$

$k =$ $\Delta k =$

$n =$ $\Delta n =$

QUESTIONS

1. Develop the determinate-error equations (7), (8), and (10) in this experiment.

2. Develop Eq. (9) by using the fact that I for a uniform solid cylinder around the same axis as the ring is $\frac{1}{2}Mr^2$.

Experiment 12.

Specific Gravity. Archimedes' Principle

Object: To determine the specific gravity of several different solids and of a liquid by the principle of Archimedes.

Apparatus: Balance with hydrostatic weighing device, solid specimens, liquid specimen, overflow vessel, sinkers, beaker, hydrometer.

Theory: Specific gravity is defined as the ratio of the density of a substance to that of some standard substance—usually water. It may be obtained by comparing the weight of the substance to the weight of an equal volume of water. This may easily be done by using Archimedes' principle which states that a body immersed in a fluid is buoyed up by a force equal to the weight of the displaced fluid. The volume of the displaced fluid, of course, will just equal the volume of the immersed portion of the body.

This principle may be used to determine the specific gravity of (1) solids more dense than water, (2) solids less dense than water, (3) liquids of any density.

1. *Solids more dense than water.* In this case the weight of the solid is obtained first in air (W_0), then in water (W_1). It is assumed, of course, that the solid will not dissolve in water. The specific gravity (S.G.) in this case is given by the formula

$$\text{S.G.} = \frac{W_0}{W_0 - W_1}. \tag{1}$$

2. *Solids less dense than water.* The solid is weighed in air (W_0). A sinker is then attached to it, and the system (solid plus sinker) is reweighed with the solid in air and the sinker in water (W_{01}). Finally the system is weighed again with both the solid and the sinker in water (W_{11}). The specific gravity of the solid is then given by the formula

$$\text{S.G.} = \frac{W_0}{W_{01} - W_{11}}. \tag{2}$$

3. *Liquids.* A solid body is weighed first in air (W_0), then in water (W_1), and finally in the liquid (W_2). The specific gravity of the liquid is then given by the formula

$$\text{S.G.} = \frac{W_0 - W_2}{W_0 - W_1}. \tag{3}$$

Method: *Part I. Solids more dense than water.* Attach the solid specimen to the pan of the balance by means of a wire and determine its weight (W_0). Fill the overflow can with water, immerse the solid in the water, and catch the overflow in a beaker. Determine the weight of the solid immersed in the water (W_1). Calculate the S.G. of the solid by means of Eq. (1), and compute the approximate error in the result. Compare the loss of weight of the solid ($W_0 - W_1$) with the weight of the water which overflowed into the beaker.

Repeat this experiment with a second solid specimen.

Part II. Solids less dense than water. Attach the solid specimen (block of wood) to the balance and determine its weight (W_0). Fasten a sinker (lead weight) to the block of wood, immerse it in water, then determine the weight of the system under these conditions (W_{01}). Finally immerse the entire system in water and determine the weight of the system (W_{11}). Calculate the S.G. of the wooden block by means of Eq. (2) and compute the approximate error.

Repeat this experiment with a second solid specimen (cork).

Part III. Liquids. Attach a small metal cylinder to the balance and determine its weight (W_0). Then immerse the cylinder in a beaker of water and redetermine its weight (W_1). Finally immerse the cylinder in a beaker of the liquid and determine its weight under this condition (W_2). Calculate the S.G. of the liquid and compute the approximate error.

Make a direct determination of the specific gravity of the liquid by means of a hydrometer and compare the two values.

Compare the values obtained with the accepted values in Table C, Appendix III.

Record: Tabulate data and results.

QUESTIONS

1. By use of Archimedes' principle develop Eqs. (1), (2), and (3).
2. Bubbles of air are likely to attach themselves to the solid specimen when immersed in a liquid. What effect in general will this have on the calculated values of S.G.?
3. A ping-pong ball floats on water in a closed vessel partially filled with water. If the air pressure in the vessel is increased by pumping more air into the vessel, will the ping pong ball rise or sink in the water? Explain.
4. Why can one neglect the buoyant effect of the air in this experiment?

Experiment 13.

Surface Tension

Object: To determine the surface tension of water (1) by a direct method, (2) by capillary action.

Apparatus: Micrometer microscope, wire frame, Jolly balance, capillary tube, cathetometer.

Theory: Because of the mutual attraction of the molecules in a liquid, those in the interior experience forces which are nearly uniform in all directions, whereas those at the surface experience a net inward force of attraction. This means that work must be done to move a molecule from the interior of a liquid to its surface, *i.e.*, the surface molecules possess more energy than those in the interior of the liquid. Thus the

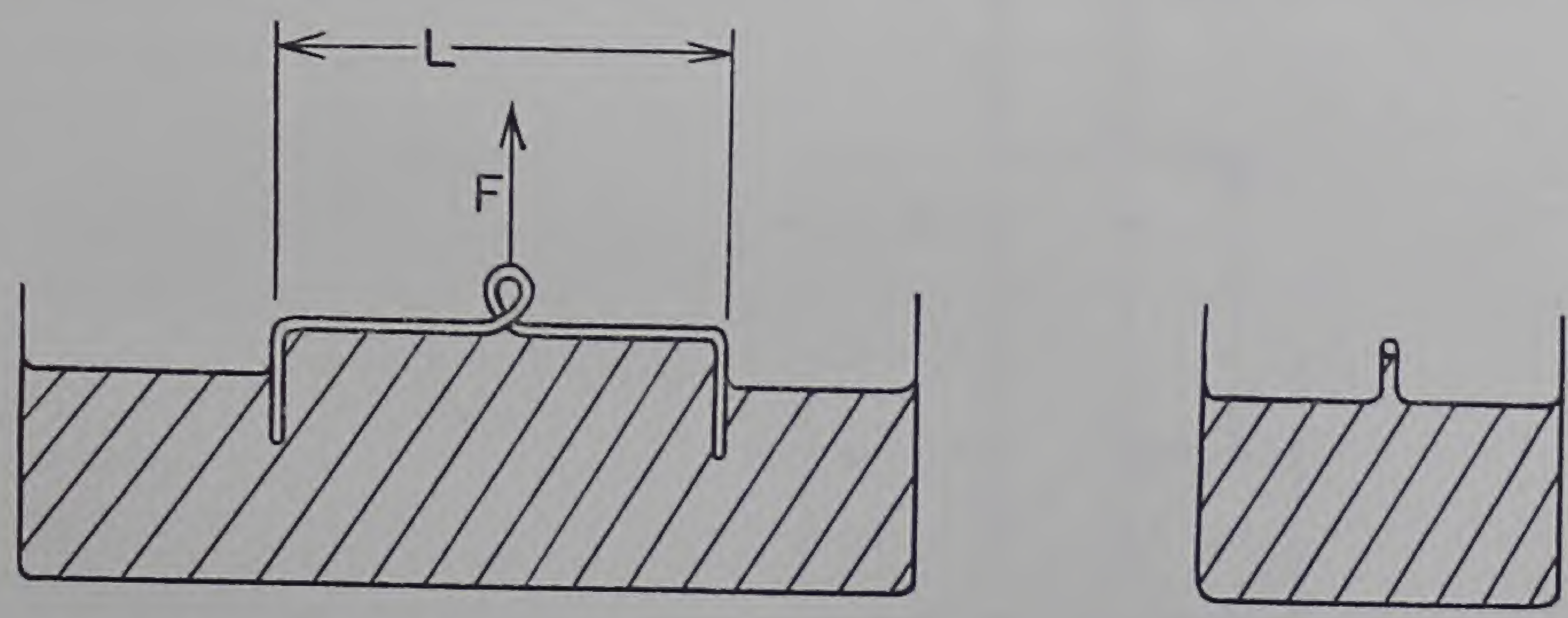


Fig. 13-1.

surface of a liquid possesses energy and hence tends to act as if it were covered by a stretched film or membrane. This phenomenon is known as surface tension. The tension T of such a film is measured by the force F which it exerts per unit length along a line l in the surface across which the measurement is made:

$$T = \frac{F}{l} \qquad \text{(dyne/cm)}. \tag{1}$$

The direct method of measuring surface tension is to lift a straight wire through the surface (Fig. 13-1). If this is done carefully the film will be pulled up with the wire. If F is the force exerted by the film (two surfaces) on the wire, and L is the length of the wire, then

$$T = \frac{F}{2L}. \tag{2}$$

The rise or fall of a liquid in a capillary tube may also be used to determine its surface tension. The surface tension in this case is given by the equation

$$T = \frac{dghr}{2 \cos \theta}, \tag{3}$$

where T = surface tension,

d = density of the liquid,

h = vertical rise of the liquid in the tube,

r = internal radius of the tube, and

θ = contact angle of the liquid with the tube wall.

See Fig. 13-2. Also see your general physics textbook for a development of Eq. (3).

Pure water in contact with clean glass has a contact angle $\theta = 0$. In this case the liquid (water) wets the glass.

***Method:** Part I. Direct Method.* Clean the wire frame by heating it to redness in an alcohol flame. *Do not handle it with the fingers but use a pair of tweezers.* Attach it beneath the weight pan and indicator on the spring of the Jolly balance as shown in Fig. 13-3, and adjust the position of the spring until the center line of the indicator is even with the center line of the glass indicator tube. This should be accomplished fairly near the low end of the scale of the balance. Record the scale reading. Place a 1-gm weight in the pan and again adjust the position of the spring until the indicator center lines coincide. Record the new scale reading. From these data the force constant of the spring may be obtained. Remove the 1-gm weight.

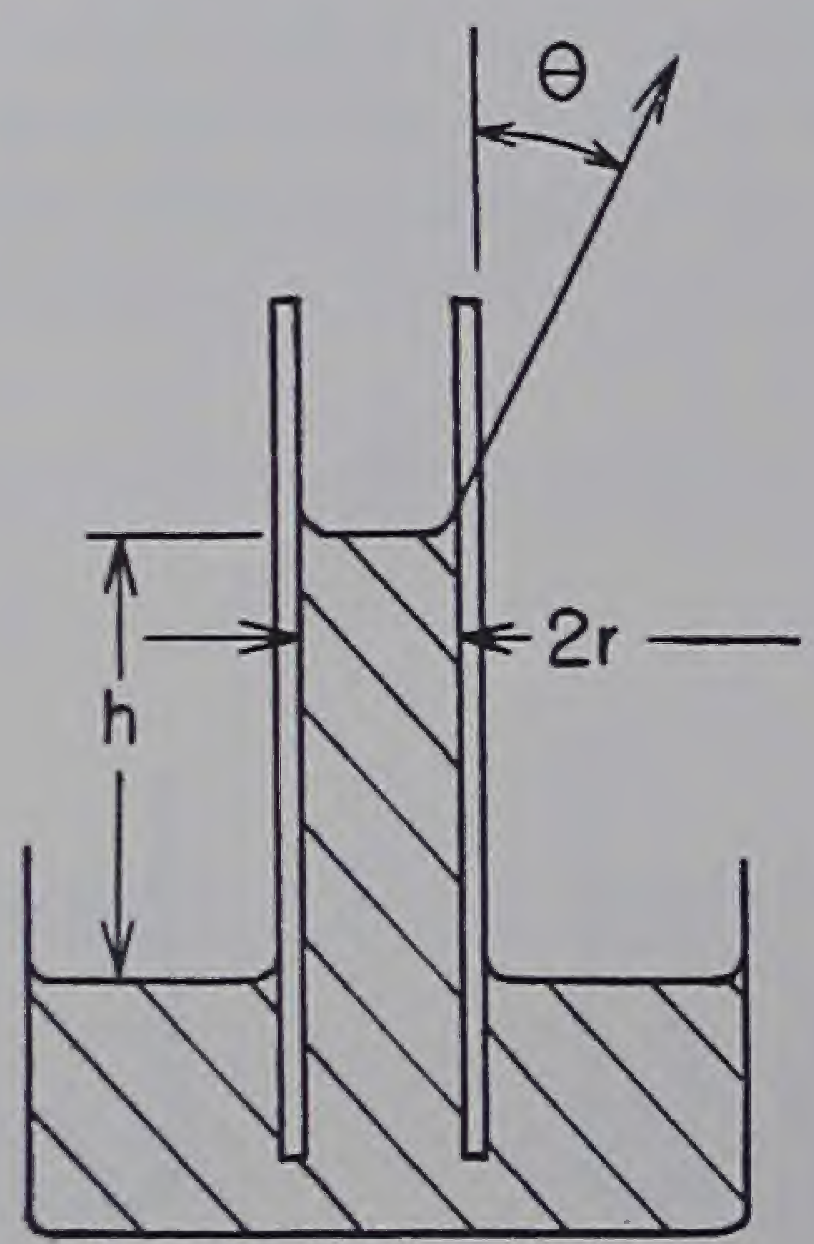


Fig. 13-2.

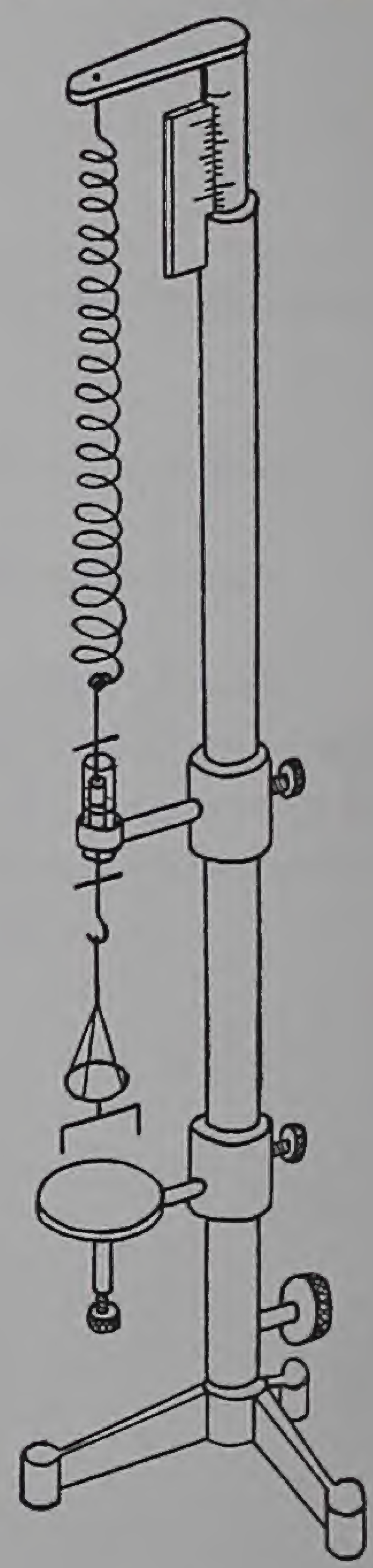


Fig. 13-3.

Place a beaker of *fresh* water on the platform, and raise the platform until the wire frame is immersed. (The tweezers may have to be used to accomplish this. NOTE: Do not put your fingers into the water. The slightest trace of oil will affect the results.) Lower the platform slowly until the frame begins to come through the surface of the water, drawing a film with it (Fig. 13-1). Continue to lower the beaker of water, but simultaneously adjust the spring tension so that the indicator center marks coincide at all times. As the wire frame draws the film farther out of the surface, a point will be reached where the wire continues to rise without any further lowering of the beaker. If the previous adjustments have been made slowly enough, this point will be easily recognized. As soon as it is reached, stop making further adjustments; the frame will gradually and then more rapidly continue to free itself. Record the setting of the scale. Repeat five times after uniform results are obtained, *i.e.*, after successive scale readings agree to within 0.2 cm.

Measure carefully and record the outside length of the wire frame using a steel scale. From the known force constant of the spring determine the force exerted in breaking the film. Compute the surface tension and its error.

Part II. Capillary-tube Method. Select a piece of capillary tubing from the beaker of chromic acid solution and rinse thoroughly with water. Dip the tube deeply in a beaker of fresh water, and then withdraw it slowly until the liquid inside the tube begins to sink. Support the tube at this position vertically in

the beaker, and measure, using the cathetometer, the height of the water in the tube above the surface of the water in the beaker. Repeat two more times.

Remove the tube and mount it under the microscope in such a way that its bottom end is above the tilted mirror of the microscope stand and its top end is in the focus of the microscope. Measure the inside diameter of the tube with the microscope (see Appendix II, Note C) taking readings when the cross hair is tangent to the bore of the tube and perpendicular to the direction of travel of the microscope. Turn the tube 90° and remeasure; use the average of the two measurements. Calculate the surface tension and its error and compare with the results from the direct method. Compare your values with that given in Table L, Appendix III.

Record:

Scale reading (no weight in pan)	______________	
With 1 gm in pan	______________	
Difference	______________	
Force constant	______________	dynes/cm
Scale readings (wire breaks through)	______________	
	______________	
	______________	
	______________	
	______________	
Average	______________	
Net spring extension to break through	______________	
Corresponding force	______________	
Length of wire frame	______________	
Surface tension	______________	Error______________

	Trial 1	Trial 2	Trial 3
Height in tube	_______	_______	_______
Height in beaker	_______	_______	_______
Height h	_______	_______	_______
Average of h	_______		
Diameter of tube:		1st meas	90° meas
Left reading		__________	__________
Right reading		__________	__________
Difference		__________	__________
Average diameter	_______		
Surface tension	_______	Error______________	

QUESTIONS

1. Frequently the capillary-tube method gives a smaller value of T than the direct method. How would you explain this? HINT: What assumption was made in using Eq. (3)?

2. Surface tension may be defined as the amount of work required to form a unit area of surface of a liquid. Show that this definition also leads to Eqs. (1) and (2) in this experiment.

3. In Eq. (3) r refers to the internal radius of the tube at what point along the tube? Explain.

Experiment 14.

Density of Air

Object: To determine the density of air.

Apparatus: Analytical balance, set of analytical weights, set of laboratory weights, spherical flask with valve, vernier caliper, vacuum pump, barometer, thermometer, drying tube, cloth. For a description of some of these items see Appendix II, Notes A1, 2, 3; D; M.

Theory: Like other matter, gases have mass. As with solids and liquids, a density may be determined for a quantity of gas; it is equal to the mass of the gas per unit volume:

$$d = \frac{m}{V}. \tag{1}$$

Unlike a solid or a liquid, however, a gas will expand to fill the entire volume of a container, no matter how small a mass of gas is present, and thus will exert a pressure on the walls of the container. The pressure exerted by a given mass of gas will depend on its temperature as well as on its volume. Under proper conditions of pressure and temperature a gas will obey the ideal-gas equation

$$pV = nRT, \tag{2}$$

where p = pressure of the gas,
V = volume of the gas,
n = number of moles (gram molecular weights) of the gas,
R = constant, the so-called gas constant, and
T = absolute temperature of the gas.

The above law holds for all pressures, volumes, and temperatures; for a given mass of gas it will hold in particular in condition 1:

$$p_1V_1 = nRT_1 \tag{3}$$

and in condition 2:

$$p_2V_2 = nRT_2. \tag{4}$$

Dividing Eq. (3) by Eq. (4) we get

$$\frac{p_1V_1}{p_2V_2} = \frac{T_1}{T_2}. \tag{5}$$

Applying Eq. (1) with proper subscripts for conditions 1 and 2, Eq. (5) becomes

$$\frac{p_1d_2}{p_2d_1} = \frac{T_1}{T_2}. \tag{6}$$

Thus if we know the density of a gas at any temperature and pressure (condition 1) we may calculate its density at any other temperature and pressure (condition 2) provided the gas obeys the ideal-gas law, Eq. (2). For our purposes, dry air behaves sufficiently like an ideal gas to enable us to use the above equations without appreciable error in this experiment.

For comparison purposes the densities of gases are generally given for *standard conditions.* It would obviously mean nothing to say that the density of carbon dioxide is 0.001965 gm/cm^3 unless the temperature and pressure at which this value is correct are also specified; for by Eq. (6) the density varies with temperature and pressure. *Standard conditions* of temperature and pressure *are taken to be zero degrees centigrade and* 760 *mm of mercury.*

Absolute temperature is related to centigrade temperature by the fact that 0°C corresponds to 273.2° Abs. The size of the absolute degree is equal to that of the centigrade degree so that we may obtain the absolute temperature by adding 273.2 to the centigrade temperature.

Thus if we wish to find the standard density of a gas, we need only measure its density under the laboratory conditions, and apply Eq. (6). Remembering that p_s = 760 mm Hg and that T_s = 273.2°Abs, Eq. (6) becomes

$$d_s = \frac{760 \times T_{\text{lab}}}{p_{\text{lab}} \times 273.2}\, d_{\text{lab}}. \tag{6a}$$

Method: Two flasks filled with dry air will be hung on the arms of an analytical balance, and enough weights will be added to the lighter side to bring the balance to a zero rest point. One of the flasks will then be evacuated and replaced on the balance. To again obtain equilibrium, weights will have to be added to the side of the evacuated flask, equal in amount to the weight of air exhausted. Knowing this weight and the volume of the flask, the density of air under laboratory conditions is easily obtained. With this, and the laboratory temperature and atmospheric pressure, Eq. (6a) may be applied to find the density of air under standard conditions.

After having read carefully the notes in Appendix II referred to above under *Apparatus,* check an analytical balance in the manner described in Appendix II, Sections M, 3(a), 1(d), and 2(e). If its unloaded rest point is within two or three divisions of the zero point, the balance is ready for use. Record the rest point. Engage the arrestment.

Check the stopcocks of the flasks for airtightness. They should turn smoothly without rasping, and with a viscous resistance. If a valve is unsatisfactory, remove the stopcock and clean both it and the stopcock seat with a cloth. Apply new stopcock lubricant *sparingly* to the stopcock and reinsert it in its seat, testing as before. These precautions are necessary to prevent leakage of air into the flask while it is being weighed.

Evacuate a flask with the vacuum pump. In using the pump, the following precautions should be observed. The pump should never be run "sucking air," that is, with its hose disconnected; running excessive amounts of air through the pump tends to oxidize the oil. Insert the neck of the flask into the pump hose, open the stopcock, and turn on the pump. When the sound of the pump indicates that the flask is evacuated (about 1 min later) close the stopcock, and *then* turn off the pump. Disconnect the flask from the hose, being careful to pull straight out so as not to break the neck of the flask.

Repeat with the other flask. While the second flask is being evacuated, connect the first flask to the drying tube and open the stopcock *gradually,* to prevent excessive cooling of the incoming air. Close the stopcock, and repeat with the second flask.

Wipe the flasks with a clean cloth to remove grease and dust, and *from this point on, handle them only with the cloth,* since oil and moisture from the fingers will change their weights. Hang the flasks by means of the loops of wire from the hooks of the analytical balance. Open the stopcocks.

Being sure to use *only the forceps* on the analytical weights, use the less accurate set to bring the balance to the zero point by means of the method outlined in Appendix II, Section M, 3(b) and (d). (See Question 1 at the end of this experiment!) Balance to the nearest milligram, and record swings to avoid error. Engage the arrestment and record the position of the rider. At this time read and record the room temperature, the barometer reading, and the temperature at the barometer.

Using a cloth, remove the lighter of the two flasks (the test flask), and close the stopcock of the other flask (the counterpoise flask). Connect the test flask to the vacuum pump and evacuate for 2 min, observing the same precautions with regard to the pump as earlier. Rehang it on the analytical balance.

Using the more accurate set of weights, bring the balance to a zero rest point once more by adding weights to the pan with the evacuated flask. Again balance to the nearest milligram, and engage the arrest-

ment. Record the amount of new weights added and the new position of the rider. Return this set of weights to its box.

Remove the evacuated flask and refill with *dry* air. Replace it on the balance, and once more bring the balance to zero. Record the amount of change, if any, from the first weighing. Use as the value of the "with air" weighing, the average of this and the first weighing.

Remove the test flask and measure the outside diameter of its spherical section, using the vernier caliper. Measure in at least 10 different representative places and find the mean of these measurements. Determine the thickness of the glass by measuring that of a sample broken flask. Then the volume of the flask may readily be calculated except for the portion in the neck. Measure the inside diameter of the neck and its length from the flask to the stopcock. Measure the dimensions of any tapered portion not properly a part of the spherical volume. Calculate the approximate volume of this extra portion and add it to the spherical volume already determined.

Using this volume and the mass of air determined above, calculate the density under laboratory conditions.

Using Table F of Appendix III, apply the proper correction to the barometer reading. Using the room temperature and the corrected barometric pressure, calculate the density of air under standard conditions. Compare this with the accepted value given in Table C.

Assuming the temperature to be known within $\pm 1°C$, the mass of the air to within ± 2 mg, and assigning reasonable values to errors in the other measured quantities, calculate the indeterminate errors in the values of d_{lab} and d_s. Does the error for d_s include the accepted value of d_s?

Record:

App. No.: Balance______
Test flask______
Counterpoise flask______
Caliper______
Weights (accurate)______

Caliper zero error______cm
Lab temperature______°C
Barometer reading______mm Hg
Barometer temp______°C
Barometer correction______mm Hg
Corrected pressure______mm Hg
Rest point of balance______div

Diameter of bulb:
1______cm
2______
3______
4______
5______
6______
7______
8______
9______
10______

Thickness of glass:
1______cm
2______
3______
4______
Ave______cm

Dimensions of neck and tapered portion:
(sketch)

Volume of neck and tapered portion: ______cm³

Ave______cm ± ______cm. Inside diameter: ______cm ± ______cm.
Volume of spherical portion: ______cm³. Total volume: ______cm³.

First balance: weights______
rider position______
Second balance: new weights______
rider position______

Third balance: weights______
rider position______
Mass of air______mg

d_{lab}______gm/cm³ ± ______gm/cm³
d_s______gm/cm³ ± ______gm/cm³

QUESTIONS

1. Why, in this experiment, is it permissible to balance the scales at the zero position instead of at the true unloaded rest point?

2. Why is it necessary to use dry air in this experiment rather than air directly from the room? Describe the effect on the value of d_s obtained (a) if some water condenses in the flask and (b) if there is water vapor present in the flask.

3. In the *Method*, the flasks were wiped dry of grease and dust after having been filled with dry air, thus giving them time to attain room temperature before weighing. Why is such a delay necessary? Why are the stopcocks opened before weighing? Why are the room temperature and atmospheric pressure read and recorded immediately after this weighing?

4. The counterpoise-flask method used in this experiment eliminates errors resulting from a change in temperature or atmospheric pressure between the first and second weighings. Explain. (HINT: Consider the buoyant effect of the air on the flasks and weights.) Why was the stopcock of the counterpoise flask closed and left closed?

5. The method of weighing used in this experiment has the features of the "substitution method," *i.e.*, weights were substituted directly in the pan from which mass had been removed (by evacuation of the flask). Which error inherent in the *balance* does the substitution method eliminate? See Appendix II, Section M, 3(d). Explain.

Experiment 20.

Linear Expansion

Object: To determine the coefficient of linear expansion of a metal.

Apparatus: Specimen whose coefficient of linear expansion is to be measured, traveling microscopes, meter stick, thermometers, boiler.

Theory: Most solids undergo a change in length upon experiencing a temperature change. The size of this change is found to depend on three factors: the amount of the temperature change, the original length of the solid, and the material composition of the solid. This relation may be expressed by the equation

$$\delta L = \alpha L_o \, \delta T, \tag{1}$$

where δL = the change in length,
L_o = the length at a standard temperature,
δT = the change in temperature, and
α = a constant depending on the material.

The constant, α, is called *the coefficient of linear expansion.* We may solve Eq. (1) for α to get

$$\alpha = \frac{\delta L}{L_o \delta T}. \tag{2}$$

Thus it is seen that α is the change in length per unit original length per degree of temperature change. In the metric system, the temperature change is measured in centigrade degrees, and the standard original length is that at 0°C. However, under the conditions of this experiment, we may use as our reference length the length at room temperature, L_r, to a sufficient degree of accuracy. Then Eq. (2) becomes

$$\alpha = \frac{\delta L}{L_r \delta T}. \tag{3}$$

Method: The solid is in the form of a tube, T, about 1 m in length. See Fig. 20-1. The tube is surrounded by another tube, S, packed with felt insulation. Near each end of the specimen tube a short brass rod, R, is soldered at right angles, and the tips of these rods are just visible through the surrounding tube. The ends of the specimen tube extend beyond the surrounding tube and are connected by rubber hoses to thermometer emplacements, which in turn are furnished with connections to running water or steam. Placed above the short brass rods are traveling microscopes, M (see Note C, Appendix II), which focus on scratches on the tips of the rods. The surrounding tube is mounted on supports, one of which is provided with a screw adjustment, A, so that the whole apparatus may be moved in the direction of its length. Thus, by means of the microscopes, small changes in the length of the specimen may be measured.

With a meter stick, measure the length of the specimen between the indexes on the brass projections. Pass a stream of cold water through the specimen (tap water will do). When the specimen has attained a uniform temperature (steady readings on both thermometers) focus the microscopes on the scratches on the brass projections and record the reading on the right-hand microscope and the readings of both thermometers. Read the thermometers to the tenth of a degree. Each student should make an independent setting of the right-hand traveling microscope to ensure accuracy. Drain out the water, being sure to empty the wells

beneath both thermometers, and connect the system to the boiler. Be sure the boiler contains sufficient water before lighting the burner. Pass steam through the system, and when the temperature becomes steady, use the adjusting screw in the left-hand support to move the scratch directly under the left-hand microscope again; read and record the right-hand microscope setting, and the readings of both thermometers. Again pass tap water through the specimen, taking readings as before.

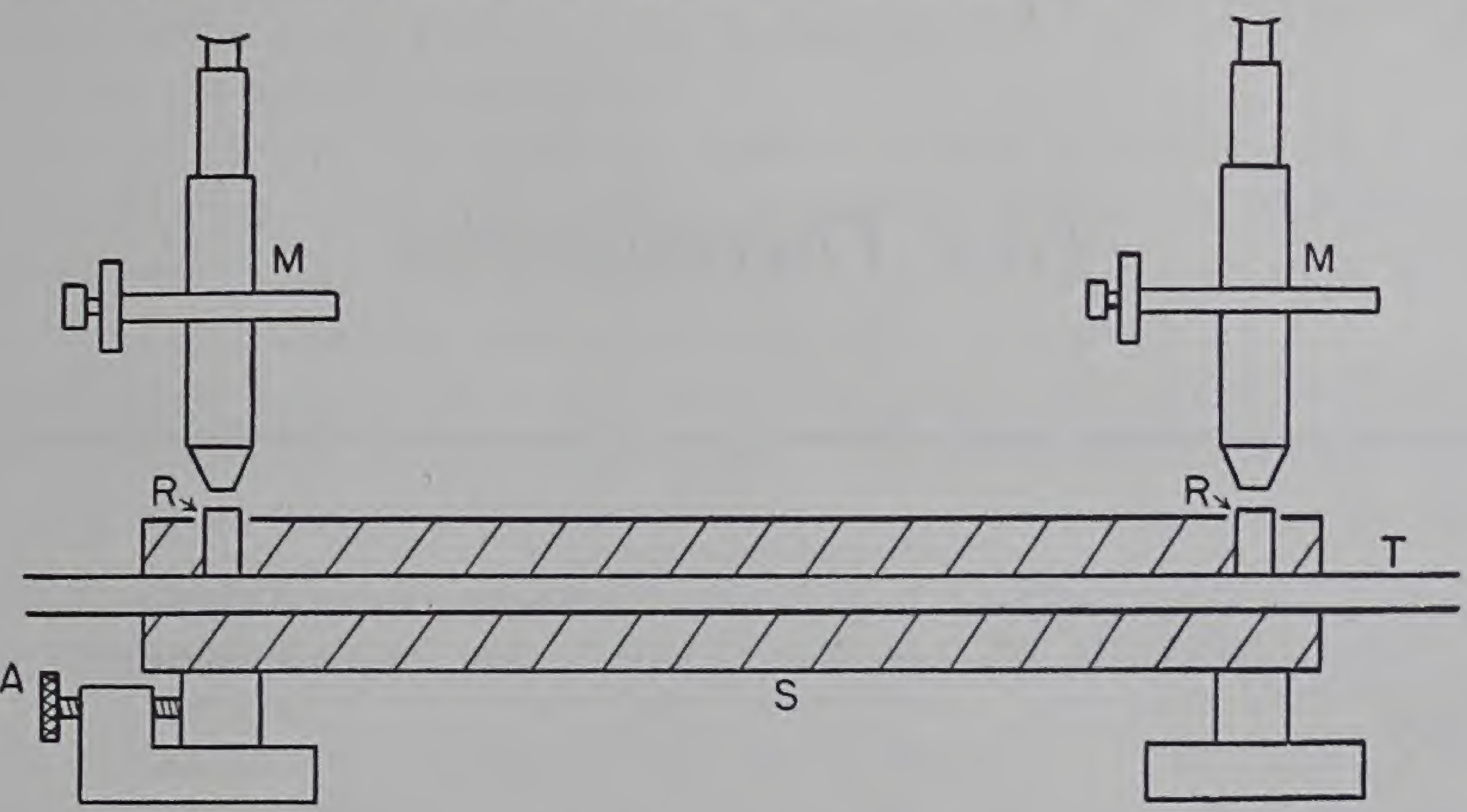


Fig. 20-1.

From these readings, using for each temperature the average value of the readings of the two thermometers, and for each micrometer reading the average of the readings taken by each student at that temperature, substitute in Eq. (3) to find α. Two separate values of α can be found from the data corresponding to the two temperature changes produced; compute them. Find the error equation corresponding to Eq. (3), and calculate the indeterminate errors in the values of α. Average the two values of α and compare with the values in Table E, Appendix III. Of what material is the specimen composed?

Record:

App. No.__________

Length between scratches__________

Room temperature__________

	Left	Right	Average
Tap-water temperature	______	______	______
Steam temperature	______	______	______
Tap-water temperature	______	______	______

Right-hand micrometer readings	Student A	Student B	Student C	Average
Tap water	______	______	______	______
Steam	______	______	______	______
Tap water	______	______	______	______

	Rising temperature	Falling temperature
$\delta T =$	______	______
$\delta L =$	______	______
$\alpha =$	______	______
Error of α	______	______
Average α	______	

Material of specimen:__________

QUESTIONS

1. What percentage error is introduced into the value of α by using L_r instead of L_o? Is this error significant in this experiment?

2. Why is it permissible to measure L_r with a meter stick whereas the increase in length must be measured with a traveling microscope?

Experiment 21.

Gas Thermometer

Object: To calibrate a constant-volume helium gas thermometer and thence to determine (1) the sublimation temperature of solid carbon dioxide (dry ice) at atmospheric pressure, (2) the temperature coefficient of pressure of helium gas.

Apparatus: Constant-volume helium gas thermometer, barometer, boiler, cracked ice, dry ice.

A sketch of the constant-volume gas thermometer is shown in Fig. 21-1. It consists essentially of a mercury manometer, one arm of which is open to the atmosphere while the other arm connects with a closed

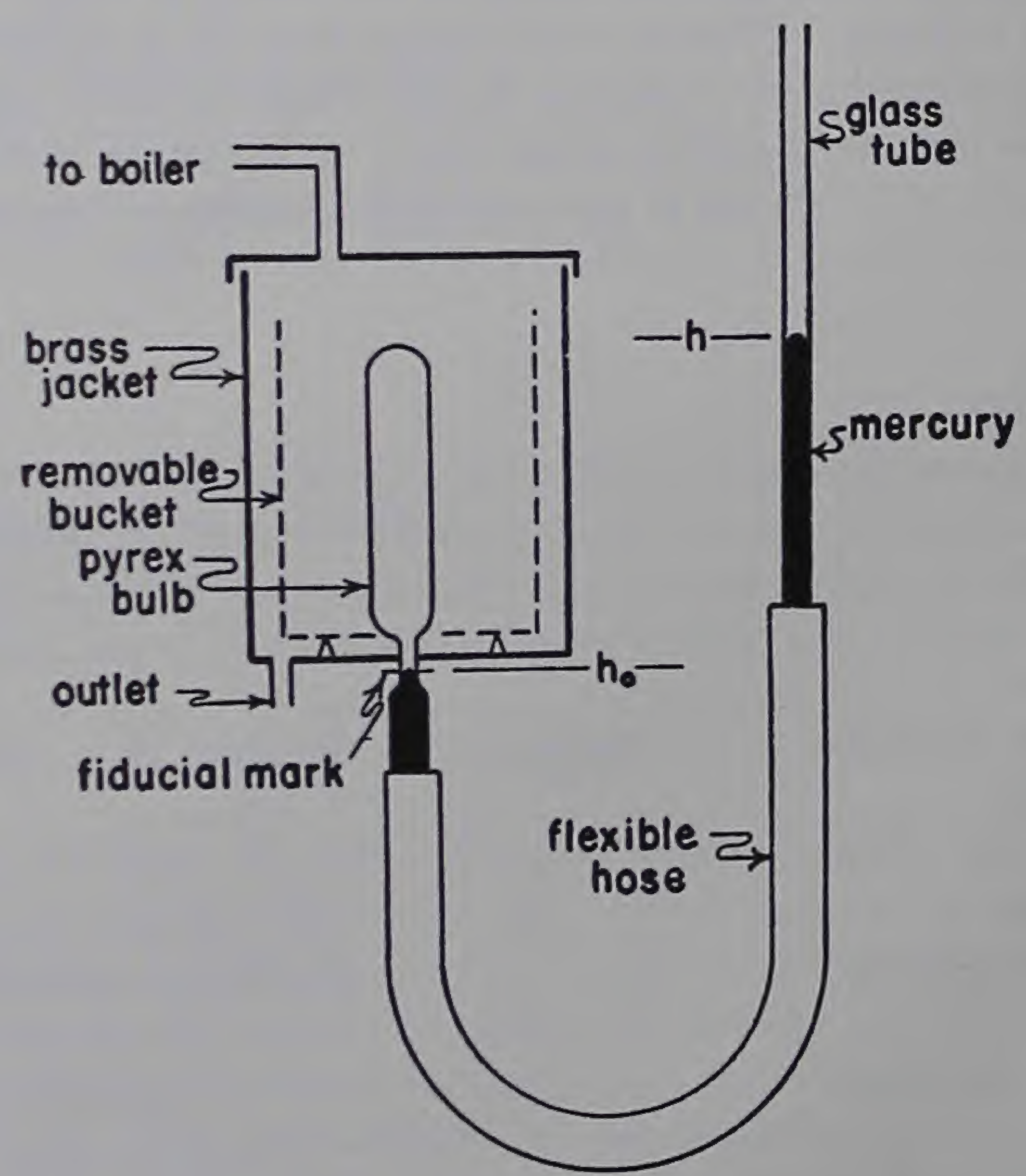


Fig. 21-1.

glass bulb containing helium gas. The glass bulb is enclosed in a brass jacket with a removable top. A stem on the top may be connected to a steam boiler for the purpose of immersing the bulb in a steam bath. Alternatively, ice or dry ice may be packed around the glass bulb in the brass jacket. A removable inner bucket facilitates the removal of the ice pack when desired. The height of either manometer arm is adjustable. To keep the gas in the bulb at constant volume, it is only necessary to keep the mercury level in the closed manometer arm at the fiducial or reference mark. This is a fine black ring on the stem of the bulb just below the brass jacket. The height of mercury in either arm may be read directly off a fixed scale on

Theory: When two systems or bodies at different temperatures are put into thermal contact with each other, heat flows from the system of higher temperature into the system of lower temperature. This flow of heat continues until the temperatures of the two systems are the same. At this point the flow of heat ceases and the systems are said to be in thermal equilibrium with each other. Since the flow of heat, or its absence, can be detected by means other than the measurement of temperatures, this method gives us a means of determining the equality or inequality of the temperatures of two systems. It gives us no information about the amount of the inequality, however.

It is a matter of experience that two systems, each of which is in thermal equilibrium with a third system, are in thermal equilibrium with each other. Hence all systems in thermal equilibrium with one another have the same temperature. Temperature therefore is a property of a system that determines whether or not this system is in thermal equilibrium with other systems.

It is well known that some properties of a system are temperature-independent, *e.g.*, its mass, while others such as its volume depend upon the temperature. Temperature-dependent properties of a system are known as thermometric properties. Any one of them may be used to indicate the temperature of the system provided some assumed rule is given correlating the thermometric property with the temperature. The simplest rule is to assume that equal temperature changes correspond to equal changes in the value of the thermometric property provided the other quantities upon which the thermometric property depends are held constant. This is equivalent to saying that the relation is linear (straight-line relation). Since two points are required to fix a straight line, so also are two fixed temperatures required to fix the temperature–thermometric property relation. The two fixed temperatures are usually taken to be (1) the temperature of melting ice under standard atmospheric pressure and (2) the temperature of boiling water under standard atmospheric pressure. These two fixed points are generally referred to as (1) the normal ice point and (2) the normal steam point. Unfortunately, temperature scales defined in this manner are, in general, different for each thermometric property of each substance used, except, of course, at the two fixed points. Gases under the proper conditions constitute an outstanding exception. The pressure of a gas at constant volume, or its volume at constant pressure, may be used as a temperature indicator. Under the appropriate conditions, these two thermometric properties of a gas give practically the same temperature scale for all gases (the gas temperature scale).

By use of the laws of thermodynamics, it is possible to define a temperature scale, the Kelvin absolute scale, which is completely independent of the nature of the substance used as a temperature indicator. It is also possible to show that this temperature scale is identical with that given by an "ideal gas." Since all gases approach the ideal state under proper conditions, all give practically the same temperature scale under these conditions. The so-called permanent gases such as hydrogen and helium are very nearly ideal over wide ranges of temperature and pressure. For these reasons the constant-volume gas thermometer (hydrogen or helium) properly calibrated is used to define a standard temperature scale which corresponds very closely with the Kelvin scale over a wide range of temperatures. For example, the constant-volume helium thermometer gives very satisfactory temperature readings over a range from -240 to 1000°C. From -100 to 200°C the deviation of the helium temperature from the Kelvin temperature is less than 0.001°C and even at -240°C the deviation is only 0.02°C.

In this experiment the thermometric property used to indicate temperature is the pressure of a fixed mass of helium gas held at *constant volume.* It is *assumed* that the change in pressure of this gas is directly proportional to its change in temperature. This means that the relation between the pressure P and the temperature t of this gas must be linear. Hence

$$t = aP + b \qquad \text{(const vol)}, \tag{1}$$

where a and b are constants. These two constants are determined by the two fixed points of the temperature scale. Calibration of the gas thermometer is essentially the determination of these constants since then a temperature t may be calculated by Eq. (1) in terms of a measured gas pressure P.

Suppose the bulb of the gas thermometer is immersed in an ice mixture and then in a steam bath. Let P_i and P_s be the measured pressures of the gas (const vol) at these two temperatures t_i and t_s. Then by Eq. (1)

$$t_i = aP_i + b,$$

$$t_s = aP_s + b.$$

These two equations solved simultaneously for a and b give values which, when substituted in Eq. (1), give

$$t = t_i + \frac{t_s - t_i}{P_s - P_i}(P - P_i). \tag{2}$$

On the centigrade scale the normal ice-point temperature is taken as 0° and the normal steam point as 100°. This assumes a standard atmospheric pressure of 76 cm of Hg on the ice and on the steam. If the atmospheric pressure under which measurements are made is not standard, then the values of t_i and t_s are not precisely 0° and 100°. It turns out that the ice-point temperature is not appreciably affected by small changes in atmospheric pressure, hence t_i may be set equal to zero even though the atmospheric pressure is not exactly 76 cm of Hg. This is evident from the fact that a *change* of 1 atm of pressure (76 cm of Hg) changes the melting point of ice by about 0.0075°C. On the other hand, the boiling point of water is quite sensitive to changes in the atmospheric pressure. Here a change of 2.7 cm of Hg in the atmospheric pressure changes the boiling point by 1°C. Hence the value of t_s should not be set equal to 100° unless the atmospheric pressure is 76 cm of Hg. The value of t_s for a large range of pressures is given in Table D, Appendix III.

For $t_i = 0$ Eq. (2) becomes

$$t = t_s \frac{P - P_i}{P_s - P_i}. \tag{3}$$

Equation (3) is the fundamental equation for a constant-volume gas thermometer. If the pressures are measured in terms of the height of mercury columns, this equation may be further simplified. For example, the pressure P_i may be expressed in the form

$$P_i = H_b dg + (h_i - h_o)dg,$$

where H_b = barometer reading in centimeters of Hg,
h_i = scale reading in centimeters of the position of the mercury meniscus in the open tube of the manometer when the bulb is packed in ice,
h_o = scale reading in centimeters of the position of the mercury meniscus at the reference mark,
d = density of mercury at room temperature, and
g = acceleration of gravity.

The pressures P and P_s may be expressed in a similar manner. When these values are substituted in Eq. (3), it becomes

$$t = t_s \frac{h - h_i}{h_s - h_i}. \tag{4}$$

Note that the barometer pressure has balanced out as well as the value of h_o in Eq. (4). But this will only be the case provided H_b and h_o remain constant during the course of the experiment. Note also that no temperature corrections on the height of the mercury columns are necessary provided the room temperature remains practically constant during the experiment. The only reason for reading the barometer, as far as Eq. (4) is concerned, is to determine the value of t_s.

Equation (3), when solved for P, gives

$$P = P_i\left(1 + \frac{P_s - P_i}{P_i\, t_s}\, t\right) = P_i(1 + \alpha_p t) \tag{5}$$

where

$$\alpha_p = \frac{P_s - P_i}{P_i t_s} = \frac{h_s - h_i}{[H_b + (h_i - h_o)]t_s}. \tag{6}$$

α_p is the temperature coefficient of pressure. Note that the barometric pressure is directly involved in Eq. (6).

Absolute zero on the centigrade scale is obtained from Eq. (3) by setting $P = 0$. This gives the value of t at absolute zero as $-1/\alpha_p$, approximately -273°C for helium. This value of absolute zero on the centigrade scale is taken as the zero of the absolute centigrade scale. Hence the relation between the centigrade temperature t and the absolute centigrade temperature T is obviously

$$T = t + \frac{1}{\alpha_p} \cong t + 273°. \tag{7}$$

On the absolute scale the pressure of an ideal gas is directly proportional to its absolute temperature as shown by Eqs. (5) and (7).

Errors: There are two determinate errors in this experiment for which it is possible to make corrections. The first of these arises because the volume of the gas is not held strictly constant during the course of the experiment, since the bulb expands or contracts with changing temperature. By a direct application of Boyle's law it is an easy matter to show that the corrected pressure P' is related to the observed pressure P by the equation

$$P' = P(1 + \gamma t) \qquad \text{(correction 1).}$$

In this equation γ is the coefficient of cubical expansion of the bulb and t is the temperature of the bulb. The reference volume is V_i, the volume of the bulb at 0°C.

The second constant error in this experiment arises because not all of the gas in the bulb is at the temperature of the bath in which the bulb is immersed. A small portion of the gas in the neck of the bulb outside the brass jacket is likely to be nearer room temperature than bath temperature. This gives an observed pressure which is too low if the bath temperature is above room temperature and too high if the contrary condition occurs. The corrected pressure P'' for this case may be worked out by use of the general gas law Let V be the total volume of the bulb of which a large part V_b is at the bath temperature (T) and a small part V_r is at room temperature T_r. Then the following equations may be written:

$$PV_b = n_bRT, \qquad PV_r = n_rRT_r, \qquad P''V = (n_b - n_r)RT.$$

In these equations n_b is the number of mols of gas at bath temperature and n_r the number of mols at room temperature. If n_b and n_r are eliminated from the third equation by use of the first two equations, then

$$P'' = P\left[1 + \frac{V_r}{V}\left(\frac{T}{T_r} - 1\right)\right] \qquad \text{(correction 2).}$$

When these two corrections are applied to all pressures that occur in the gas-thermometer equations, it may be shown that the right-hand members of Eqs. (3) and (4) should be multiplied by the factor

$$1 + \frac{P - P_s}{P_i}\left(273\gamma + \frac{273}{T_r}\frac{V_r}{V}\right); \tag{8}$$

and that the right-hand member of Eq. (6) should be multiplied by the factor

$$1 + \frac{P_s}{P_i}\left(273\gamma + \frac{273}{T_r}\frac{V_r}{V}\right). \tag{9}$$

In these factors γ is the coefficient of cubical expansion of the bulb, T_r is the absolute room temperature, and V_r/V is the fraction of the total volume of the bulb exposed to room temperature.

Let us now examine the effect on t and on α_p of making small errors in the measurement of h, h_i, and h_s. We may safely neglect the error in t_s since it is likely to be less than 0.1°C. Furthermore it is certainly not necessary to consider errors in the correction factors (8) and (9). Under these conditions the determinate-error equations corresponding to Eqs. (4) and (6) are

$$\frac{\Delta t}{t} = \frac{\Delta(h - h_i)}{h - h_i} - \frac{\Delta(h_s - h_i)}{h_s - h_i}, \tag{10}$$

and

$$\frac{\Delta\alpha_p}{\alpha_p} = \frac{\Delta(h_s - h_i)}{h_s - h_i} - \frac{\Delta[H_b - (h_i - h_o)]}{H_b - (h_i - h_o)}. \tag{11}$$

The indeterminate-error equations may be got from these in the customary manner.

Method: Adjust the manometer arms of the gas thermometer so that, at room temperature, the mercury level in the open arm is about in the middle of the scale. Since h is a linear function of t, this initial adjustment permits a range of temperature measurements extending both below and above room temperature by

about equal amounts. Read the height of the fiducial mark on the scale. Record this as h_o. All subsequent adjustments in the experiment are made leaving this arm *fixed* and adjusting the height of the open manometer arm.

Read the barometer (see Note D, Appendix II). Record the temperature at the barometer. Correct the barometer reading by use of Table F, Appendix III. Use this corrected barometer reading to determine the steam-point temperature t_s (Table D, Appendix III).

Pass steam through the jacket surrounding the glass bulb until thermal equilibrium is reached. This may take 10 or 15 min. Raise the open arm of the manometer in order to bring the mercury level up to the fiducial mark. If the mercury level stays there after this adjustment, thermal equilibrium between the bulb and steam bath has been reached; otherwise further adjustments are necessary. When a steady state is achieved, read the height of the mercury level h_s in the open arm.

Detach the boiler and pack the brass jacket with cracked ice. Bring the mercury level to the fiducial mark by lowering the open arm. When thermal equilibrium obtains, read the height of the mercury level h_i in the open arm. Remove the ice pack around the bulb by lifting out the inner bucket. Drain off any water left in the brass jacket.

CAUTION: Before proceeding to the next part of the experiment (dry-ice bath), *lower the open arm of the manometer at least* 30 *cm*. It is essential that the mercury not rise above the fiducial mark; otherwise it might enter the bulb and freeze, causing considerable delay and inconvenience. Mercury freezes at $-40°C$, a temperature much higher than that of dry ice.

Powder a quantity of dry ice, put the inner bucket in place, and pack it with dry ice. Wait several minutes for thermal equilibrium to be reached, then bring the mercury level in the closed arm up to the fiducial mark by carefully raising the open arm. Be very careful not to get the mercury much above the fiducial mark. When equilibrium is reached, determine h.

Remove the dry ice. Then slowly raise the open arm as the gas pressure in the bulb builds up. Failure to do this may force mercury out of the open arm.

Calculations: By use of Eqs. (4) and (6) calculate uncorrected values of t and α_p. Use the uncorrected barometer reading in Eq. (6). By use of (8) and (9) determine the corrections for t and α_p. Note that the values for the quantities involved in these correction factors need not be known accurately; rough estimates will suffice. Estimate the fraction of the total volume of the bulb exposed to the room. The value of γ is three times the linear coefficient of expansion of the bulb. Since pressures are proportional to absolute temperatures,

$$\frac{P - P_s}{P_i} \simeq \frac{T - 373}{273} \qquad \text{and} \qquad \frac{P_s}{P_i} \simeq \frac{373}{273}.$$

Obtain the corrected values of t and α_p. Estimate the errors involved in measuring h_o, h, h_i, h_s, and H_b. Hence calculate the indeterminate errors in t and α_p.

Record:

App. No.__________

Barometer__________ Room Temp__________

Corrected barometer__________

Height:

h_o__________ Δh_o__________ $\dfrac{V_r}{V}$__________

h_s__________ Δh_s__________

h_i__________ Δh_i__________ γ__________

h__________ Δh__________

Uncorrected	Correction factors	Corrected
t__________	__________	__________
α_p__________	__________	__________

Abs Zero__________

Δt__________

$\Delta\alpha_p$__________

Experiment 22.

Mechanical Equivalent of Heat

Object: To determine the heat equivalent of mechanical energy, Joule's constant.

Apparatus: The mechanical equivalent-of-heat apparatus as shown in Fig. 22-1 consists essentially of a conical metal cup, A, mounted in an insulated support which can be rotated by a hand crank; and a friction cone, B, that fits into cup A. A flanged wheel, C, is attached to cone B, and is weighted by a heavy metal ring, D. A cord is wound about the wheel C and supports a mass m. The friction cone B is hollow and contains water. Its temperature is determined by a thermometer, and it is kept in thermal equilibrium by stirring. The number of turns made by the conical cup is recorded on a revolution counter.

Theory: Both heat and mechanical work are forms of energy, and hence a simple relationship exists between their units of measurement: the calorie and the erg. Most physical processes consist of the transformation of one kind of energy into another; in this experiment mechanical energy will be transformed into heat. Measuring both the amount of mechanical work W and the amount of resultant heat H, the ratio between them may then be determined:

$$J = \frac{W}{H}, \tag{1}$$

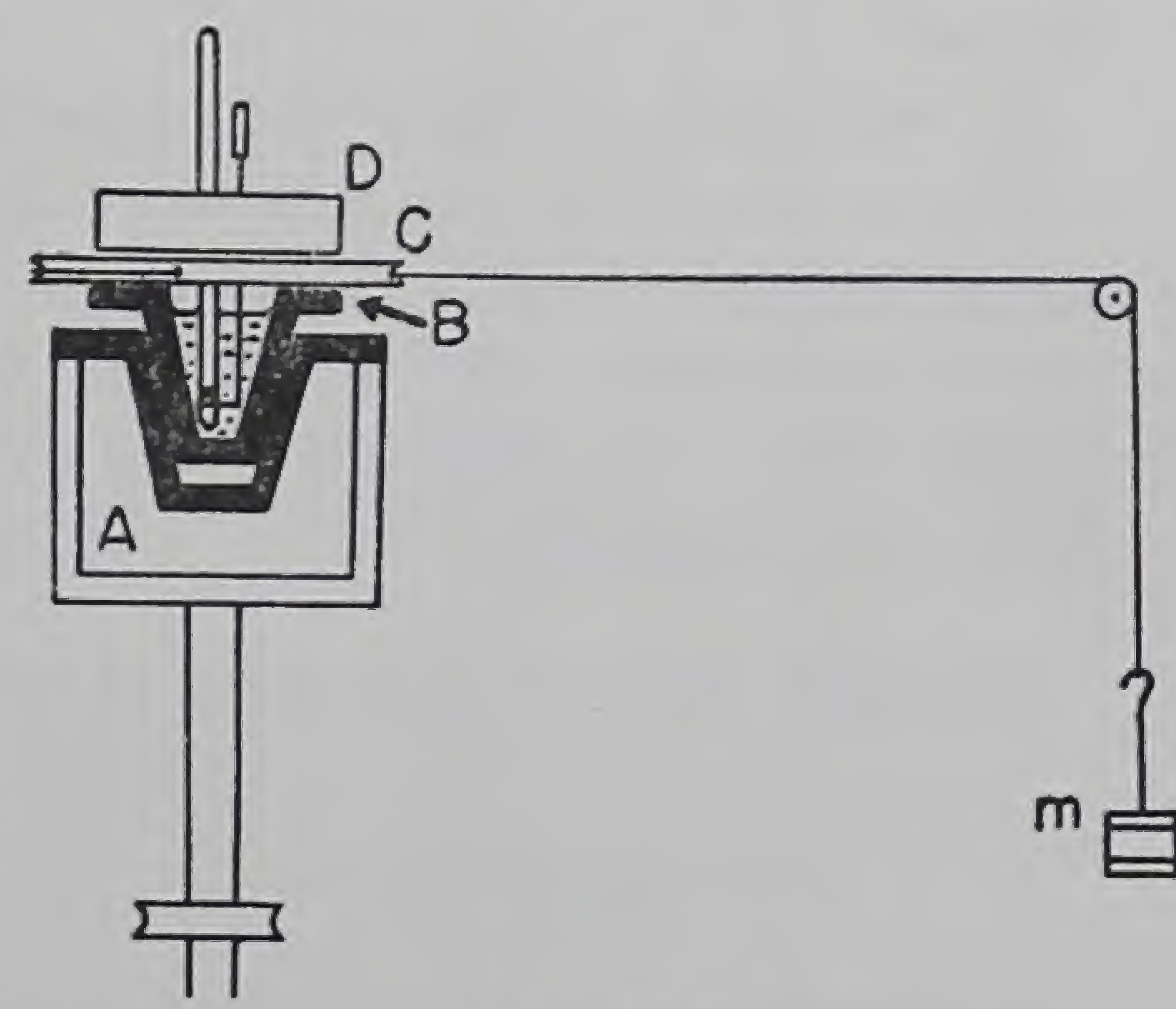


Fig. 22-1.

where J is Joule's constant, the mechanical equivalent of heat.

The weight mg exerts a torque on the friction cone B by means of the flanged wheel, C, just sufficient to prevent the cone from turning as the conical cup beneath it is rotated. It is apparent that the same effect would be obtained by holding the conical cup fixed, and allowing the mass to fall through a certain height, thus rotating the friction cone. This equivalent height is equal to the product of the circumference of the flanged wheel and the number of rotations of the friction members with respect to each other. Recalling that the work done by a falling body of mass m is given by mgh, where h is the distance of fall, the mechanical work done against the friction is evidently

$$W = mg\pi dn, \tag{2}$$

where g = acceleration due to gravity,

d = diameter of the flanged wheel (at the *bottom* of the groove), and

n = number of revolutions of the conical cup.

The amount of heat necessary to raise the temperature of the cup, friction cone, water, stirrer, and thermometer is

$$H = (M_w + Ms + e)(T_2 - T_1), \tag{3}$$

where M_w = mass of the water inside the friction cone,
M = mass of the conical cup, the friction cone, and the stirrer,
s = specific heat of the cup, cone, and stirrer,
e = water equivalent of the thermometer,
T_2 = final temperature, and
T_1 = initial temperature.

Substituting Eqs. (2) and (3) in Eq. (1), we may find the value of the mechanical equivalent of heat.

Method: Weigh the conical cup, the friction cone, and the stirrer (metal parts only). Add an amount of cold water (about 10° below room temperature) sufficient to fill the friction cone nearly to the top of the tapered portion, and weigh again to determine the mass of water added. Assemble the equipment, and make a brief trial run to determine whether the mass m is supported steadily at a constant height while turning at a reasonable speed. If not, change the amount of oil between the friction members until the adjustment is satisfactory. A fair-sized mass should be supported by the turning so as to get rapid rise in temperature, thus decreasing the errors due to radiation between the cup and its surroundings.

When the above adjustment has been satisfactorily made, the temperature should still be several degrees below room temperature. If not, replace the water with some that is colder, and wait several minutes for the cones to assume equilibrium temperature. Record the reading of the revolution counter. Then begin to stir the water steadily, and record its temperature each 30 sec for 3 min. At this time begin to turn the crank, lifting the mass to some convenient height, and thereafter turn the crank steadily so that the mass remains at just the same height. Stir the water continuously, and record its temperature every 30 sec. When the temperature has risen as much above room temperature as it started below, stop turning, but continue to stir the water and record its temperature every 30 sec for another 3 min.

Record the diameter of the bottom of the groove of the flanged wheel, the value of m, the final reading of the revolution counter, and the volume of the thermometer which is below the surface of the water. The water equivalent of the thermometer may be taken as 0.46 times the volume in cubic centimeters which is immersed.

Make a second run of this experiment.

Plot a graph of temperature versus time for each run, indicating room temperature by a dotted line. Take as temperature T_1 the point on the curve just before starting to crank; as temperature T_2, the highest point on the curve. Substitute the proper values into your equation and determine the value of J. Compare this with the accepted value of 4.185×10^7 ergs/cal. Compute the indeterminate error. Does this include the standard value?

Record: Record your data in tabular form

QUESTIONS

1. Consider the following: flanged wheel not uniform in diameter, friction in the pulley supporting mass m, friction members not as cool as water at start of turning, a few drops of water spilled during the run, mass m not held at same height during the run. Discuss in each case whether the item will cause an error in the results, whether the error is determinate or indeterminate, and whether the resulting value of J will be smaller or larger. Justify your answers.

2. Suppose that the oil between the friction members changed during the experiment resulting in a gradually changing friction. What effect on the results would this have? Discuss fully.

3. Discuss a possible method of using a spring balance instead of a weight mg to counteract the friction torque. Suppose that this apparatus were transported to the moon along with an apparatus as used in the experiment. What precautions would need to be taken in using Eq. (2) in each case?

4. Using estimated errors in each of your measured quantities, what is the indeterminate error in your result? (HINT: use $\Delta(T_2 - T_1)$ and $\Delta(M_w + Ms + e)$ rather than ΔT_2, ΔT_1, etc., in developing your error equation.)

Experiment 23.

Heat of Fusion

Object: To determine the heat of fusion of ice.

Apparatus: Ice, calorimeter, watch, thermometers, balance, paper towels. The calorimeter consists of a metal cup which may be placed inside a larger vessel consisting of an insulated water jacket which surrounds the metal cup. (See Fig. 23-1.) The purpose of the water jacket is to insulate the calorimeter cup from the effects of drafts or sudden changes in temperature of the surroundings.

Theory: The latent heat of fusion is the energy required to change a unit mass of a substance in its solid state to its liquid form, the temperature remaining constant at the melting point. When a substance makes such a change, it absorbs heat; when the reverse happens, heat is set free.

It should be noted that the heat of fusion is distinct from the specific heat of a substance involving a change in temperature.

In this experiment the latent heat of fusion of ice (the number of calories necessary to melt 1 g of ice at its melting point) is to be found.

The method of calorimetry will be used; a mass of water will be placed in a calorimeter cup and the change in its temperature as ice is added and melted will be noted. The cup is placed in the insulating jacket. In order to counteract the effects of radiation of the cup, to and from the water jacket, the water in the cup will at the beginning of the experiment be a few degrees warmer than that in the water jacket, and enough ice will be added so that at the finish, the water in the cup will be an *equal* number of degrees *cooler*. Thus on the average, equal amounts of heat will be radiated from the cup to the jacket, and from the jacket to the cup, during the experiment. These amounts, then, may be ignored.

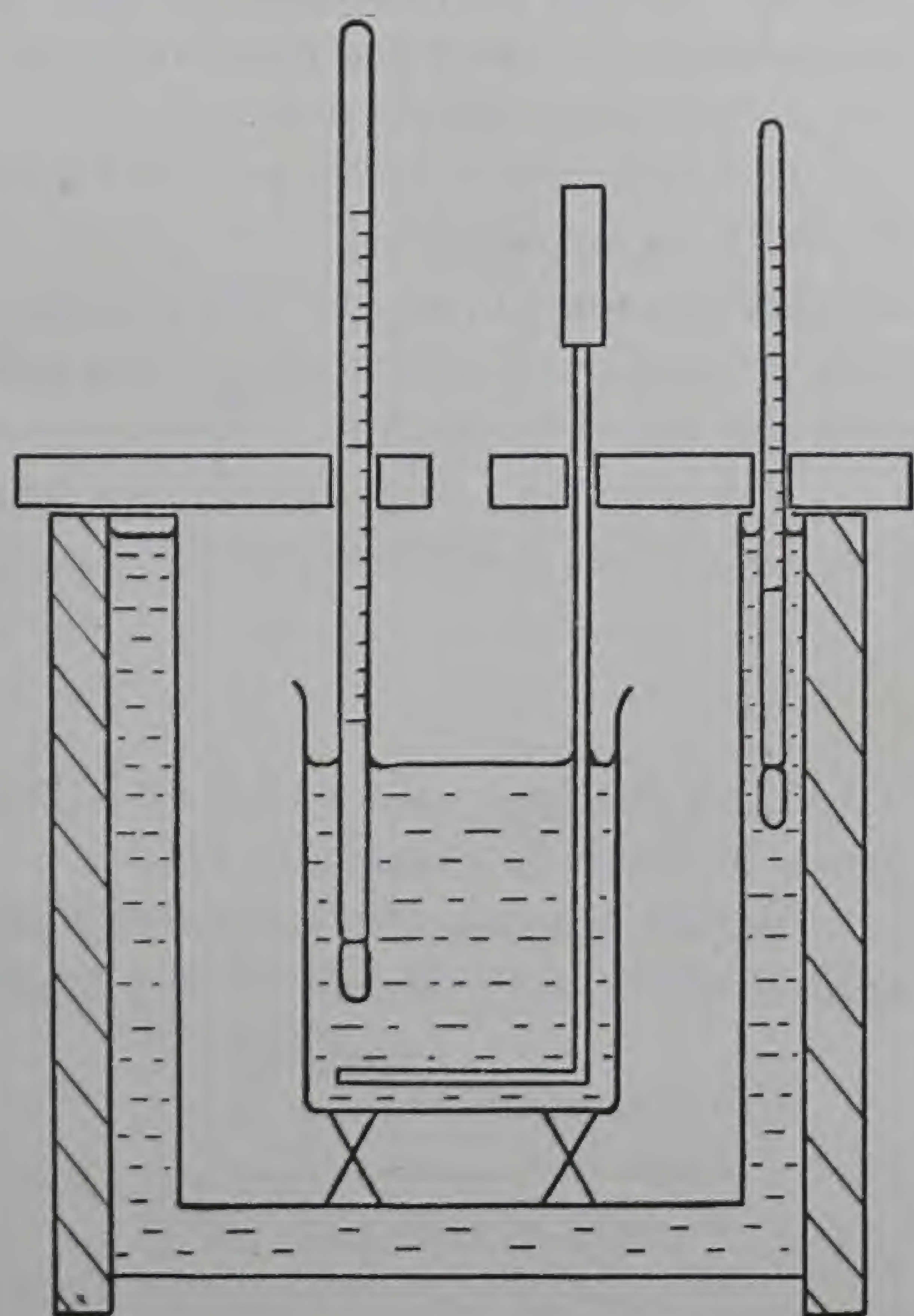


Fig. 23-1

Let M = initial mass of the water,
M' = mass of ice added,
m = mass of the calorimeter cup plus stirrer,
s = specific heat of the cup and the stirrer (Table E, Appendix III),
e = water equivalent of the thermometer (use 0.46 times the volume of the thermometer which is immersed),
t' = original temperature of the cup and the water,
t = final temperature of the whole system,
L = latent heat of fusion of ice.

Heat lost equals heat gained. Thus

$$M(t' - t) + ms(t' - t) + e(t' - t) = LM' + M'(t - 0). \tag{1}$$

From this the latent heat of fusion is

$$L = \frac{(M + ms + e)(t' - t) - M't}{M'}. \tag{2}$$

Method: Weigh the *dry* calorimeter cup and stirrer to the nearest tenth of a gram; add about 1500 g of water at a temperature approximately 5° above jacket temperature, and weigh again. Place the calorimeter inside the water jacket upon the insulating base provided; fasten the cover in place; and insert the thermometer so that it is immersed 2 or 3 cm deep in the water. Obtain a piece of ice weighing about 150 g (what will be the dimensions of such a piece?), and place it on a paper towel near the calorimeter ready for use.

Take the temperature of the water to the nearest 0.1° every 30 sec for 3 min, stirring steadily. Then remove the cover, laying it on edge so as not to break the thermometer. Wipe the piece of ice with a paper towel as dry as possible (why?) placing it in the calorimeter *without touching the ice with the fingers and without splashing*. *Note the time at which the ice is placed in the water.* Replace the cover. This entire operation must not take more than 30 sec. Continue to stir and take readings every 30 sec. Continue in this manner for 3 min after the ice has all melted as evidenced by the reaching of a minimum temperature. Remove the calorimeter cup and carefully weigh again to determine the mass of ice added. Be sure to make this final weighing under the same conditions as the initial weighings were made, using the same balance and weights.

Repeat the experiment.

Plot a curve of temperature versus time for each run. The initial portion of each curve (first 3 min) should be a straight line with a slight negative slope. In order to determine the temperature at the instant the ice was added, extend this straight line to a point corresponding to the time at which the ice was added. The temperature indicated by this point is the temperature t'. Mark this point with a special symbol and pass the curve through it. The minimum point on the curve represents the temperature t.

The error in L is primarily due to errors in the measurements of $t' - t$ and M'. Hence the determinate-error equation is approximately

$$\frac{\Delta L}{L} = \frac{\Delta(t' - t)}{t' - t} - \frac{\Delta M'}{M'}. \tag{3}$$

Find the two values of L and their errors. Average these values and compare with the accepted value given in Table L, Appendix III.

Before leaving the laboratory, empty the calorimeter cup, but not the water jacket. Wipe up any spilled water, and dispose of wet towels.

Record:

App. No.__________

	TRIAL I		TRIAL II
Weight of cup and stirrer	__________	$= m$	__________
Weight of cup and stirrer + water	__________		__________
Weight of cup and stirrer + water + ice	__________		__________
Material of cup and stirrer	__________		
Specific heat of these	__________	$= s$	
Initial mass of water	__________	$= M$	__________
Mass of ice added	__________	$= M'$	__________
Volume of thermometer immersed	__________		__________
Water equivalent of thermometer	__________	$gm = e$	__________
Jacket temperature	__________		__________
Initial temperature	__________	$= t'$	__________
Final temperature	__________	$= t$	__________
Latent heat of fusion	__________	$= L$	__________
Error in L	__________	$= \Delta L$	__________

Heat lost equals heat gained. Thus

$$M(t' - t) + ms(t' - t) + e(t' - t) = LM' + M'(t - 0). \tag{1}$$

From this the latent heat of fusion is

$$L = \frac{(M + ms + e)(t' - t) - M't}{M'}. \tag{2}$$

Method: Weigh the *dry* calorimeter cup and stirrer to the nearest tenth of a gram; add about 1500 g of water at a temperature approximately 5° above jacket temperature, and weigh again. Place the calorimeter inside the water jacket upon the insulating base provided; fasten the cover in place; and insert the thermometer so that it is immersed 2 or 3 cm deep in the water. Obtain a piece of ice weighing about 150 g (what will be the dimensions of such a piece?), and place it on a paper towel near the calorimeter ready for use.

Take the temperature of the water to the nearest 0.1° every 30 sec for 3 min, stirring steadily. Then remove the cover, laying it on edge so as not to break the thermometer. Wipe the piece of ice with a paper towel as dry as possible (why?) placing it in the calorimeter *without touching the ice with the fingers and without splashing*. *Note the time at which the ice is placed in the water.* Replace the cover. This entire operation must not take more than 30 sec. Continue to stir and take readings every 30 sec. Continue in this manner for 3 min after the ice has all melted as evidenced by the reaching of a minimum temperature. Remove the calorimeter cup and carefully weigh again to determine the mass of ice added. Be sure to make this final weighing under the same conditions as the initial weighings were made, using the same balance and weights.

Repeat the experiment.

Plot a curve of temperature versus time for each run. The initial portion of each curve (first 3 min) should be a straight line with a slight negative slope. In order to determine the temperature at the instant the ice was added, extend this straight line to a point corresponding to the time at which the ice was added. The temperature indicated by this point is the temperature t'. Mark this point with a special symbol and pass the curve through it. The minimum point on the curve represents the temperature t.

The error in L is primarily due to errors in the measurements of $t' - t$ and M'. Hence the determinate-error equation is approximately

$$\frac{\Delta L}{L} = \frac{\Delta(t' - t)}{t' - t} - \frac{\Delta M'}{M'}. \tag{3}$$

Find the two values of L and their errors. Average these values and compare with the accepted value given in Table L, Appendix III.

Before leaving the laboratory, empty the calorimeter cup, but not the water jacket. Wipe up any spilled water, and dispose of wet towels.

Record:

App. No.________

	Trial I		Trial II
Weight of cup and stirrer		$= m$	
Weight of cup and stirrer + water			
Weight of cup and stirrer + water + ice			
Material of cup and stirrer			
Specific heat of these		$= s$	
Initial mass of water		$= M$	
Mass of ice added		$= M'$	
Volume of thermometer immersed			
Water equivalent of thermometer		$gm = e$	
Jacket temperature			
Initial temperature		$= t'$	
Final temperature		$= t$	
Latent heat of fusion		$= L$	
Error in L		$= \Delta L$	

QUESTIONS

1. If there is a *constant* error of 0.5°C in the thermometer used in this experiment, what error will this introduce into the value of L? Explain.

2. If the ice is wet when placed in the calorimeter so that the mass consists of 99% ice and 1% water, what constant fractional error would be introduced into the value of L?

3. What constant fractional error in L would be introduced by neglecting to take into account the water equivalent of the thermometer? Is this error significant in this experiment?

4. Why is it necessary to be careful about the weight determinations in this experiment?

Experiment 24.

Hygrometry

Object: To determine the dew point and the relative humidity of the atmosphere.

Apparatus: Dew-point apparatus, sling psychrometer, hygrodeik.

Theory: Hygrometry is the measurement of the amount of water vapor present in a given space. According to the law of partial pressures, the total pressure exerted by a mixture of gases is equal to the sum of the individual pressures which would be exerted if each gas occupied the same volume alone. In any given volume, therefore, *the individual partial pressures are directly proportional to the amounts* (*i.e., the masses*) *of the gases present.*

The amount of water vapor which a space will contain has a maximum value which increases with increasing temperature. Introducing more vapor than this saturation value, or decreasing the temperature of a space which is already saturated, will cause the total amount of vapor to decrease, the excess being noticed in the form of condensation. The temperature at which this condensation begins is called the *dew point.*

This phenomenon of condensation is manifested almost daily in most parts of the world, in the form of dew, fog, or rain. In these cases the air is nearly saturated, and a drop in temperature, or a moist wind blowing in, causes droplets to form.

A useful concept is that of *relative humidity,* which is defined as the ratio of the mass of water vapor actually present in a given volume (its *absolute* humidity) divided by the mass required to produce saturation at that temperature. Thus a relative humidity of 100% indicates saturation conditions; further, it is clear that when a mass of air reaches its dew point, its relative humidity is 100%. Tables have been prepared showing the amount of water vapor necessary for saturation at various temperatures in terms of vapor pressures. See Table D, Appendix III.

There are two general methods of determining the relative humidity. One is the dew-point method, which depends on the fact that because of local cooling, condensation will appear on a surface which is cooler than the dew point of the atmosphere. The other is the psychrometer method which depends on the fact that the rate of evaporation of water into an atmosphere depends on the amount of water *vapor* already present. It is found that the actual vapor pressure p in millimeters of Hg is given approximately by the simple equation

$$p = p_w - 0.50(t - t_w), \tag{1}$$

where t and t_w are the dry- and wet-bulb temperatures respectively, and p_w is the saturated vapor pressure corresponding to t_w.

Method: *Part I. The Dew-point Method.* Fill the small nickel-plated container about three-quarters full of ether, benzol, or other volatile liquid, and replace the cap with the thermometer. Upon subjecting the liquid in the container to a partial vacuum, some of it evaporates, cooling the rest of the liquid and the container. Record the air temperature, t, and, at the instant that dew begins to form on the bright surface of the container, record the temperature of the metallic surface, as indicated by the inner thermometer. Nickel surfaces are provided at either side of the container, and these remain at room temperature for com-

parison purposes, so that the slightest film on the container surface will be easily seen. Release the vacuum control, and allow the container to warm up. Record the container temperature the instant the dew disappears. If the cooling is stopped as soon as dew appears, the two values of the dew-point temperature should not differ by more than 1°C. Repeat several times, and obtain the mean value of the dew-point temperature. CAUTION: Keep as far away from the container as convenient; even the faintest breath you exhale will cloud the surface. Do not confuse this effect with that of the genuine dew point. When finished, return the unused liquid to the bottle.

Since the water vapor in the atmosphere is not confined to a definite volume, its pressure will not change as the temperature is lowered. Therefore the saturation vapor pressure at the dew point will be the same as the vapor pressure in the warmer air. Another way of saying this is that until condensation actually starts, the amount of vapor present in a given mass of air at the dew point is the same as the amount in the same mass of warmer air. From Table D in Appendix III, the saturation vapor pressure at the dew point, (*i.e.*, the actual vapor pressure in the atmosphere) can be found. Further, the saturation vapor pressure at *room* temperature can be found from the same table. The ratio of these two is equal to the relative humidity.

Part II. Psychrometer Method. The apparatus consists of two thermometers mounted side by side, the bulb of one of them being covered by a cloth which is kept wet with distilled water. See Fig. 24-1. The instrument is swung about the handle so as to pass a current of air over the bulbs at a rate of about 3 m/sec (*i.e.*, a little over two swings per second). CAUTION: *Stand clear of furniture, walls, and other persons;* with proper care there can be no excuse for breakage!

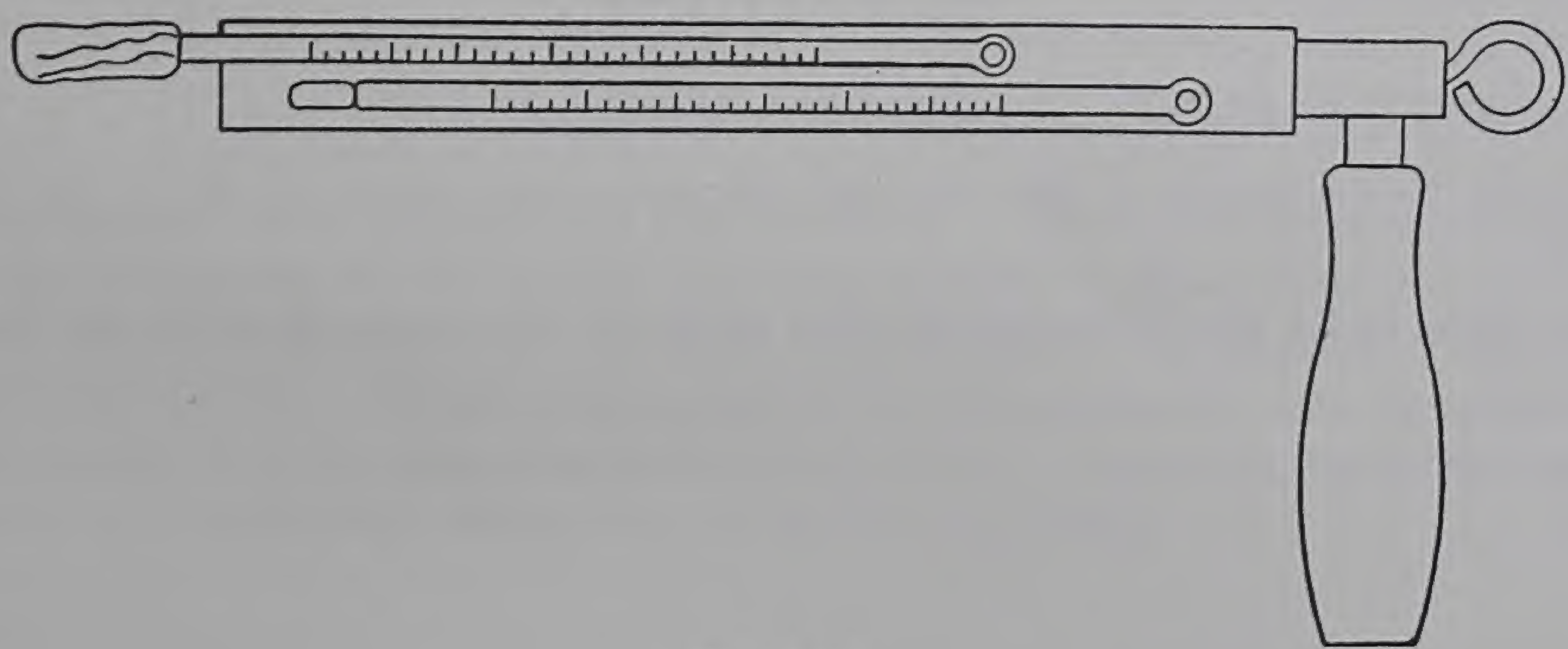


Fig. 24-1.

Saturate the cloth with distilled water, and swing the instrument. Read both thermometers, and again whirl the psychrometer. Continue in this manner until the readings become steady. Record these steady values. Calculate the actual vapor pressure by use of Table D and Eq. (1). Determine the relative humidity.

Part III. The Hygrodeik. This is a direct-reading instrument on the psychrometer principle. Directions for its use are printed upon it. Like the sling psychrometer it reads correctly only when air moves past it at 3 m/sec. Fan it vigorously until the readings reach a steady value. Record the wet- and dry-bulb readings, and use the chart and swinging arm to find the relative humidity, the absolute humidity, and the dew point.

***Record:** Part I. Dew Point.*

	Air temp, t	Dew appears	Dew disappears
Average			

Vapor pressure__________
Saturated vp__________
Rel Humidity__________

Part II. Sling Psychrometer.

	Wet bulb	Dry bulb
Wet and dry readings		
Average readings		
Difference		
Actual vapor pressure		
Sat vp		
Rel humidity		

Part III. Hygrodeik.

Wet bulb________
Dry bulb________
Rel humid.________
Abs humid.________
Dew point________

QUESTIONS

1. Suppose Part II of this experiment is performed in a perfectly dry room ($p = 0$). What will be the reading of the wet-bulb thermometer if the dry-bulb thermometer reads (a) 20°C, (b) 38°C? HINT: Use Eq. (1) and Table D.

2. What would have to be the temperature of a perfectly dry room ($p = 0$) for the value of t_w to be 0°C?

Instructions for Electricity Laboratory

The General Instructions, Section D of the Introduction, hold equally for all experiments in the course. In addition, for electricity laboratories, the following points should be borne in mind:

The equipment in electricity laboratories is in general more subject to injury than that used in other parts of the laboratory course. A very small mistake in procedure can result in severe damage. *Therefore, the source of electric power is never to be connected into the circuit until the instructor has checked the wiring.* When the source of power is a wall outlet, the plug is not to be inserted until the inspection has been made. If the power source is a battery (and this includes standard cells) neither wire is to be connected to the terminals before the inspection. Following this rule rigorously will prevent damage to equipment in most cases.

Most items of equipment have their ratings shown clearly. These ratings must never be exceeded. In the case of meters, the ratings are shown by the full-scale readings. Rheostats have the maximum current marked on the name plates or stamped on the ends of their mounting boards. Other instruments have ratings shown on the name plates. When using a piece of equipment for the first time, look for these ratings. Read the notes in Appendix II if the apparatus is described there. Above all, be as well informed as possible about the experiment being performed before coming to the laboratory.

Experiment 30.

Electric and Magnetic Fields

Object: (1) To map the equipotential lines of an electric field and hence determine the electric lines of force. (2) To map the magnetic lines of force of a magnetic field and hence determine the traces of the equipotential surfaces.

Apparatus: Electric-field apparatus consisting of battery, leads, electrodes, sensitized conducting paper, probes, and table galvanometer. Bar magnet, small magnetic compass, board, and paper.

Theory: *Fields of Force.* A field of force is simply a region in which forces exist. These forces may be electrical, magnetic, or gravitational in character. The strength or intensity of the field (electric, magnetic, or gravitational) at any point is, by definition, the force in dynes acting upon a unit quantity (of positive charge, of north pole, or of mass). For example, the strength of an electric field at a given point in space is said to be six units if there is a force of 6 dynes acting upon a unit positive charge placed at that point. Since force is a vector quantity having direction as well as magnitude, it is clear that a field of force also has direction as well as magnitude at any point. Its direction is just the direction of the force exerted upon the unit charge, pole, or mass.

It is convenient to graphically represent a field of force by means of lines of force. A line of force is simply the path along which the unit quantity (positive charge, north pole, mass) would move if it were constrained to move with a very small velocity. These lines of force essentially give the direction of the field at every point in space. It is customary to imagine as many lines of force drawn through any unit area (square centimeter) taken perpendicular to the lines as there are dynes of force acting upon the unit quantity at that point. Thus, in the example given in the preceding paragraph, there were 6 dynes of force acting upon the unit positive charge at a certain point in the field. In this case the electric field around this point would be represented by six lines per square centimeter drawn in the direction of the force. By this artifice one can represent both the magnitude and the direction of the field at any point in space.

With these conventions it is clear that a uniform field is represented by a set of parallel lines of force. A convergent set of lines of force represents a field of increasing strength; a divergent set of lines represents a field of decreasing strength. It is important, of course, that the student recognize the conventional character of lines of force. They have no objective reality.

Potential. Two different points in a force field are said to have a difference of potential (1) if work is required to carry the unit quantity (positive charge, north pole, mass) from the one point to the other, and (2) if this work is independent of the path between the two points. The fields produced by charged bodies, permanent magnets, gravitational masses, etc. have this character. Under these circumstances the term potential difference (P.D.) means the amount of work in ergs required to carry our unit quantity between the two points under consideration.

If some one point in the field is chosen as a base point or point of zero potential (frequently the point at infinity), then the P.D. between this point and any other point in the field may be determined. This P.D. is simply called the potential of the other point. Thus the potential at any point in the field is, by definition, the amount of work that must be done on the unit quantity in order to carry it from the base point to the point in question.

Equipotential Surfaces. An equipotential surface is the locus of a set of points all of which have the same potential. From this definition it is evident that our unit quantity could be moved over such a surface without having any work done on it. But in order for this to occur it is necessary and sufficient that this surface be everywhere perpendicular to the lines of force in the field. Thus, from a geometrical point of view, the equipotential surfaces in a force field are a set of surfaces which are everywhere perpendicular to the lines of force of the field. It is possible, therefore, to describe a field of force either in terms of a set of lines of force or in terms of a set of equipotential surfaces. In the case of an electric field it is generally easier in the laboratory to determine the equipotential surfaces than it is to determine the lines of force. In the case of the magnetic field just the opposite is true.

Potential of a Conductor. If an electrical conductor, charged or uncharged, is placed in an electric field, the free charges within it will move under the action of this field until all points in the conductor reach the same potential. When this occurs further motion of the free charges will cease because then there will be no net electric field inside the conductor. This reasoning holds for hollow as well as solid conductors. Hence it is a usual laboratory practice to screen delicate electrostatic instruments from external electric fields by surrounding them with a wire screen or gauze connected to ground.

Since all points of a conductor have the same potential when it is in electrical equilibrium, it is quite customary to talk of the potential of a conductor. Furthermore, it is clear that the surface of such a conductor must be an equipotential surface. This means, of course, that the electric lines of force always leave or enter the conductor at right angles to its surface.

These conclusions cannot be applied to a magnetized body since magnetic conduction does not exist, *i.e.*, there is no transfer of magnetic charge.

Mapping Lines of Force and Equipotential Surfaces. As an example we show in Fig. 30-1 the traces of the lines of force and equipotential surfaces for two equally charged conducting spheres A and B whose centers are in the plane of the paper. Notice that no lines of force appear inside either conducting sphere and that the lines of force leave both spheres at right angles to their surfaces. Also note that the traces of the equipotential surfaces (dotted) are everywhere perpendicular to the lines of force and that the spherical surfaces of the two conductors are equipotential surfaces.

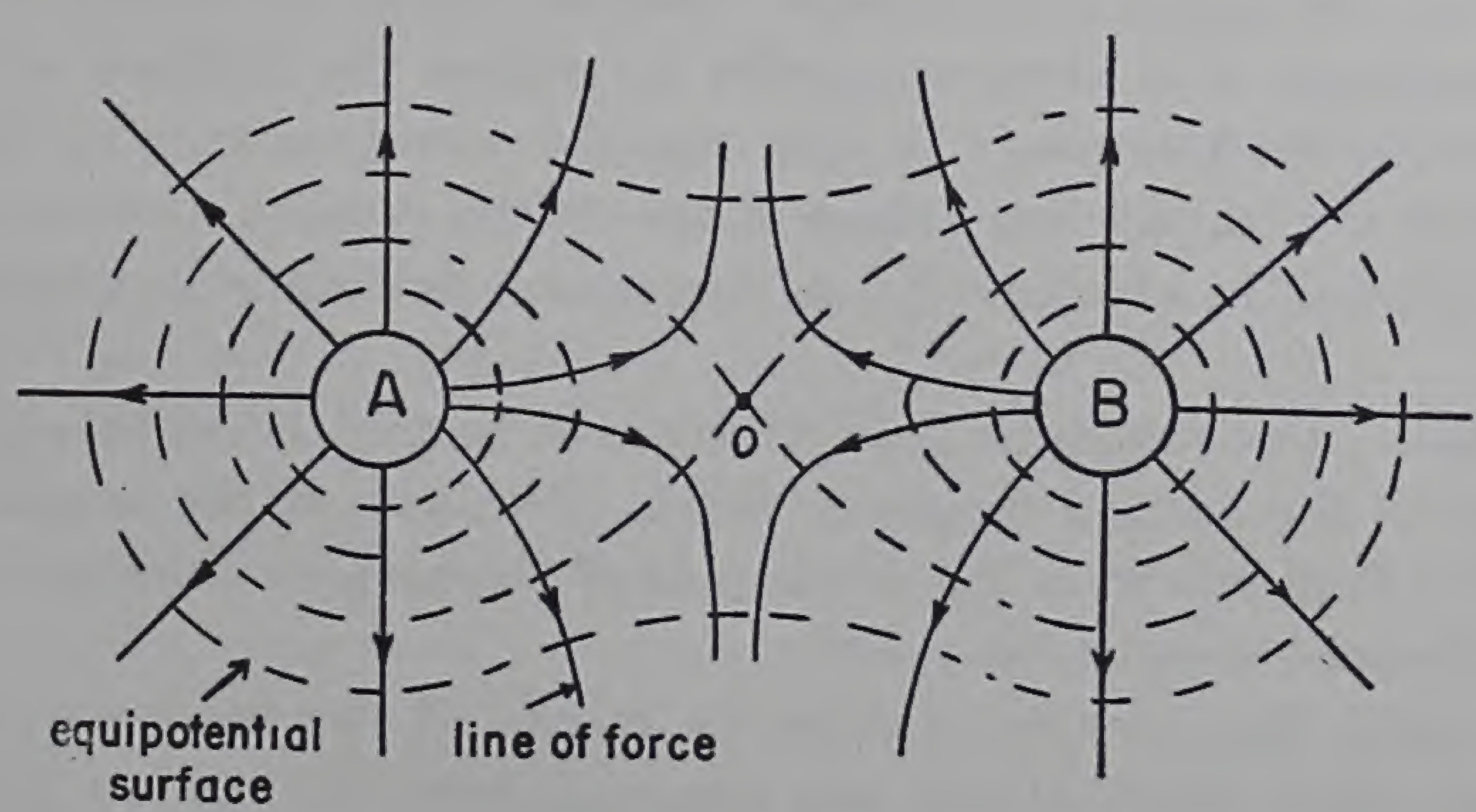


Fig. 30-1.

The point O halfway between the centers of the two spheres is a critical (neutral) point at which point the intensity of the electric field is zero, *i.e.*, a unit + charge placed there would have zero electrical force acting upon it and, therefore, would be in equilibrium. This equilibrium would be stable as far as motion along the line AB is concerned but would be unstable for motion perpendicular to the line AB.

***Method:** Part I. Electric Field.* The apparatus in this part of the experiment consists essentially of a piece of paper coated with colloidal graphite and mounted on the underside of a board. Two electrodes are put into contact with this paper at opposite ends of the paper. When a voltage is applied across these electrodes by means of a battery, charges flow between the electrodes across the resistance paper along the lines of force of the electrical field established between the electrodes. Two metal probes attached to a

galvanometer may be put into contact with the paper at any two points on the paper. If these two points do not lie on the same equipotential line, a deflection of the galvanometer will occur; whereas, if they do, no galvanometer deflection will occur. An equipotential line may then be determined by fixing the position of one of the probes, and moving the other in such manner that no galvanometer deflection results. The second probe under these conditions traces out an equipotential line. By moving the first probe to a new position on the paper and repeating the procedure, a second equipotential line may be determined, etc.

In order to record the positions of the equipotential lines, a piece of ordinary graph paper is mounted on the top surface of the board. Dummy electrodes and dummy probes are used on this paper which match the position of those on the resistance paper. Hence the equipotential lines on the resistance paper may be directly transferred to the graph paper.

Clamp a set of electrodes and their dummies to the board and connect the electrodes to a 6-volt battery through a knife switch. Connect the probes to the galvanometer. Outline the dummy electrodes on the graph paper. Clamp one of the probes near one of the electrodes. Determine an equipotential line by moving the other probe in such a way that no galvanometer deflection occurs. Instead of attempting to get a continuous trace of this line, it is better to determine a set of points on this line by recording with small circles on the graph paper the successive positions of the second probe which give zero galvanometer deflections.

When a sufficient set of points (five to ten) have been determined to locate one equipotential line, move the first probe to a new position farther away from the electrode and repeat the process. Continue until about 11 equipotential lines have been determined. These lines should be equally distributed over the entire surface of the paper. Special care should be exercised in getting the equipotential lines in the neighborhood of the electrodes.

Remove the graph paper and the first set of electrodes. Replace them with a second sheet of graph paper and a second set of electrodes of different shape from the first set. Determine the equipotential points for this second electric field by the method given above.

Draw smooth curves through the points of equal potential on each sheet of the graph paper. These curves are the traces of the equipotential surfaces for each field. With a red pencil draw a set of lines (lines of force) everywhere perpendicular to the equipotential lines. These lines of force should start on the positive electrode and end on the negative electrode. Indicate their directions by means of arrows.

In this part of the experiment it is always possible to replace the battery with an audio-oscillator and the galvanometer with a pair of headphones. In this case the sound emitted by the headphones will vanish when the two probes to which the phones are attached are on the same equipotential surface. The absence of sound from the phones replaces the absence of a galvanometer deflection as a criterion for equal potentials.

Part II. Magnetic Field. The magnetic field which is to be plotted in this experiment is that due to a permanent bar magnet and to the earth's magnetic field. The axis of the magnet is to be placed parallel to the direction of the earth's field but with the north pole of the magnet directed toward the south, and the south pole of the magnet directed toward the north.

Determine the north-south line at your station by means of the magnetic compass needle. Be sure that the bar magnet and all other pieces of iron are removed from the station in this process. Then place the board, with the permanent magnet in the center of the board, in such a position at your station that the north pole of the magnet is directed toward the south. Cover the board with a large piece of paper. Outline the position of the permanent magnet under the paper and indicate its polarity and the direction of the north-south line on this paper. Place the compass near the north pole of the magnet and make dots as near each end of the needle as possible and in line with it. Then move the compass in the direction in which its north pole points until the south pole is above the dot previously made at the north pole. Make another dot at the north pole of the compass in this new position. Continue in this fashion until the series of dots leads to the south pole of the bar magnet, or to the edge of the paper. Draw a smooth curve through the points and indicate, by arrows, the direction of the field. It is clear that this line is a line of force of the magnetic field. In a similar manner trace other lines of force until the field is clearly represented on all sides of the bar magnet. It is not necessary to start in each case from the north pole of the magnet. It is just as well to start from the south pole or from some point away from the magnet.

Two regions will probably be found where the direction of the compass needle is indeterminate—points of neutral or critical equilibrium. These points are called neutral points because the earth's field at these points is just balanced by the field of the bar magnet. The region in the neighborhood of these points should be mapped with considerable care. Since the field in this neighborhood is very weak, it may be necessary to gently tap the compass each time a direction is taken in order to overcome the effect of friction at the pivot of the compass.

After you have mapped the lines of force of the magnetic field (black pencil), draw in the equipotential lines (red pencil). This may be done outside of the laboratory period.

Record: The record of this experiment will consist of:

(a) The graphs of the two electric fields and the one magnetic field properly titled and labeled.
(b) A schematic drawing of the electric-field apparatus.
(c) Answers to the questions given below.

QUESTIONS

1. Show that equipotential surfaces always cut lines of force at right angles in any field of force.

2. Can two different lines of force or two different equipotential surfaces ever cross each other? Explain.

3. Are there any neutral or critical points in the electric fields such as are present in the magnetic field in this experiment?

4. Would it be possible to produce a field such as that shown in Fig. 30-1 by means of the electric-field apparatus that you used in this experiment? If so, how would you do it?

5. It may be shown that the strength of the field produced by a bar magnet at a point on its extended axis a distance r from the *center* of the magnet is approximately $2M/r^3$ provided r is fairly large compared to the length of the magnet. M is the magnetic moment of the magnet. At either one of the neutral points in Part II of this experiment $2M/r^3$ must just be equal and opposite to the strength of the earth's horizontal magnetic field. Taking the value of this field as 0.17 oersted, calculate the magnetic moment M of the magnet used in this experiment.

Experiment 31.

Condenser Capacitance

Object: To determine the capacitance of two condensers, when taken singly and when connected in parallel and in series, by the comparison-of-deflection method with a ballistic galvanometer.

Apparatus: Ballistic galvanometer, damping key, two paper condensers, one standard mica condenser, two double-pole double-throw switches, dry cell. For a description of some of these items see Appendix II, Sections G1, 4; H1, 3, 4.

Theory: The capacitance C of a condenser is defined as the ratio of its charge Q to the potential difference V between its plates, that is,

$$C = \frac{Q}{V}. \tag{1}$$

When Q is expressed in coulombs and V is expressed in volts, then C is given in farads. A capacitance of 1 farad is *extremely* large. It is customary, therefore, to express capacitance in microfarads, *i.e.*, millionths of a farad.

A simple and direct method of comparing the capacitances of two condensers is to compare their charges when both have the same potential difference across their plates. Suppose, for example, that a condenser of unknown capacitance C_x is charged to a potential V thus accumulating a charge Q_x. Similarly, a standard condenser of known capacitance C_o is charged to the same potential V, thus accumulating a charge Q_o. Then from Eq. (1) we have

$$\frac{C_x}{C_o} = \frac{Q_x}{Q_o}. \tag{2}$$

If now each condenser is discharged through a ballistic galvanometer, the two deflections (or throws) obtained will be in the same ratio as Q_x and Q_o. See Appendix II-H4 for the action of a ballistic galvanometer. Thus Eq. (2) may be replaced by the equation,

$$\frac{C_x}{C_o} = \frac{D_x}{D_o}, \tag{3}$$

where D_x and D_o are the two deflections. Hence in order to determine C_x it is only necessary to observe the deflections D_x and D_o and to know the value of C_o.

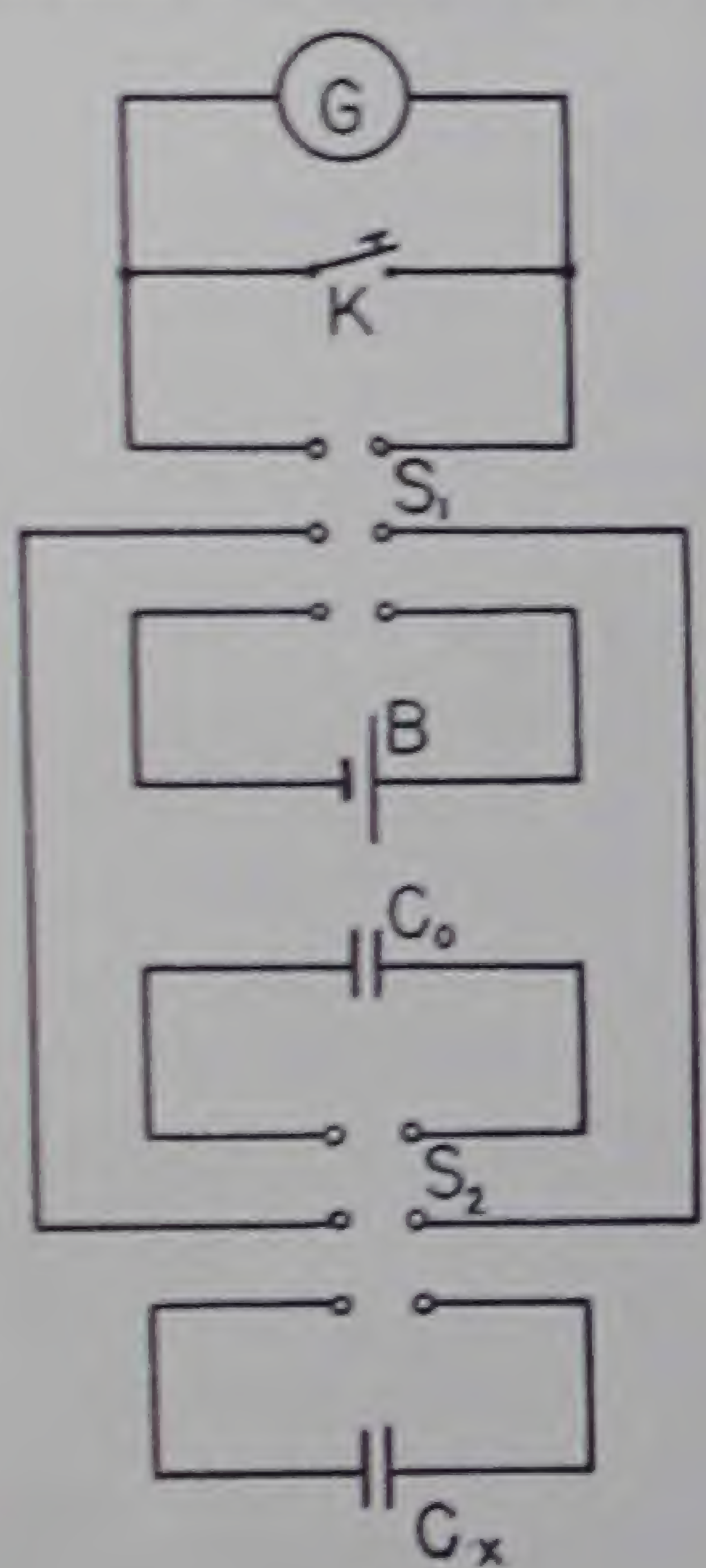


Fig. 31-1.

A satisfactory setup for determining C_x, as shown schematically in Fig. 31-1, consists of a galvanometer G, damping key K, battery B, condensers C_o and C_x, and double-pole switches S_1 and S_2. By appropriate manipulation of S_1 and S_2 either condenser may be charged to battery emf (assumed constant) and then discharged through galvanometer G.

Parallel and Series Connections. Two condensers of capacitances C_1 and C_2 may be connected in parallel as shown in Fig. 31-2. The combined capacitance C_p of this parallel combination is

$$C_p = C_1 + C_2 \text{ (parallel)}, \tag{4}$$

since the charges are added but the potentials are the same.

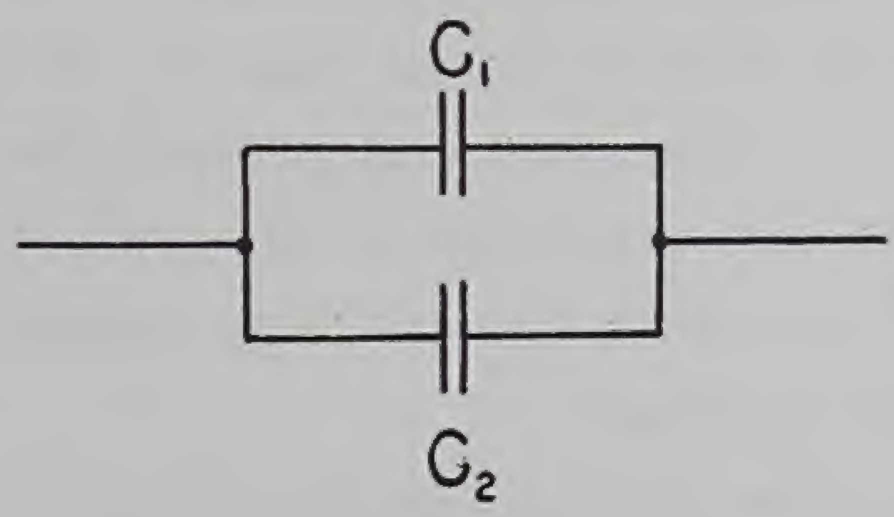


Fig. 31-2.

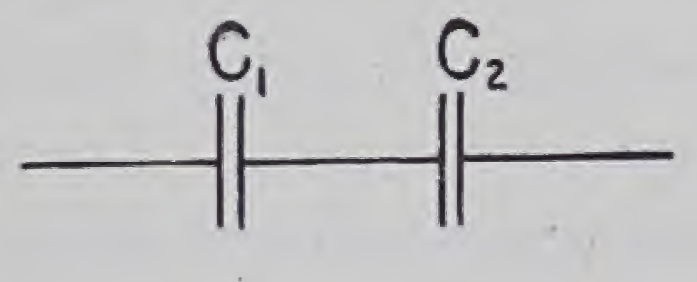


Fig. 31-3.

Two condensers may also be connected in series as shown in Fig. 31-3. The combined capacitance of this series combination is given by the equation

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{(series)}, \tag{5}$$

since the potentials are added but the charges are the same.

The determinate-error equations corresponding to Eqs. (3), (4), and (5), respectively, are:

$$\frac{\Delta C_x}{C_x} = \frac{\Delta C_o}{C_o} + \frac{\Delta D_x}{D_x} - \frac{\Delta D_o}{D_o}, \tag{3a}$$

$$\Delta C_p = \Delta C_1 + \Delta C_2, \tag{4a}$$

and

$$\frac{\Delta C_s}{C_s} = \frac{C_2\,\Delta C_1}{(C_1 + C_2)C_1} + \frac{C_1\,\Delta C_2}{(C_1 + C_2)C_2}. \tag{5a}$$

Time of Charging. When a condenser is connected across the terminals of a battery, the charge Q on its plates at any time t is given by the relation,

$$Q = Q_{\max}(1 - e^{-t/RC}). \tag{6}$$

This relation indicates that full charge is only obtained after an infinite amount of time. However at the time $t = RC$ the condenser is about 63% charged. If t is 10 times this value, we may assume that the condenser is fully charged. The value RC is known as the *time constant* of the circuit. Here R is the resistance in the circuit and C is the capacitance. Generally RC is quite small, *e.g.*, 10^{-6} sec, hence instantaneous charging takes place for all practical purposes. Even if R is very large (10^6 ohms) the time constant of a 1-μf condenser is only 1 sec. Thus one is generally safe in assuming that after 10 sec such a condenser connected across a battery will be fully charged.

Method: Before attempting to perform this experiment the student should carefully read Appendix II, Notes F; H1, 3, 4 concerning the type of galvanometer used.

Make the connections as shown in Fig. 31-1 but *do not connect* the cell until the instructor has checked your wiring. Use either one of the unknown condensers as C_x and the standard 0.5-μf condenser as the known C_o. With both switches S_1 and S_2 open, adjust the reading telescope and scale on the galvanometer so that the cross hairs of the telescope coincide with the zero of the scale. Excessive motion of the galvanometer coil may be stopped by closing the damping key K.

Connect C_o into the circuit by means of switch S_2 and the cell into the circuit by means of switch S_1. The cell is now connected across C_o thus charging it. Allow this charging process to go on for about 10 sec. Sometimes a high resistance is connected in series with the cell in order to protect the galvanometer in case the cell is accidentally connected across the galvanometer. This may prolong the time required for completely charging the condenser.

Check the galvanometer to see that it gives a steady zero reading. Then reverse switch S_1 so that C_o

is now connected across the galvanometer G. Observe the maximum deflection of the galvanometer in millimeters, estimating to a half millimeter. Repeat this process of charging and discharging C_o using a longer charging time, say 20 sec. If there is no appreciable increase in the maximum deflection for the longer charging period of time, then the shorter charging period is sufficient and may be used in the remainder of the experiment. If, however, there is an appreciable increase in deflection for the longer period, it will be necessary to still further increase the charging period in order to find the proper minimum time.

After observing and recording the deflection D_o for C_o, reverse switch S_2 so that C_x is in the circuit. Charge and discharge C_x using the same charging period as for C_o. Observe and record the deflection D_x for C_x. Then go back to C_o. Proceed in this manner taking *alternate* readings of D_o and D_x until five D_o and four D_x readings have been taken. Record these readings and compute the average values of D_o and D_x. Estimate the error in each of these averages and record it with the average.

Determine by means of Eq. (3) the unknown capacitance C_x; also determine the error in C_x. Assume that the error in C_o may amount to as much as $\pm 1\%$ of the rated value.

Replace C_x by the other unknown condenser and repeat the above procedure for determining its capacitance.

Connect the two unknown condensers in *parallel* and insert this combination into the circuit as C_x. Determine the capacitance of this combination and compare it with the value calculated by Eq. (4). In each case calculate the errors involved.

Finally connect the two unknown condensers in *series* and insert this combination into the circuit as C_x. Proceed as above in the determination of capacitance.

Record: Give apparatus numbers of galvanometer, standard condenser, and unknown condensers. Tabulate your data. Express your results in the following form:

Item	C_x	% error
Condenser No. 1		
Condenser No. 2		
Parallel (exp)		
Parallel (calc)		
Series (exp)		
Series (calc)		

QUESTIONS

1. Develop the indeterminate-error equations for this experiment by use of the rules given in Section A4 of the Introduction. How do they differ from Eqs. (3a), (4a), and (5a)?

2. If the galvanometer deflections in this experiment are too small for accurate measurement, how may they be increased without using a more sensitive galvanometer?

Experiment 32.

Joule's Law

Object: To determine the heat equivalent of electrical *energy*, Joule's constant.

Apparatus: Calorimeter, heating coil (about 1-ohm 20 nichrome wire), clock, thermometer, ammeter (#0 to 5 d.c.), 45-ohm rheostat with switch, dropping resistance (600-watt heating element), insulating board, source of 115 volt d.c. See Appendix II, Sections J2; G1, 3 on the ammeter and rheostat.

Theory: Energy exists in many different forms, and physical processes usually involve a conversion of one form of energy into another form. In this experiment electrical energy is converted into heat energy, and we wish to determine the number of joules of electrical energy that will produce 1 cal of heat energy. If W represents the electrical energy, expressed in joules for example, and if H represents the heat energy produced, expressed in calories, then the ratio of W to H is constant. This constant is represented by the symbol J and is frequently called Joule's constant, or the mechanical equivalent of heat. In equation form

$$J = \frac{W}{H}. \tag{1}$$

In order to determine J it is necessary to measure W and H.

The amount of electrical energy W which is dissipated in the form of heat by a current I in a resistance R during a time t is given by the equation

$$W = VIt, \tag{2}$$

where V is the potential drop across the resistance. By Ohm's law, $V = IR$. We may substitute for V in Eq. (2) and get

$$W = I^2Rt. \tag{3}$$

If I is measured in amperes, R in ohms, and t in seconds, then Eq. (3) will give W in joules.

Suppose that the heat generated in this resistance (heating coil) is measured calorimetrically by immersing the coil in a calorimeter cup filled with water. As a result the temperature of the calorimeter cup and its contents will rise. This temperature rise may be noted on a thermometer immersed in the water and, hence, the amount of heat H given to the calorimeter may be computed.

An appropriate setup for this experiment is shown in Fig. 32-1. It consists essentially of a calorimeter with stirrer and thermometer. The calorimeter cup is partially filled with water. The heating coil is immersed in the water as shown. This coil is connected through ammeter A, switch Sw, dropping resistance R_D, and rheostat Rh to the power supply (115 volts d.c.).

When the circuit is closed electrical energy in the heating coil is converted into heat energy absorbed by the calorimeter. The amount of heat absorbed by the calorimeter is given by the equation

$$H = (M_w + M_cS + M_E)(T_2 - T_1), \tag{4}$$

where H = heat in calories,
M_w = mass of water in grams,
M_c = mass of calorimeter cup in grams,
S = specific heat of cup,
M_E = water equivalent in grams of heating coil, stirrer, and thermometer, and
$T_2 - T_1$ = rise in temperature in °C of calorimeter.

The quantities on the right side of Eq. (4) may be measured and, thus, H may be determined.

If we substitute the values of W [Eq. (3)] and H [Eq. (4)] in Eq. (1), we get

$$J = \frac{I^2Rt}{(M_w + M_cS + M_E)(T_2 - T_1)}. \tag{5}$$

The principal errors in this experiment occur in the measurements of I and $T_2 - T_1$. The *determinate-error* equation involving these quantities is

$$\frac{\Delta J}{J} = \frac{2\,\Delta I}{I} - \frac{\Delta T_2}{T_2 - T_1} + \frac{\Delta T_1}{T_2 - T_1}. \tag{5a}$$

The errors in the masses, the resistances, and the time are negligible compared to the other errors in this experiment. In addition there is the error, ever present in calorimeter experiments, introduced by heat

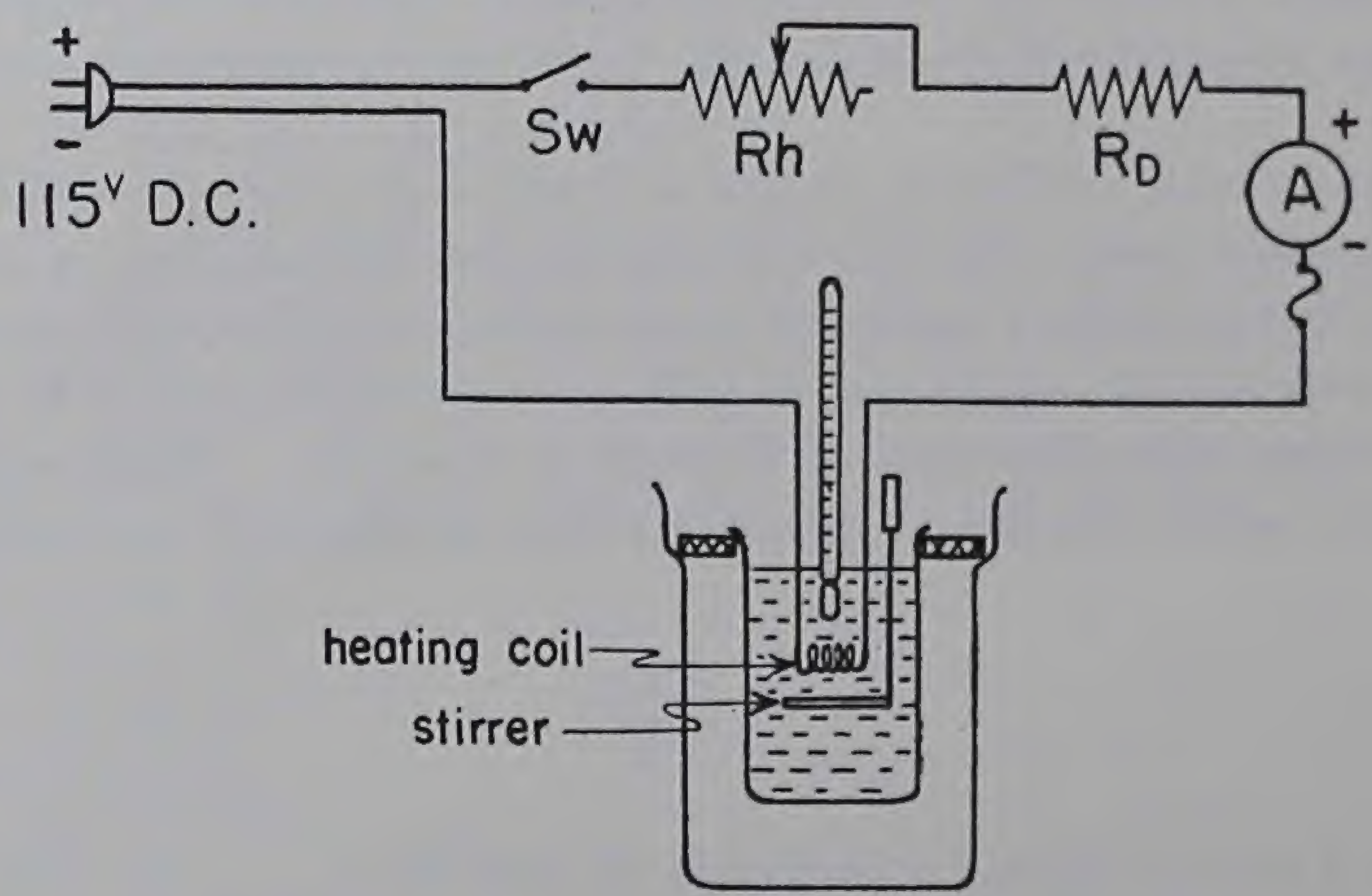


Fig. 32-1.

exchange between the calorimeter and its surroundings during the course of the experiment. Since this heat exchange is generally proportional to the time, it is advisable to reduce the time of an experimental run to a minimum. Also it is advisable to have the initial temperature T_1 of the calorimeter as much below room temperature as the final temperature T_2 is above room temperature. In this way the net heat exchange between the calorimeter and its surroundings during the experiment may be reduced to a minimum.

Method: Connect the apparatus as shown in Fig. 32-1 but do not plug in on the power line (115 volts d.c.) until the instructor has checked your connections.

Weigh the calorimeter cup when empty and dry. Add cold water from the tap until the cup is about three-quarters full, and weigh again. The water should be 3° or 4° below room temperature. Place the calorimeter in the metal jacket and insert the heating coil, stirrer, and thermometer. Handle the thermometer with great care. It is easily broken and hard to replace. Close the switch and *quickly* adjust the current by means of the rheostat to a value between 3 and 4 amp, limited however by the current rating of the rheostat. Open the switch.

Place the insulating board in such a position that it shields the calorimeter from the heat radiated by the rheostat and the dropping resistance.

Start stirring the water and after 30 sec read the thermometer to the nearest 0.1°. Start your time and temperature measurements from this point. Read the thermometer every 30 sec thereafter while *stirring continuously*. At the beginning of the third minute close the switch Sw and read the ammeter as well as the thermometer. It is necessary to keep the current constant at this value during the run by continual

adjustment of the rheostat. One student should perform this adjustment and record all data while the other stirs and reads the clock and the thermometer. The temperature will now begin to rise rapidly because of the heat supplied by the coil. Stir continuously and record readings on both ammeter and thermometer every 30 sec. When the temperature of the calorimeter has risen to a value approximately as much above room temperature as its initial temperature was below, open the switch *immediately* after reading the ammeter and thermometer. Continue to stir and to take temperature readings for an additional 2 min.

Plot a time (abscissa) versus temperature (ordinate) curve. Indicate on this curve, along with the usual items, the room temperature and the exact times at which the current was turned on and turned off. The interval in seconds between these two times is the value of t in Eq. (5). For I use the average ammeter reading during the run. For T_1 use the temperature of the calorimeter at the instant the current was turned on. For T_2 use the maximum temperature on the time-temperature curve. The resistance R of the coil is given. The water equivalent of the thermometer, heating coil, and stirrer may be taken as 6 gm.

Compute by means of Eq. (5) the value of J in joules per calorie. Also compute the *indeterminate* error in J by use of Eq. (5a). Assume that the error in the ammeter may be $\pm 1.5\%$ of full-scale reading and that the errors in T_1 and T_2 may amount to $\pm 0.05°C$. Compare your value of J with the commonly accepted value 4.18 joules/cal.

Make a second run of this experiment.

Precautions:

1. Handle the thermometer with care.
2. Do not turn on the current unless the heating coil is immersed in water. It may burn out otherwise.

Record:

App. No. Calorimeter________________
Ammeter________________
Heating coil________________

Resistance of heating coil:	R = (	) ohm
Mass of calorimeter empty:	M_c = (	) gm
Mass of calorimeter + water:	(	) gm
Mass of water:	M_w = (	) gm
Specific heat of calorimeter:	S = (	)
Water equiv of coil, etc.:	M_E = 6.0 gm	
Room temperature:	(	) °C

Tabulate your time, temperature, and current data.

From temperature vs. time curve:

Time of heating:	t = (	) sec
Initial temp of water:	T_1 = (	) °C
Final temp of water:	T_2 = (	) °C
Experimental value of J:	J = (	) joule/cal
	ΔJ = (	) joule/cal
Accepted value of J		4.18 joule/cal
% difference	(	)

QUESTIONS

1. What constant percentage error would be introduced into your value of J by failing to take into account the water equivalent of the thermometer, heating coil, and stirrer?

2. If the thermometer you used had a constant error of 0.5°C, what error would this introduce in your value of J?

3. What is the purpose, if any, of plotting a heating curve in this experiment? Would it not suffice to take only two temperature readings?

4. If the resistance of the heating coil was not known, how could a voltmeter be used in this experiment to complete the data? What change in equation (5) would result?

Experiment 33.

Electrolysis

Object: To determine the electrochemical equivalent of copper by means of a coulometer.

Apparatus: Copper coulometer, ammeter, dropping resistance, rheostat, source of d.c., balance, heater. The circuit is set up as shown in Fig. 33-1.

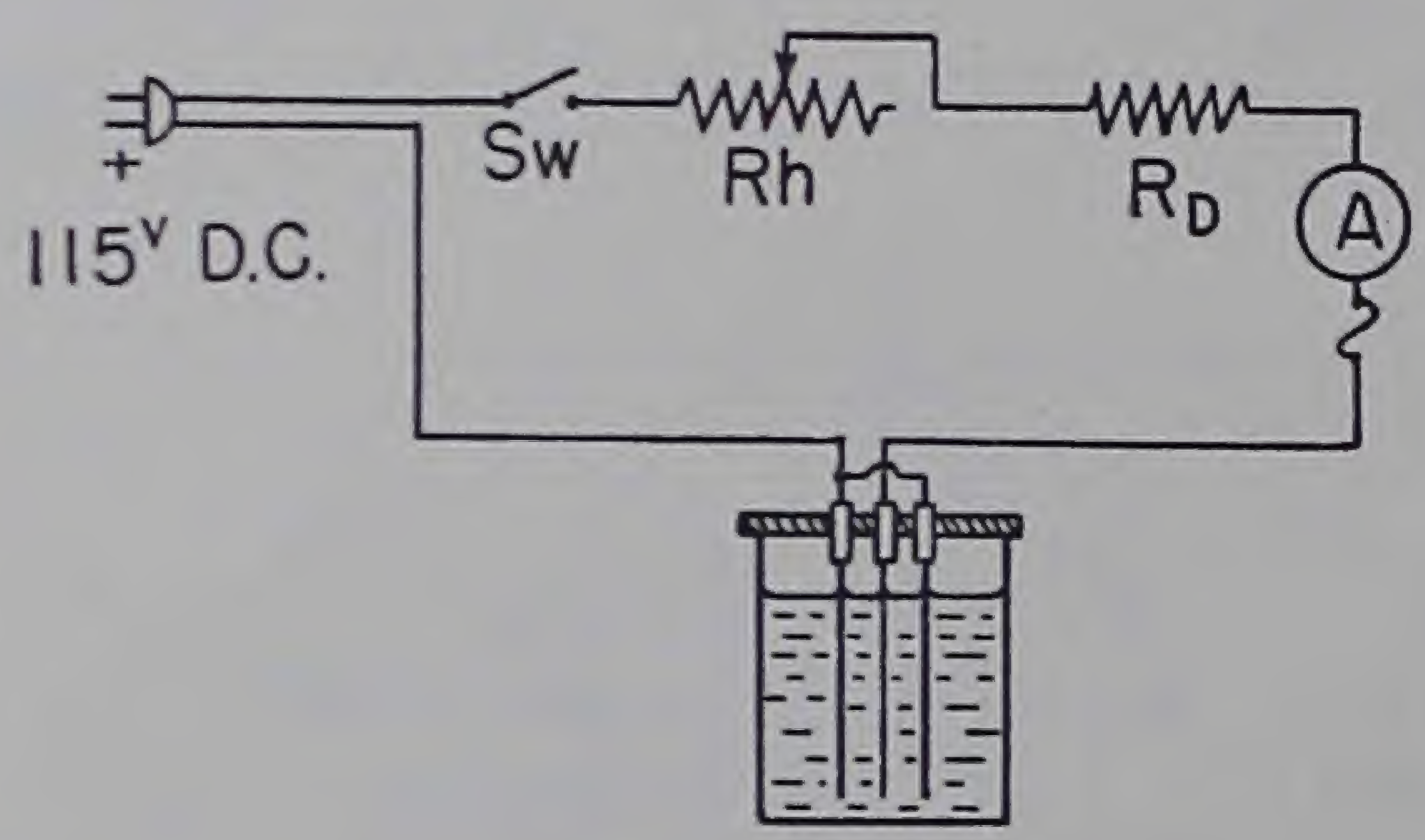


Fig. 33-1.

Theory: Consider two copper plates immersed in a solution of $CuSO_4$. The salt in solution is ionized, with Cu^{++} and SO_4^{--} ions present. If now a potential difference is established between the plates, the ions will move in the solution, the positive ions toward the cathode (cations) and the negative ions toward the anode (anions). When the copper ions reach the cathode, their charge will be neutralized by the acquisition of two electrons each, and they will be deposited on the cathode. The SO_4^{--} ions will give up their excess electrons at the anode and combine with atoms of copper there, and then go into solution as $CuSO_4$, to be ionized again. Thus the quantity of copper deposited on the cathode (or removed from the anode) depends on the quantity of electricity that has passed through the cell. For every two electronic units of charge, one atom of copper is deposited on the cathode.

Faraday, in the days before the discovery of the electron, formulated his famous laws of electrolysis: that the amount of any substance deposited from a solution by an electric current is directly proportional to the size of the current and to the time during which the current exists.

$$M = kIt, \tag{1}$$

where M = number of grams of metal deposited,
I = current in amperes,
t = time in seconds, and
k = proportionality constant, the *electrochemical equivalent.*

This, of course, agrees with the conclusions of the ionic concept discussed above; the product of current and time is charge.

The value of the electrochemical equivalent may be computed from theoretical considerations, since the

quantities that enter into it have been determined with good accuracy. Let e represent the size of the electron charge, N_o represent Avogadro's number, and A represent the atomic weight of copper.

Referring to Eq. (1), k is seen to represent the number of grams of the metal in question deposited by the passage of a coulomb of charge. For copper, it is required that $2e$ coulomb be passed for each atom. For a gram atomic weight of copper, N_o times this amount must be passed. Thus the value of k for copper is seen to be given by

$$k = \frac{A}{2eN_o}. \tag{2}$$

It is to be emphasized that the coulometer is a quantity-measuring device, and is most frequently so used. In this experiment, however, it will be used to determine the electrochemical equivalent of copper.

Method: Several precautions must be observed in this experiment. The electrolyte used is an acid solution of copper sulfate, commonly called "vitriol," and is very corrosive; if it is spilled on skin or clothing, large quantities of water should be used to flood the areas. Further, only the copper plates used as electrodes are to be immersed in it, since it will attack and corrode the clamps holding the plates in place. When the experiment is at an end, the electrodes are to be washed in running water immediately.

The electrolyte is to come within about 2 cm of the clamps of the electrodes; measure the area of the cathode (center electrode) taking into account both sides, and compute the proper current to be used—about 20 ma/cm^2. Make a short trial run, in order to adjust the current to the proper value. The cathode should be clean and free of oxide for this run. If necessary, it may be cleaned with a piece of sandpaper. After the trial run, clean the cathode carefully under running water, avoiding any roughness which may knock off some of the flaky deposit. Dry the plate by holding it a foot or two above a heater until no sign of moisture remains. Weigh it on the balance, obtaining as accurate a weight as possible. Note which weights are used so that the same ones may be used later to minimize errors in the weights. (The *difference* in weight is the subject of interest.)

Replace the cathode in its holder, and begin the run immediately. The current should be kept *constantly* adjusted to the value chosen, and should be started and stopped accurately on the second. A 30-min run will suffice for the first trial. Remove and wash as before, being extremely careful not to lose any of the deposit. Dry thoroughly, and again weigh carefully. Determine the mass of copper deposited on the cathode, and using this and the values of current and time, determine the electrochemical equivalent. Compare this with the accurate value (use four-place logarithms) computed from Eq. (2). The atomic weight of copper is 63.57 grams per gram molecular weight; see Table L, Appendix III, for values of e and N_o. Write the determinate-error equation corresponding to Eq. (1) as transposed to find k. Find the indeterminate error in your experimental value of k. Do the experimental and computed values agree within this error?

Repeat the experiment once, making a longer run if time permits.

Record: Tabulate all appropriate data and results.

QUESTIONS

1. What is the greatest source of error in this experiment? Give reasons for your answer. What are some other sources? Why is it desirable to make as long a run as possible?

2. Assuming that the values given for e, N_o, and A have an error of no more than two units in the last significant figure given, how accurately is the theoretical value for k for copper known? Is this significant in comparison with your experimental error?

3. How could this coulometer be used to standardize (calibrate) an ammeter?

4. Suppose that two coulometers were placed in series, the first being a copper coulometer as used in this experiment, and the second being a silver coulometer, using $AgNO_3$ as the electrolyte. Given that the atomic weight of silver is 107.88 gm/mole, in which coulometer would the greater weight of metal be deposited in 30 min with an average current of 2.00 amp? What will be the difference in the added weights?

Experiment 34.

Ohm's Law: Resistance of a Carbon and a Tungsten Lamp

Object: To determine the resistances of a carbon and a tungsten lamp using an ammeter and a voltmeter.

Apparatus: A-c ammeter (0—1), a-c voltmeter (0—150), variable autotransformer, 100-watt carbon lamp, 100-watt tungsten lamp, mounted socket.

Theory: Ohm's law states that

$$E = IR, \tag{1}$$

where R is the size of a resistance in ohms, I is the current in amperes through the resistance, and E is the potential difference in volts across the resistance. In order to avoid confusion when there is more than one resistance in the circuit, subscripts should be used with Eq. (1), which holds *only* when the subscripts on E, I, and R are all the same, *e.g.*,

$$E_t = I_t R_t.$$

In this way, the error of applying the total emf to only one particular resistance in the circuit, or some similar error, need never be made.

The power P in watts dissipated by a resistance as a result of a current through it is given by the equation

$$P = EI, \tag{2}$$

where again E is the potential difference in volts across the resistance and I is the current in amperes through it.

Method: First read the following items in Appendix II for descriptive material about operation and precautions in the use of the various instruments listed above: Note J, Sections 1 and 2 on the voltmeter and the ammeter; and Note G, Section 5 on the autotransformer.

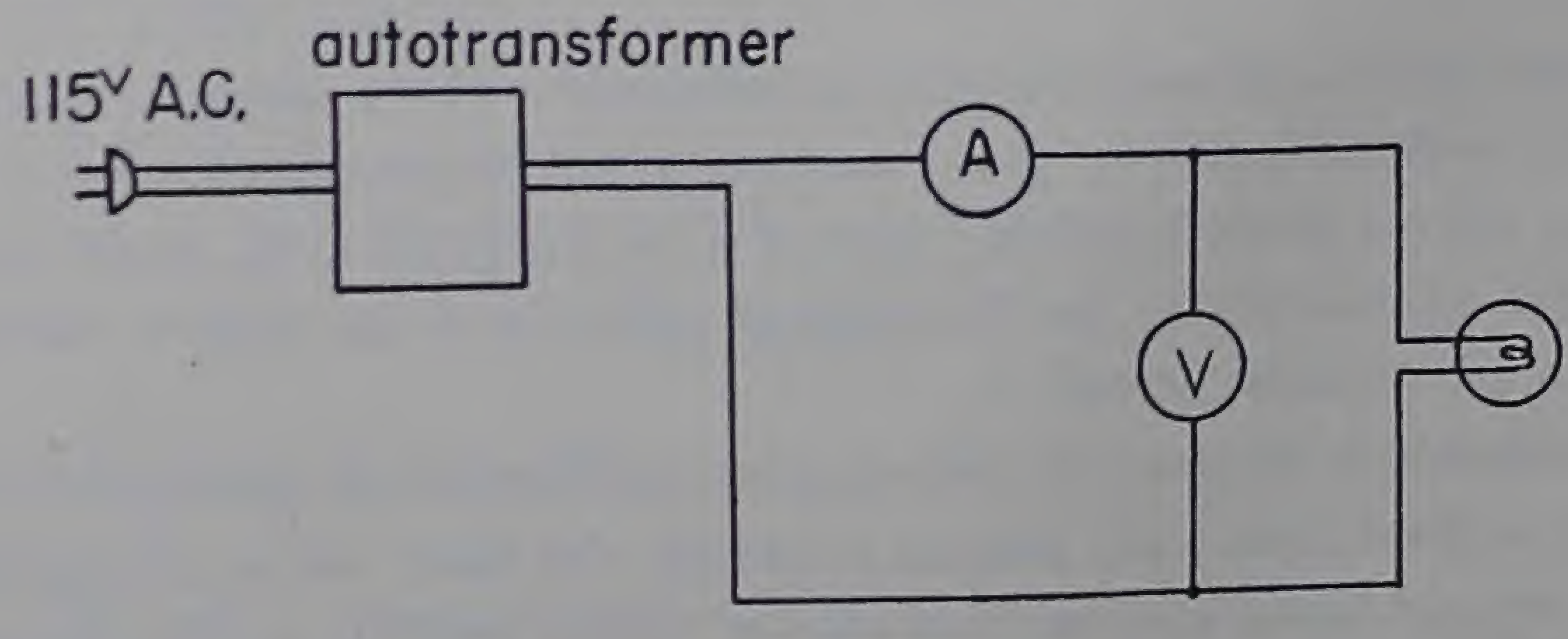


Fig. 34-1.

Connect the circuit as shown in Fig. 34-1, remembering the precautions in the Instructions for Electricity Laboratory. When the instructor has checked the circuit and connected the power, take a set of data for the

carbon lamp, recording the current for several different voltages in 10-volt steps, from 10 volts up to a maximum of 120 volts (the highest voltage being determined by the capacity of the ammeter). (Do not use voltages higher than 120 volts since the lamp will "burn out" rapidly at currents higher than that for which it is designed.)

Repeat for the tungsten bulb.

Note that the a-c meters used in this experiment read correctly when lying flat on their backs.

Calculate the resistance of each lamp for each pair of current and voltage values. Calculate the power consumption of each lamp at each point. Compute the indeterminate error in R and in P for each of three representative points for each bulb. The error of each meter may be taken as $\pm 0.5\%$ of the full-scale reading.

On one sheet of graph paper, plot the resistance as a function of the current for each bulb. On a second sheet, plot the resistance as a function of the power dissipation for each bulb.

Record: Arrange data and results in tabular form.

QUESTIONS

1. What do your curves show as to the relation between the resistance and the temperature in the case of each lamp? Which of the two sets of curves shows this variation most directly? Why?

2. In this experiment the ammeter reads the current in the lamp plus the current in the voltmeter. What error is introduced in the determination of the resistance of the lamp as a result of this "incorrect" reading of the ammeter? The resistance of the voltmeter is 10,000 ohms.

3. Would it be better in this experiment to connect the voltmeter across both lamp and ammeter? In this case the voltmeter reading is "incorrect." Explain. The ammeter resistance is 0.5 ohm.

Experiment 35.

Measurement of Resistance by the Wheatstone Bridge Method

Object: To determine the resistances of two coils, when taken singly and also when connected in series and in parallel, by means of (1) a slide-wire Wheatstone bridge, (2) a dial-type Wheatstone bridge.

Apparatus: Double resistance coil, dial resistance box, rheostat, switch, battery, galvanometer, slide-wire Wheatstone bridge, dial Wheatstone bridge. See Appendix II, Notes H1, 2; G2 on the galvanometer and on the dial box.

Theory: The resistance R of a conductor is defined by Ohm's law as the ratio of the potential difference V across the conductor to the current I in the conductor, *i.e.*, $R = V/I$. If V and I are expressed respectively in volts and in amperes, then R will be expressed in ohms.

There are several different methods available for measuring resistance. The most direct method is the ammeter-voltmeter method as used in Experiment 34. For accurate measurements by this method the ammeter and the voltmeter must have appropriate ranges, they must read correctly, and the resistance of one of them must be known.

The Wheatstone bridge method, used in this experiment, possesses distinct advantages over the ammeter-voltmeter method in that it is both a null and a comparison method. The unknown resistance is compared with a standard known resistance by getting a zero (null) deflection in a galvanometer connected in the bridge circuit.

The slide-wire Wheatstone bridge, as shown schematically in Fig. 35-1, consists of a uniform resistance wire, usually 1 m in length, stretched between points A and B. The unknown resistance X is connected between A and D and a dial resistance box R is connected between B and D. A battery, with switch Sw and rheostat Rh connected across AB, furnishes current for the bridge. A galvanometer G is connected

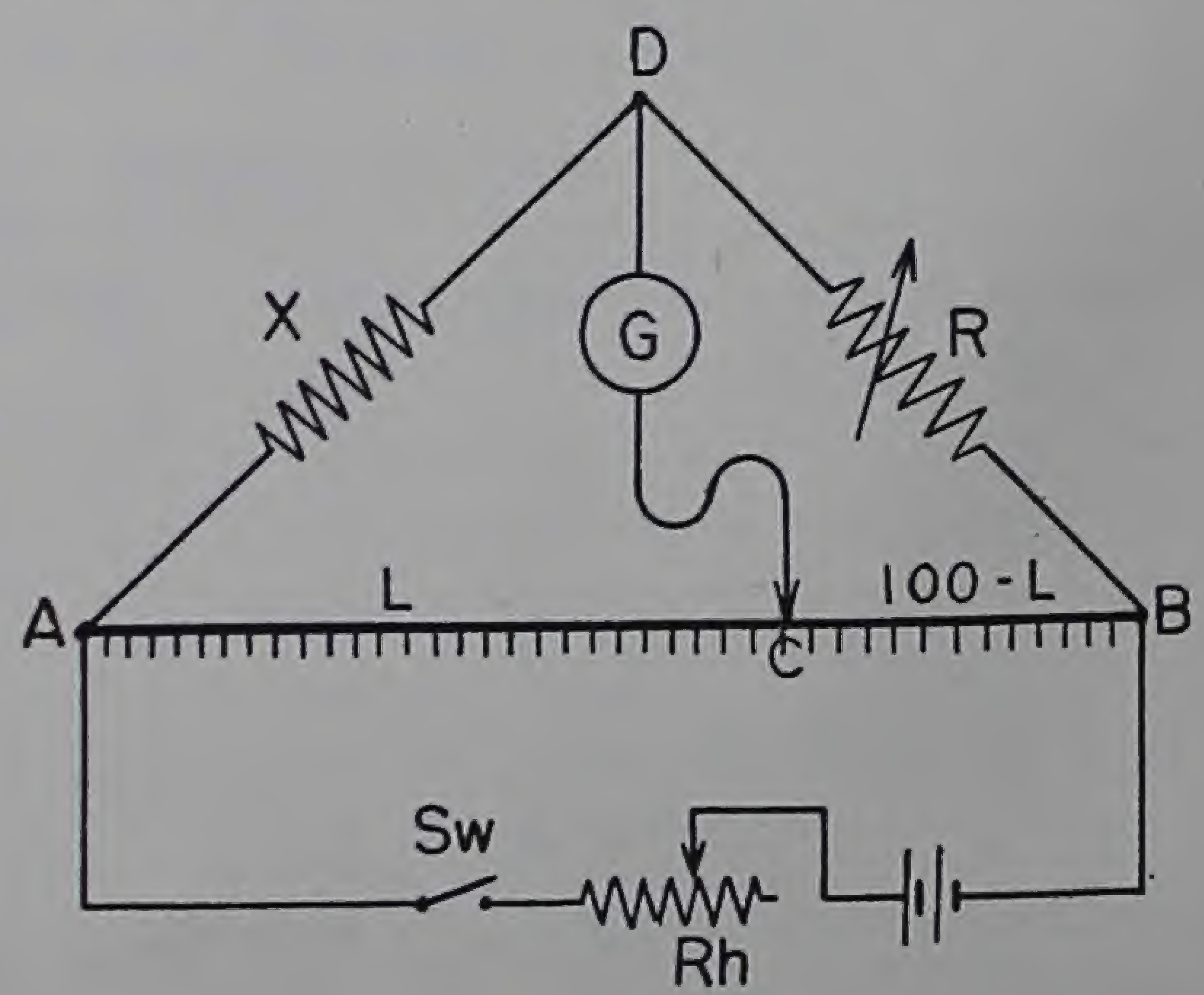


Fig. 35-1.

between D and C. By means of a sliding contact, the position of point C is variable and may be anywhere in the interval AB, *i.e.*, one terminal of the galvanometer may be connected to any point on the bridge wire.

When the battery switch is closed, current will exist in all arms of the bridge including the galvanometer arm. However it is possible, under normal conditions, to find some one position C for the slider on the bridge wire such that the current in the galvanometer arm is zero, *i.e.*, the galvanometer deflection will be zero when contact is made with the bridge wire at this point. Under this condition the bridge is said to be balanced.

Since for a balanced bridge the galvanometer current is zero, it follows from Ohm's law that the potential difference between D and C must be zero. Hence the potential difference between A and D must equal that between A and C; and the potential difference between D and B must equal that between C and B.

Let I_1 be the current in X and in R (why is this current the same?) and let I_2 be the current in the bridge wire. Further let R_{AC} be the resistance of the bridge wire between A and C, and let R_{CB} be the resistance of the bridge wire between C and B. Then,

$$I_1X = I_2R_{AC} \qquad \text{and} \qquad I_1R = I_2R_{CB}.$$

If we divide the first equation by the second and solve for X, we get

$$X = \frac{R_{AC}}{R_{CB}} R. \tag{1}$$

Since the bridge wire is assumed to be uniform, it follows that

$$\frac{R_{AC}}{R_{CB}} = \frac{\text{length } AC}{\text{length } CB} = \frac{L}{100 - L}$$

where L is the length in centimeters of the section AC. It is assumed that the wire is 100 cm long.

Equation (1) may then be written in the form

$$X = R\frac{L}{100 - L}. \tag{2}$$

The error equation corresponding to Eq. (2) is

$$\frac{\Delta X}{X} = \frac{\Delta R}{R} + \frac{100\,\Delta L}{(100 - L)(L)}. \tag{2a}$$

It may be shown that the coefficient of ΔL in this equation has a *minimum* value when $L = 50$ cm, *i.e.*, when the balance point is at the center of the bridge wire. Hence the fractional error in X is reduced to a minimum when the bridge is balanced in this manner. This means, of course, that R should be chosen as nearly equal to X as possible for greatest accuracy.

Equation (2) enables us to compute the value of X in terms of a known resistance R and the position of the balance point on the slide-wire. A very wide range of resistances may be measured by this method, *i.e.*, resistances ranging from one to several thousand ohms, provided that one has a variable standard R. For the measurement of very low resistances or very high resistances, the bridge method must be modified.

In a *balanced* Wheatstone bridge it is possible to exchange the positions of the battery and of the galvanometer without disturbing the balance equation. Can you prove this? However, analysis shows that the bridge is more sensitive when the galvanometer, rather than the battery, is connected between the junction of the high-resistance arms and that of the low-resistance arms. In this case X and R are the high-resistance arms. It can also be shown that the sensitivity of the bridge increases with increasing battery current. However, it is necessary to keep the current in the arms of the bridge from exceeding the current capacity of the arms.

SERIES AND PARALLEL CONNECTIONS

When two resistances are connected in series, as shown in Fig. 35-2, the combined resistance X_s is given by the equation

$$X_s = X_1 + X_2 \qquad \text{(series)}. \tag{3}$$

When two resistances are connected in parallel, as shown in Fig. 35-3, the combined resistance X_p is given by the equation

$$\frac{1}{X_p} = \frac{1}{X_1} + \frac{1}{X_2} \qquad \text{(parallel)}. \tag{4}$$

Fig. 35-2.

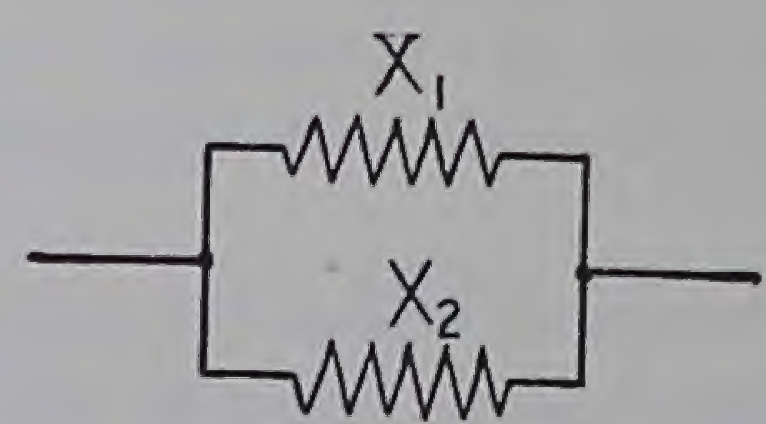


Fig. 35-3.

The corresponding error equation for this case is

$$\frac{\Delta X_p}{X_p} = \frac{X_2}{X_1 + X_2}\,\frac{\Delta X_1}{X_1} + \frac{X_1}{X_1 + X_2}\,\frac{\Delta X_2}{X_2}. \tag{4a}$$

In the dial Wheatstone bridge, the bridge wire is replaced by a set of ratio coils so that the ratio R_{AC}/R_{CB} is given directly on a dial as some decimal multiple of 1 such as 0.1 or 100. If this ratio is represented by Q, then Eq. (1) becomes

$$X = RQ. \tag{5}$$

By varying R and Q a wide range of X may be measured. In this type of bridge the ratio coils and the variable resistance R are contained in a single box with dials for reading R and Q. There are three sets of terminals on the box for connecting the battery, the galvanometer, and the unknown resistance. In some types the galvanometer and the battery are incorporated in the bridge so that one needs only to connect in the unknown resistance.

***Method:** Part I. The Slide-wire Bridge.* Examine the bridge and note how it may be utilized to conform with the schematic diagram in Fig. 35-1. For example, points A, B, and D in the diagram correspond in the real bridge to heavy metal strips with negligible resistance. Connections may be made at any of the terminals on these strips.

Connect the unknown resistance X_a and a dial resistance box to the bridge by means of short heavy leads. The unknown resistances consist of a set of two resistors, a and b, with independent terminals. Be certain that all the connections are tight. Loose connections are the cause of much trouble in electrical measurements. Make all the other connections called for in Fig. 35-1 *except the two connections* at the battery. Have your instructor check your setup before making the connections at the battery terminals.

After all connections have been made, set the rheostat at about 40 ohms, set the dial resistance box at 10 ohms, depress the low-sensitivity button, L, on the galvanometer, and close the battery switch. Set the slider near the one end of the slide wire, *e.g.*, at the 5-cm mark, and momentarily depress it to make contact with the wire. Ordinarily a large deflection of the galvanometer will occur to right or left. Raise the slider, move it up scale about 10 cm, and try again. The galvanometer deflection should be smaller, thus indicating that you are moving the slider toward the balance point. Continue this process until the galvanometer deflects in the opposite direction indicating that you have passed the balance point. Reverse the direction of motion of the slider and, using smaller intervals, seek the balance point. In the immediate neighborhood of the balance point it will be necessary to use the high sensitivity range of the galvanometer by depressing the H button.

In carrying out the above process do not scrape the slider along the wire, and do not press it down with such force that it dents the slide-wire.

In case you are not able to find a balance point anywhere on the slide-wire either because the galvanometer deflections are always in the same direction or because there is no galvanometer deflection, look for an open circuit or a loose connection. If this fails, call the instructor.

After you have found this preliminary balance point, note the position of the slider with reference to the center of the bridge wire. If it is more than 10 cm away from the center, adjust R in order to bring the balance point within 10 cm of the center of the bridge wire. This adjustment is necessary if an accurate

value of X_a is to be obtained as indicated in the theory of this experiment. Determine the position of this final balance point C as accurately as possible by using the high sensitivity range of the galvanometer. Make an estimate of the error in the position of this balance point by finding how far the slider may be moved away from this balance point without giving a noticeable galvanometer deflection. Record $L(= AC)$, $100 - L(= CB)$, the final reading of the dial box R, and the error in the balance-point position L. By use of Eqs. (2) and (2a) determine the resistance X_a and the percentage error in X_a. The percentage error in R may be taken as 0.25%.

Proceed in a similar manner to determine the resistance of the second resistor, the resistance of the two resistors in series, and the resistance of the two resistors in parallel. Also determine the percentage errors in each case.

Make a second determination of each of the above resistances by *reversing* the positions of X and R in the bridge. Use the same value of R for each resistance that was used in the first determination. *Note well* that the reversal of the positions of X and R in the bridge will produce a new balance point C' on the bridge wire which will fall on the opposite side of the center from that of C and at very nearly the same distance from the center. For this reversed position of X and R, balance equation (2) must be modified to read

$$X = R\frac{100 - L'}{L'}, \tag{2'}$$

where $L' = AC'$ and $100 - L' = C'B$. It is highly advisable that the student redraw Fig. 35-1 with X and R reversed in positions and develop Eq. (2′).

Compute the average value of these two determinations for each resistance and use these averages as your final result for measurement of resistance with a slide-wire Wheatstone bridge.

Check the series and parallel resistances against the values calculated by use of Eqs. (3) and (4). They should check within the limits of the errors in each case.

Part II. The Dial Wheatstone Bridge. Use the dial bridge to remeasure all of the resistances used in Part I of this experiment. Set Q in these measurements to a value such that four significant figures are obtained for X. In each case estimate the error in the setting of R for a balance and compute the corresponding percentage error in X. Assume that the error in Q is negligible.

Record: Give the apparatus numbers of the resistance a and b, slide-wire bridge, galvanometer, dial box, and dial bridge. Tabulate your data. Express your results in the following form:

Item	Slide-wire bridge		Dial bridge		% difference
	X (ave)	% error	X	% error	
Resistance a					
Resistance b					
Series (exp)					
Series (calc)					
Parallel (exp)					
Parallel (calc)					

QUESTIONS

1. Show that it is possible to interchange the positions of the battery and galvanometer in Fig. 35-1 without changing balance equation (2).

2. Show that the coefficient of L in Eq. (2a) has a minimum value when $L = 50$. (Calculus required.)

3. Develop the error equation (4a) by applying the rules given in Section A-4 of the Introduction.

4. If d equals the *shift* of the *balance point* to the *left* in Fig. 35-1 when X and R are reversed in position, show that $X/R = (100 + d)/(100 - d)$.

Experiment 36.

Galvanometer Sensitivity

Object: To determine the current sensitivity of a galvanometer.

Apparatus: Reflecting galvanometer (approximate resistance 150 ohms), two dial resistance boxes, one low resistance shunt, damping key, reversing switch, voltmeter (0 to 3), and battery. See Appendix II, Notes H1, 3; J1; G2, 7 concerning the galvanometer, voltmeter, dial box, and reversing switch.

Theory: The *current sensitivity* of a galvanometer is the current in amperes necessary to produce a unit deflection on the scale. If the scale is in millimeters and at a distance of 1 m from the galvanometer, this sensitivity is called the *figure of merit* of the galvanometer. In this experiment the current sensitivity will be considered to be the current in amperes per millimeter deflection at the fixed distance of the mounted scale, in this case 50 cm. Since the galvanometer is constructed so that the scale reading d is very closely proportional to the galvanometer current I_g, the current sensitivity K will be given by the relation

$$K = \frac{I_g}{d}. \tag{1}$$

Hence it is only necessary to observe the deflection d corresponding to a known I_g in order to determine K. However, the galvanometer currents which will give satisfactory deflections are extremely small—of the order of 10^{-8} amp in this experiment. Thus it is necessary to use special methods in order to obtain and to determine these small currents.

The arrangement that is used to accomplish this purpose is shown in Fig. 36-1. In this figure G is the galvanometer, DK a damping key, D a dial resistance box, S a low-resistance shunt, R a dial resistance

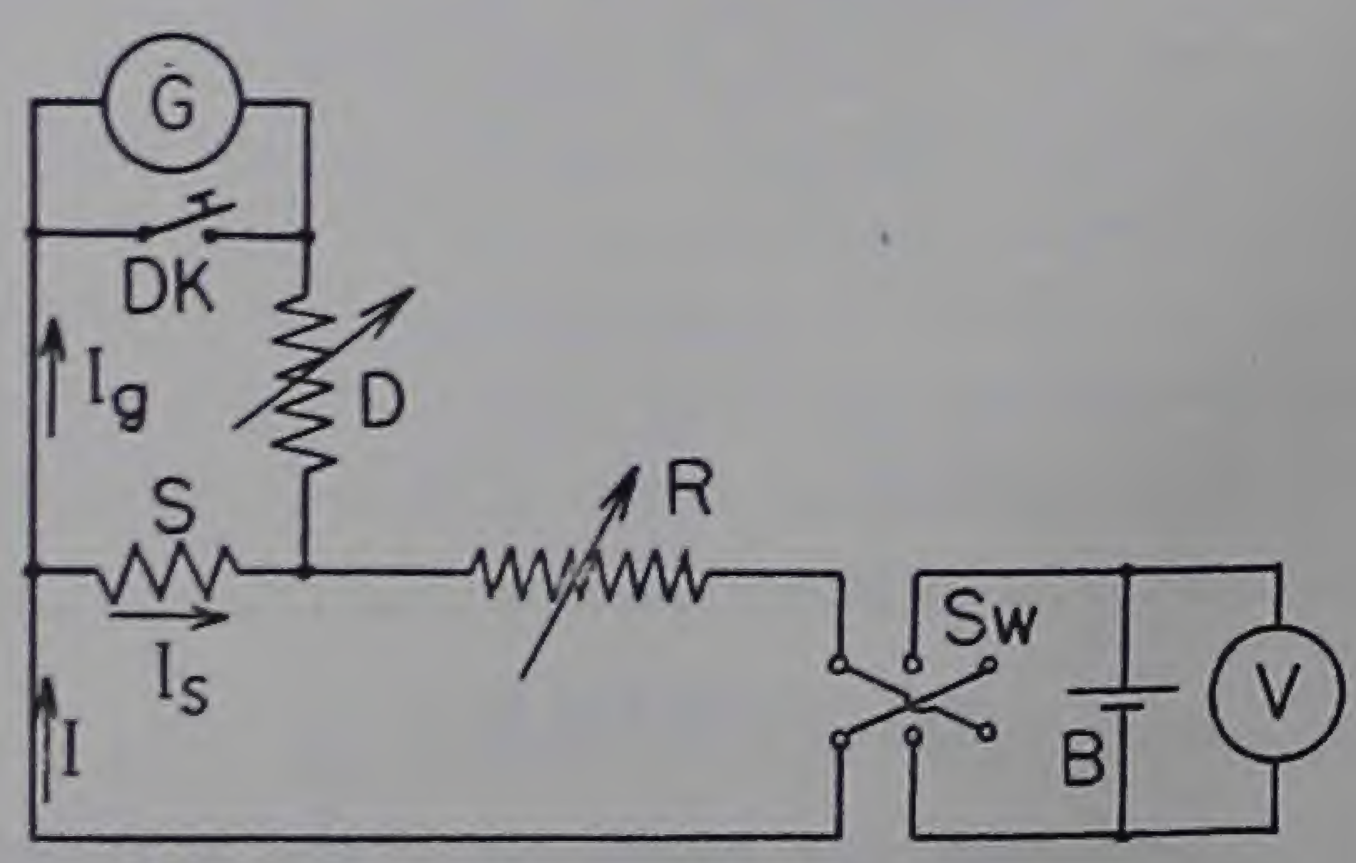


Fig. 36-1.

box, Sw a reversing switch, V a voltmeter, and B a dry cell. When the reversing switch is closed the battery sends a small current I through the main circuit since R is set at a relatively high value of resistance. But only a small fraction of this main current passes through the galvanometer, most of it by-passing the galvanometer through the low-resistance shunt S.

Galvanometer Current. Let the symbols R, D, and S represent the actual resistances of the two dial boxes and the shunt. Let G represent the resistance of the galvanometer. Let V represent the potential difference impressed across the main circuit, *i.e.*, the reading of the voltmeter, when the battery is supplying current. By Ohm's law the current I is equal to V divided by the total resistance of the external circuit. This total resistance is

$$R + \frac{S(G + D)}{S + G + D},$$

hence

$$I = \frac{V}{R + \dfrac{S(G + D)}{S + G + D}}. \tag{2}$$

Also it is clear that the potential drop across S must equal the sum of the potential drops across G and D. Hence, by Ohm's law,

$$I_g(G + D) = I_s(S) = (I - I_g)(S), \tag{3}$$

where I_g is the galvanometer current, I_s is the current in S, and I is the current in the main circuit. Let us solve Eq. (3) for I_g, substituting for I its value given by Eq. (2). We get

$$I_g = \frac{S}{S + G + D} I = \frac{S}{(S + G + D)} \frac{V}{R + \dfrac{S(G + D)}{S + G + D}} = \frac{SV}{(R + S)(G + D) + RS}. \tag{4}$$

Since S has a value of only a few tenths of an ohm as compared to several hundred ohms for R and $G + D$, the above expression for I_g may be reduced, without appreciable error, to the expression

$$I_g = \frac{SV}{R(G + D)}. \tag{5}$$

By means of Eq. (5), the galvanometer current may be determined provided S, D, R, G, and V are known. D and R are dial-resistance-box readings and hence are known, the value of the shunt resistance S is stamped on its base, and V is the voltmeter reading. However, the galvanometer resistance G in this experiment is not given and must be determined.

Galvanometer Resistance. The method of determining G is as follows: Set $D = 0$ and adjust R until a reasonable galvanometer deflection occurs, say about 100 mm. Call this deflection d_o. Now increase D to about 150 ohms, leaving R unchanged. There will be less current in the galvanometer and its deflection will decrease to some value d_1. Since virtually a constant potential is applied across the galvanometer circuit $(G + D)$ in this process, it follows that

$$(G + 0)d_o = (G + D)d_1.$$

Hence,

$$G = D\frac{d_1}{d_o - d_1}. \tag{6}$$

This method of determining the resistance of a galvanometer is sometimes called the "partial deflection method." It is an easy and convenient method to use but will only give satisfactory results provided S is very much smaller than G.

Galvanometer Sensitivity. We are now in a position to determine the galvanometer sensitivity. It is just the ratio between I_g and the corresponding galvanometer deflection d, and is given by the equation

$$K = \frac{I_g}{d} = \frac{SV}{R(G + D)}\frac{1}{d}. \tag{7}$$

Error Equations. The *determinate*-error equation for the galvanometer resistance G may be obtained from Eq. (6) in the customary manner. It is

$$\frac{\Delta G}{G} = \frac{\Delta D}{D} + \frac{d_o}{d_o - d_1}\left(\frac{\Delta d_1}{d_1} - \frac{\Delta d_o}{d_o}\right). \tag{6a}$$

It is advisable to make d_1 about half the value of d_o.

The *determinate*-error equation for the galvanometer constant is

$$\frac{\Delta K}{K} = \frac{\Delta S}{S} + \frac{\Delta V}{V} - \frac{\Delta R}{R} - \frac{\Delta G + \Delta D}{G + D} - \frac{\Delta d}{d}. \tag{7a}$$

The errors in R, S, and D may be taken as $\pm 0.25\%$ each. The error in V may be taken as $\pm 1\%$ of the full-scale deflection of the voltmeter.

Method: Make the connections as shown in Fig. 36-1 except for the battery connections. Have the instructor check your wiring before making these connections. Be certain that all connections are tight, especially those at the shunt S. Poor connections here may lead to a burned-out galvanometer. Why? Adjust the galvanometer so that the scale reading is zero.

Part I. Determination of Galvanometer Resistance. Set R at its maximum value (about 10,000 ohms), set $D = 0$, close the damping key K, and then close the reversing switch Sw. The battery is now in the circuit but no deflection of the galvanometer should be observed since it is "shorted out" by the closed damping key. Any appreciable deflection of the galvanometer under these conditions is a danger signal that something is wrong with the circuit and the reversing switch should be opened immediately, disconnecting the battery. If, however, no galvanometer deflection occurs, the damping key may be opened *momentarily*. There should then be a small deflection of the galvanometer which may be rather slow because of large damping. If this occurs the damping key may be left open. Decrease R carefully until the *steady* galvanometer deflection is about 100 mm. This is the deflection d_o. Read and record this deflection, estimating to a half of a millimeter. Next increase D from 0 to 150 ohms, leaving R unchanged. The galvanometer deflection should now decrease to the value d_1—about one-half of its original value. Read and record this value. Compute, by means of Eq. (6), the galvanometer resistance G. Also compute by use of Eq. (6a) the indeterminate error in G.

Part II. Determination of Galvanometer Sensitivity. Allow D to remain constant at 150 ohms during this part of the experiment. By carefully varying R, obtain a set of corresponding galvanometer deflections. Choose R so that the deflections are roughly 30, 60, 90, and 110 mm. For each value of R get deflections in both the red and black portions of the scale by use of the reversing switch Sw. Be certain that the voltmeter is reading during this process. Record the values of R and of d, along with D, S, and V. Compute by means of Eq. (7) the value of K for each R. Also compute the indeterminate error in K by use of Eq. (7a). These values of K should be constant within the limits of the errors.

Record:

App. No. Galvanometer______________
Shunt______________
D res box______________
R res box______________
Voltmeter______________

Part I.

D = 150 ohms
d_o = (   ) mm $\Delta d_o = \Delta d_1$ = (   ) mm
d_1 = (   ) mm
G = (   ) ohms ΔG = (   ) ohms

Part II.

G = (   ) ohms
V = (   ) volts
S = (   ) ohms
D = 150 ohms

	R, ohms	Red, d, mm	Black d, mm	Ave d, mm	$K, \frac{\text{amp}}{\text{mm}}$
1					
2					
3					
4					
				Ave	

QUESTIONS

1. What is the relation between the current sensitivity K of the galvanometer as obtained in this experiment and the figure of merit of the galvanometer?

2. What constant percentage error is introduced into the value of K by using Eq. (5) instead of (4) for I_g? Use typical values of S, V, R, G, D in computing this error.

3. How could the setup for this experiment be used to determine an unknown low resistance, if K for the galvanometer is known?

4. Why is it necessary for S to be much smaller than G in order for the "partial-deflection method" to work?

Experiment 37.

Potentiometer

Object: To measure with a slide-wire potentiometer the emf of two different cells when taken singly and then when taken in opposition. To determine the error in a laboratory voltmeter when it reads 1.00 volt by determining the "true" potential difference with the potentiometer.

Apparatus: Slide-wire potentiometer, storage battery, rheostat and switch, double-pole double-throw switch, standard cell, two unknown cells, table galvanometer and tap key, two dial resistance boxes, voltmeter. See Appendix II, Sections H1, 2; G6; J1 for descriptions of the galvanometer, standard cell, and voltmeter.

Theory: Although the most direct and convenient method of measuring a potential difference is by means of a direct-reading voltmeter, this method has the distinct disadvantage that the voltmeter draws some current in the measurement. In doing so it changes, materially in many cases, the potential difference which one wishes to measure.

The potentiometer method of measuring potential difference avoids this difficulty since the potentiometer draws no current when it is *balanced* for a reading of potential difference, *i.e.*, the method is a null method. It is also a comparison method because the unknown potential difference is compared with the known emf of a standard cell. As a result, very precise and accurate determinations of potential difference can be made with this method.

In one of its simplest forms, the slide-wire form, the potentiometer consists of a long uniform wire AB in which a constant current I is maintained by means of a battery connected to the ends of the wire. See Fig. 37-1. There is thus a uniform drop in potential along the wire as one goes from the positive terminal of the wire A to the negative terminal B.

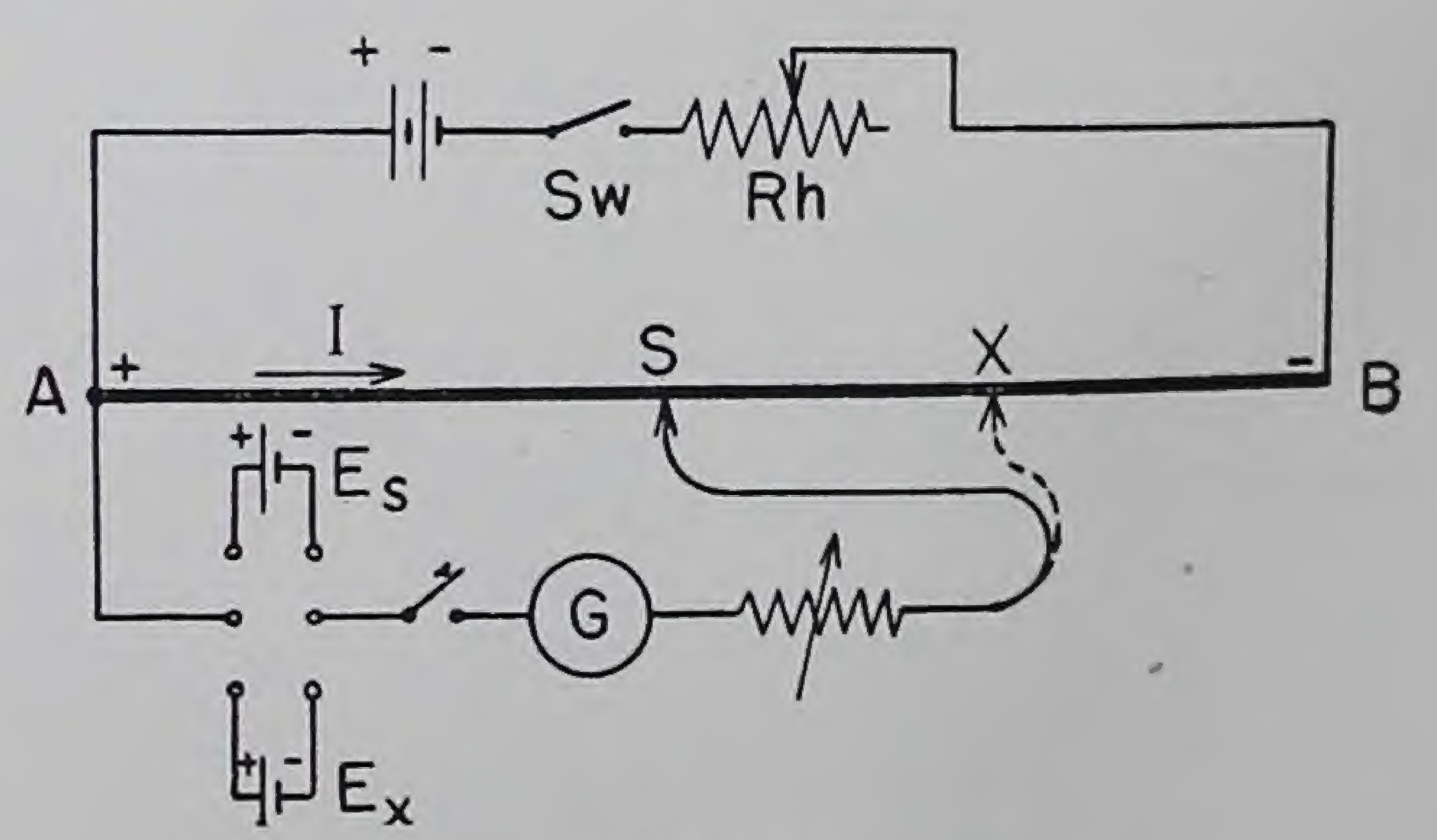


Fig. 37-1.

Suppose that a standard cell of known emf E_s, and a cell whose emf E_x is to be determined, are connected as shown so that either positive pole may be connected through the double-pole switch to A and either negative pole may at the same time be connected through the galvanometer G to some point on the slide-

wire. Consider the standard cell to be in the circuit first. It will be possible to find some point S along the slide-wire such that the drop in potential along the wire from A to S is just equal to E_s, *i.e.*, provided E_s is less than the total drop in potential from A to B. If then the galvanometer circuit is closed at this point S by means of a sliding contact, there will be *no* current in the galvanometer and standard cell because the emf of the standard cell just "balances" the potential difference from A to S. Under these conditions the potentiometer is said to be balanced for the standard cell and $E_s = V_{AS}$. By Ohm's law V_{AS}, the potential drop from A to S, is equal to IR_{AS} where R_{AS} is the resistance of the bridge wire between A and S. Hence

$$E_s = IR_{AS}. \tag{1}$$

In a similar manner we may consider E_x to be connected in the circuit. There will be a point X on the wire for which V_{AX} just equals E_X, provided $E_x < V_{AB}$. Hence

$$E_x = IR_{AX}. \tag{2}$$

If we divide Eq. (2) by Eq. (1) and solve for E_x, we get

$$E_x = \frac{R_{AX}}{R_{AS}} E_s = \frac{L_X}{L_s} E_s \tag{3}$$

since R is proportional to length L for a uniform wire.

Equation (3) enables us to determine E_x in terms of E_s and the ratio L_x/L_s. E_s is known and the two lengths, L_x and L_s, may be determined experimentally by "balancing" the potentiometer, *i.e.*, finding points S and X for which one obtains zero galvanometer deflections.

When two cells are connected in *opposition*, as shown in Fig. 37-2, the combined emf is the *difference* between the separate emf's, *i.e.*,

$$E_x = E_1 - E_2. \tag{4}$$

The positive terminal of this combination is the positive terminal of that cell which has the larger emf. In the figure shown it is assumed that E_1 is greater than E_2.

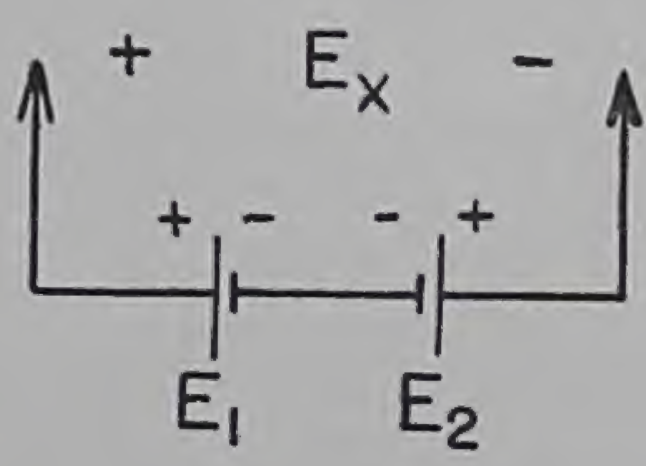


Fig. 37-2.

Calibration of Voltmeter. The potentiometer may be used to calibrate a voltmeter and is often so used. It is only necessary to determine the potential difference across the terminals of a voltmeter with a potentiometer, and to compare this potential difference with the voltmeter reading. Suppose the voltmeter V is connected to a battery B through a variable resistance R as shown in Fig. 37-3. The voltmeter reading may be varied from 0 to almost the full emf of the battery by adjustment of R. If at the same time the potentiometer reading across V is taken, the two values may be compared.

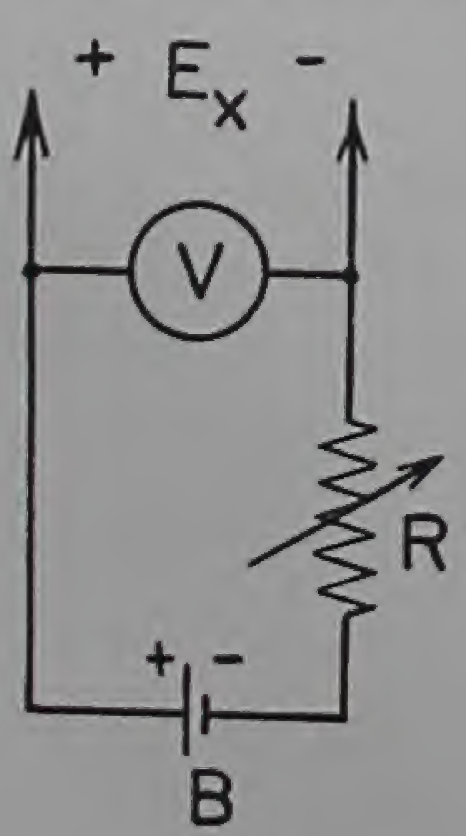


Fig. 37-3.

Error Equations. The *determinate*-error equation corresponding to Eq. (3) is

$$\frac{\Delta E_x}{E_x} = \frac{\Delta E_s}{E_s} + \frac{\Delta L_x}{L_x} - \frac{\Delta L_s}{L_s}. \tag{3a}$$

Generally the error in E_s is much smaller than those in L_x and in L_s. Hence, in this experiment, this error may be neglected.

Standard Cell. The standard cell used in this experiment is of the Weston unsaturated type. See Appendix II, Section G6. Its emf is given on a tag attached to it. It should be handled with great care. To prevent changes in its emf it should *never* be used to supply currents in excess of 10^{-4} amp.

***Method:** Preliminary.* Make the connections as shown in the upper part of Fig. 37-1 (line AB and above). Connect the voltmeter between the terminal A and the slider S. Set the slider at point B. Close the battery switch and note the reading of the voltmeter. Adjust the rheostat until this reading is well over 1.5 volts. Note the voltmeter reading. Now set the slider at position A and note the voltmeter reading. Move the slider 20% of the length AB and again note the voltmeter reading. Repeat this process for slider positions at 40, 60, 80, and 100% of the length AB. Observe that the voltmeter readings increase

uniformly with the length AS and are in fact directly proportional to this length. This means that there must be a uniform drop in potential along the wire from A to B, any portion of which may be "tapped off" by use of slider S. Disconnect the battery and remove the voltmeter from the circuit.

Part I. Emf of Cells. Complete the connections as shown in Fig. 37-1 but do not connect in any battery or cell until the instructor has checked your wiring. *This is especially important for the standard cell.* A wrong connection may ruin it. In finally making battery and cell connections be very careful to arrange the polarities to correspond with the figure. Otherwise it may not be possible to find balance points. Why not? As point A, it will be convenient to use the zero end of the meter sticks.

Set the slider at approximately 120 cm (on the second section of the slide-wire; distance measured from point A), set the battery rheostat at about half resistance, set the dial resistance box in series with the galvanometer at 9000 ohms, close the battery switch, and then close the standard-cell switch. A galvanometer deflection should occur. Try to reduce this deflection to zero by varying the resistance in the battery rheostat. When this has been done, reduce the dial-resistance-box reading to zero and then determine the exact balance point S by shifting the slider until the galvanometer deflection is zero. In this process do not rub the slider along the wire but rather lift it up before moving it. If there is an appreciable interval of the slide-wire over which the galvanometer deflection appears to be zero, determine the end points of this interval (small but opposite deflections at the end points) and take the middle point of this interval as the balance point for the standard cell. This apparent lack of sensitivity when the standard cell is used is probably due to a high protective resistance incorporated in the standard cell.

Immediately after determining the balance point S for the standard cell, set the dial box at 9000 ohms, throw the double-pole switch for E_x and determine the new balance point X by shifting the position of the slider. For final adjustment, reduce the dial-box setting to zero. *Do not disturb the battery rheostat in this process.*

Then switch back to the standard cell and redetermine its balance point. Any shift in the balance point of the standard cell indicates a change in the current through the potentiometer wire. Continue this process of switching back and forth between E_s and E_x until three L_s and two L_x have been determined. Use the average values of L_s and L_x for the determination of E_x.

Replace the first unknown cell by the second unknown cell in the potentiometer circuit. Determine as above the emf of this second cell.

Connect the two unknown cells in opposition and use this combination as E_x. Determine as before this value of E_x and compare it with the calculated value.

The error in the position of the balance point may be taken as one-fourth the interval over which no appreciable galvanometer deflection occurs.

Part II. Error of Voltmeter. Connect the voltmeter in series with a dial resistance box and a good dry cell as shown in Fig. 37-3. Use this combination as E_x. Adjust the dial resistance box in series with the voltmeter until the voltmeter reads exactly 1 volt, *i.e.*, as closely as one can judge. Determine E_x across the voltmeter with the potentiometer in the manner outlined in Part I. Be certain that the voltmeter is reading (1 volt) while this determination is being made. The error in the voltmeter is the difference between E_x (potentiometer) and the voltmeter reading (1 volt). Determine this error.

Record: Give the apparatus numbers of the potentiometer, standard cell, galvanometer, dial box, and voltmeter. Tabulate your data making separate tables for each determination of an unknown emf or potential difference. Summarize your results in the following form:

Item	Use	E, volts	% error
Std. Cell	Standard emf		....
Cell No. 1	Measured emf		
Cell No. 2	Measured emf		
Opposition	Measured emf		
Opposition	Calculated emf		
Voltmeter	Calibrated at 1 volt		

QUESTIONS

1. Why is it necessary that the battery furnishing the current for the potentiometer have a larger emf than any emf or potential difference to be measured with the potentiometer?

2. In Part II of this experiment on the calibration of a voltmeter, why is it necessary that the voltmeter be reading 1 volt when the calibration is made? Suppose R in Fig. 37-3 is set so that the voltmeter reads 1 volt when connected into the circuit. If now the voltmeter is disconnected and the potentiometer is used to determine E_x, what will be the value of E_x?

3. Suppose an ammeter is available in this experiment along with the other apparatus already specified. How could one proceed to determine the resistance per unit length of the potentiometer wire AB in Fig. 37-1?

4. How would one proceed to make this slide-wire potentiometer a direct-reading potentiometer, *i.e.*, one in which the position of the balance point in meters would equal the measured potential difference in volts?

Experiment 38.

Thermocouple

Object: To calibrate a copper-constantan thermocouple for low-temperature measurements and to determine the freezing point of mercury.

Apparatus: Copper-constantan thermocouple (24 ga. Cu; 20 ga Const.), reflecting galvanometer, Ayrton shunt, low-resistance shunt, decade resistance box, damping key, reversing switch, 20-ohm rheostat with switch, Dewar flasks (thermos bottles), ice, solid carbon dioxide, alcohol, storage battery, voltmeter, mercury, iron test tube. See Appendix I, Section H5 on the Ayrton shunt.

Theory: When a circuit is formed of two wires of different metals with the two junctions at different temperatures, an electromotive force is produced in the circuit. This is known as the thermoelectric effect and was discovered by Seebeck in 1821. Such a device, known as a thermocouple, may be employed in the measurement of temperature. When properly calibrated, the thermocouple becomes a convenient and sensitive thermometer.

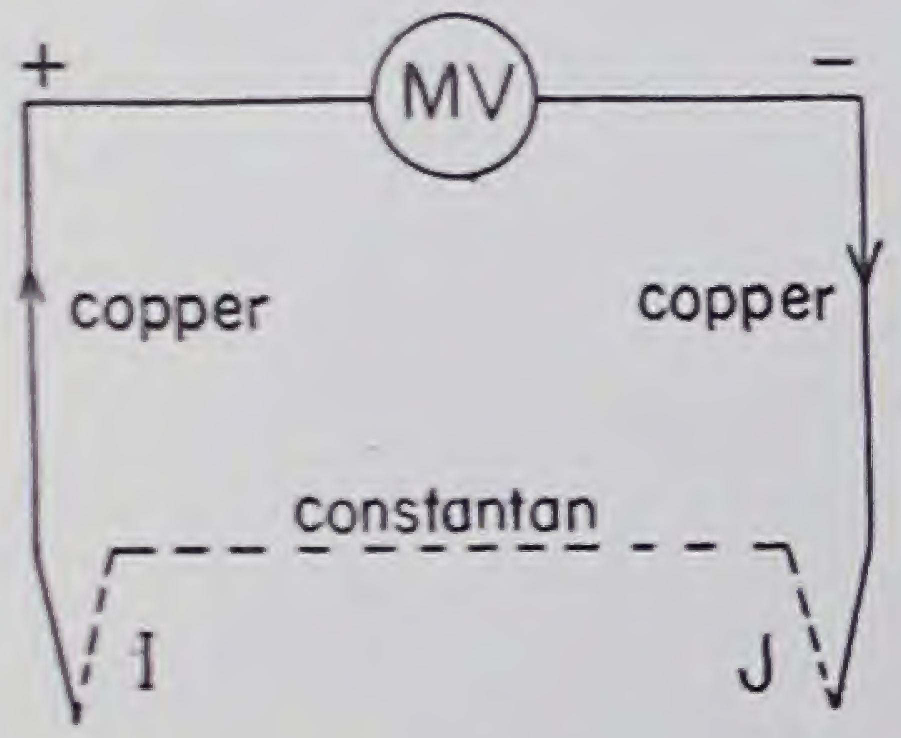


Fig. 38-1.

A pair of metals frequently used as a thermocouple is copper and constantan. Constantan is an alloy of 60% copper and 40% nickel. This couple may be used to measure a wide range of temperatures (−200 to 350°C) when properly calibrated. Figure 38-1 shows a copper-constantan thermocouple with its two junctions, I and J, and a millivoltmeter for measuring the thermal emf. If junction I is at a higher temperature than J, then the current is in the direction indicated.

In using the thermocouple for the measurement of temperature, it is customary to keep one of the junctions at the fixed temperature of 0°C by immersing it in a mixture of ice and water. Let this be the junction I in Fig. 38-1. The other junction J is then placed in the region the temperature of which is to be found. After thermal equilibrium is established, the emf produced by the thermocouple is measured. This emf is a function of the temperature of junction J. The functional relationship may usually be well represented by the empirical equation

$$E = at + bt^2, \tag{1}$$

where E is the emf produced by the couple, t is the temperature of junction J, and a and b are empirical constants to be determined by experiment. The derivative of E with respect to t, *i.e.*, $\frac{dE}{dt}$, is called the thermoelectric power of the couple and is approximately a linear function of t.

Tables have been prepared for a number of different standard thermocouples giving the relationship between E and t. A table of values for the copper-constantan thermocouple is given in Appendix III, Table K. In using this table of values, based upon a standard copper-constantan thermocouple, it is generally necessary to correct the reading of the laboratory copper-constantan thermocouple in order to make it correspond with the tabulated values. It has been found that the correction is small and is very nearly

proportional to the observed E of the thermocouple over a wide temperature range. This means that the corrected value of the emf, say E', is directly proportional to the observed value E. Hence

$$E' = KE, \tag{2}$$

where K is very nearly equal to one. If E_o' and E_o represent respectively the corrected value and the observed value of the emf at some known temperature t_o, then K may be determined and Eq. (2) may be written in the form

$$E' = \left(\frac{E_o'}{E_o}\right) E. \tag{3}$$

In this experiment the calibration temperature t_o is the sublimation temperature of solid CO_2 (dry ice). It has the value -78.5°C at which temperature the tabulated value of the emf of a standard copper-constantan couple is $E_o' = 2.72$ mv. E_o in Eq. (3) is determined by immersing the junction J of the laboratory couple in a mixture of solid CO_2 and alcohol and observing the emf developed. The couple could then be used to measure any temperature between -78.5 and 0°C by observing E at that temperature, calculating E' by means of Eq. (3), and using Table K to find the temperature which corresponds to E'. It will be used in this experiment to find the freezing point of mercury.

In Fig. 38-1 a millivoltmeter is shown for the purpose of measuring the emf of the thermocouple. This is not a very satisfactory method of measuring the emf of the couple, since the millivoltmeter draws some current and hence measures the potential difference (P.D.) across the terminals of the couple rather than its total emf.

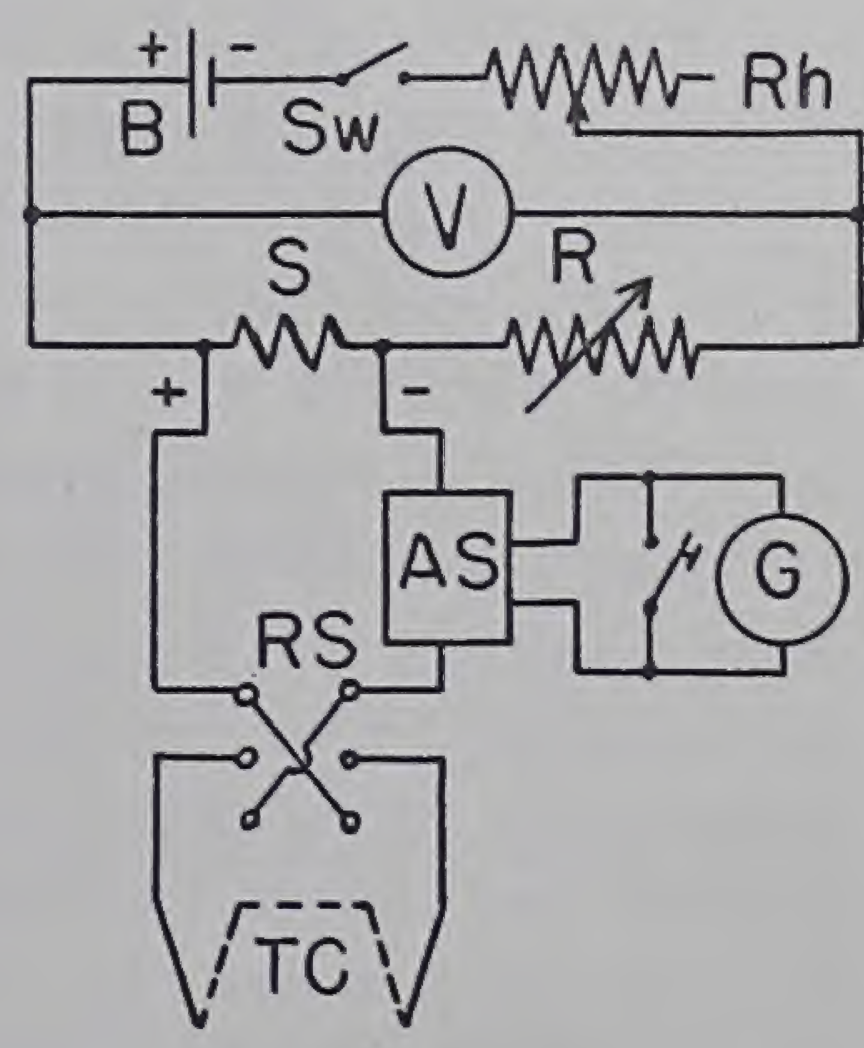


Fig. 38-2.

A much better, but more complicated, procedure is to replace the millivoltmeter with some potentiometer arrangement which will balance the emf of the couple against a known P.D. In this way no current is drawn from the couple when its emf is being measured.

In Fig. 38-2 is shown a satisfactory arrangement for achieving this result. The battery B supplies current through a rheostat Rh to a low resistance S and a decade resistance box R. A voltmeter V reads the P.D. across S and R. The thermocouple TC and galvanometer G are connected in series across the low resistance S. The galvanometer is protected with an Ayrton shunt AS and a damping key. A reversing switch RS is placed in the thermocouple circuit so that connections to the junctions may be reversed, if need be, for balancing.

The emf of the couple is balanced against the P.D. across the resistance S. This is accomplished by varying R until the galvanometer reads zero. Under these conditions it is evident that E for the couple is given by the equation

$$E = V\frac{S}{R + S}, \tag{4}$$

where V is the reading of the voltmeter. Since R is in general quite large compared to S, it is possible to write Eq. (4) in the form

$$E = V\frac{S}{R}. \tag{5}$$

The final working equation for this experiment may be obtained by combining Eqs. (3) and (5). We get

$$E' = E_o' \frac{V}{V_o}\frac{R_o}{R}, \tag{6}$$

where the symbols with the subscript correspond to values at the calibration temperature t_o, and those without the subscript correspond to values at the unknown temperature.

The errors in this experiment are rather difficult to estimate without a more complete experimental analysis because of the nature of the assumption concerning the correction to be made on the observed emf

of the laboratory thermocouple. For precise work the couple should be calibrated at a number of different known temperatures in the range in which the couple is to be used.

As far as Eq. (6) is concerned, the indeterminate error in E' may be computed in the ordinary manner. It should be noted that since V and V_o are practically equal to each other in this experiment, any constant error in the voltmeter reading will contribute little or nothing to the error in E'. The value of E_o' taken from the table is probably good to within ± 0.005 mv. Finally, the readings R and R_o from the decade resistance box are accurate to within $\pm 0.25\%$.

Once the error in E' has been determined the error in the temperature to which it corresponds may be obtained by use of Table K.

Method: Connect the apparatus according to Fig. 38-2 except for the battery. After the instructor has checked the circuit, connect the battery but leave all switches opened. Set the rheostat at about 20 ohms. Set the decade resistance box at 9900 ohms and the Ayrton shunt at zero. Fill a thermos bottle with a mixture of ice and water. Place both junctions of the thermocouple in this mixture, and wait 3 or 4 min for thermal equilibrium to be established. Then close the reversing switch but not the battery switch. Change the Ayrton shunt setting step by step from zero to unity through the intermediate settings of 0.001, 0.01, and 0.1. Stop the adjustment if there is any appreciable galvanometer deflection. Such deflection under these circumstances indicates trouble. It should be corrected before proceeding with the experiment.

Part I. Calibration of Thermocouple. Half fill a second thermos bottle with alcohol or acetone and *slowly* add solid CO_2 until the flask is almost full. The mixture will bubble over if the CO_2 is added too rapidly. Take one of the junctions of the thermocouple out of the mixture of ice and water (leave the other in the ice), wipe it dry, and then place it in the CO_2 mixture. After thermal equilibrium has been established, the two junctions will be at 0° and −78.5°C, respectively.

With the Ayrton shunt set at 0.001, close the battery switch and the reversing switch. See that the voltmeter is reading. Then balance the emf of the thermocouple against the P.D. across S by reducing R from its original setting of 9900 ohms. Balance is indicated by zero deflection of the galvanometer. If a decrease in R does not reduce the galvanometer deflection, reverse switch RS and try again. *For a final balance, use the full sensitivity of the galvanometer by changing the Ayrton shunt setting to* 1. Before taking this reading, be sure to stir both the ice mixture and the CO_2 mixture. Record the decade-box reading R_o to the nearest ohm and the voltmeter reading V_o at the balance point. Take four more readings of R_o and V_o, stirring the mixtures before each reading. Any constant change of R_o in these five readings indicates a lack of thermal equilibrium. In this case the process should be continued until a constant value of R_o is obtained. Stirring the mixture before each reading is important. Set the Ayrton shunt to zero and open battery and reversing switches after these readings.

Compute the average value of R_o and of V_o. For the error in average R_o use either the mean deviation or 0.25%, whichever is the larger. The value of E_o' at −78.5°C may be obtained from Table K.

Part II. Freezing Point of Mercury. Remove the junction from the CO_2 mixture and wipe it off with a paper towel. Place it in the tube containing the mercury. Then carefully place this tube in the mixture of CO_2 and alcohol. Do not allow the mixture to boil over in this process. Since mercury freezes at about −40°C, it should freeze in this mixture in 5 or 10 min and eventually reach the temperature of the mixture.

After the mercury seems to have frozen, close the battery switch and reversing switch; then balance the circuit as in Part I by varying R. Stir both mixtures before taking readings. Repeat this process as time goes on until a steady value of R is obtained. This will indicate that the mercury is in thermal equilibrium with the CO_2 mixture. The value of R should be very nearly equal to R_o found in Part I of the experiment provided the voltmeter reading V has not changed appreciably. At this point the tube containing the mercury and thermocouple junction should be removed from the CO_2 mixture.

As soon as the tube of mercury is taken out of the CO_2 mixture, its temperature will rise very rapidly until the mercury reaches its melting point. At this point the melting mercury absorbs its latent heat of fusion, thus keeping the temperature constant during the process of fusion. After all of the mercury has melted, its temperature will again rise until it reaches room temperature. Hence the melting point of mercury may be determined by use of the heating curve (temperature versus time) for mercury.

In order to obtain this heating curve it is necessary to determine the temperature of the mercury as a function of the time. The total time interval, of course, must be large enough to include both the solid and liquid states of the mercury. This may be done in the following manner.

Immediately after the tube of solid mercury is taken from its CO_2 bath, its temperature should be taken every 30 sec by means of the thermocouple. This can be done by taking a set of R and V values at half-minute intervals until the temperature of the mercury is well above its melting temperature. Since the mercury heats up very rapidly at first, it is advisable to keep the circuit in *continual* adjustment for balance by constant adjustment of R. In this process the ice mixture should be stirred continuously to ensure a constant temperature of 0°C for the other junction of the couple. It will be noted that while the mercury is melting the value of R will remain practically constant, indicating a constant temperature for the mercury. After melting has taken place, the temperature of the mercury will again rise, requiring an increasing value of R for balance. Continue taking R and V values every half-minute until the value of R attains the maximum resistance of the decade box. At this point the switch RS should be reversed and readings discontinued for 2 or 3 min (keep track of the time). After this time has elapsed, take five or more readings at half-minute intervals. Then set the Ayrton shunt to zero and open the battery switch and the reversing switch.

By means of Eq. (6) and Table K compute the set of temperatures for the mercury. Plot these temperatures (ordinate) against the times (abscissa). By use of this heating curve, determine the melting temperature of mercury to the nearest half-degree. Determine the error in this temperature. Compare this observed temperature with the accepted melting point of mercury (−38.9°C).

Record: Record the apparatus numbers of the important components of your equipment. Tabulate your data. Summarize your results.

QUESTIONS

1. What will be the value of R in Part II of this experiment when the mercury reaches the temperature of 0°C?

2. Why was it necessary to reverse RS at a certain point in the heating curve and discontinue readings for a short time?

3. How could one determine the latent heat of fusion of mercury by use of the heating curve? What additional data would be necessary for this determination?

4. Suppose one junction of this thermocouple were held at room temperature, say 20°C, instead of at the ice point. Would it still be possible to measure temperature with this couple by using Table K? Explain.

Experiment 39.

Electromagnetic Induction

Object: To study some of the phenomena of electromagnetic induction; particularly to note the emf induced in a secondary coil when the magnetic flux linking it is changed. This magnetic flux is produced either by a current-carrying primary coil or by a permanent magnet.

Apparatus: Table galvanometer, primary coil (200 turns, No. 20 wire), secondary coil (350 turns, No. 24 wire), battery, switch, permanent bar magnet, brass rod, iron rod, high resistance (20,000 ohms).

Theory: Whenever the magnetic flux (magnetic lines of force) linking a coil of wire *changes*, there is an emf induced in the coil which is proportional to the *time rate* of *change* of this magnetic-flux linkage. The magnetic flux linking the coil may arise from any source whatever, *e.g.*, permanent magnet, earth's field, current in another coil, current in the coil itself. But the induced emf is independent of the nature of the source; it depends only upon the time rate of change of magnetic-flux linkage, whatever the source. This is known as Faraday's law of electromagnetic induction (1831) and is the foundation principle on which much of our modern-day electrical machinery is based.

The direction of the induced emf is given by Lenz's law, that is, the induced emf is always in such a direction as to set up electrical conditions—induced currents—which tend to *oppose* the *change* which is bringing them about. This is in complete accord with the law of the conservation of energy.

A simple diagram will serve to illustrate these principles. In Fig. 39-1 is shown a loop of wire with some magnetic flux linking it. As long as this magnetic flux is constant, there is no induced emf in the wire. Sup-

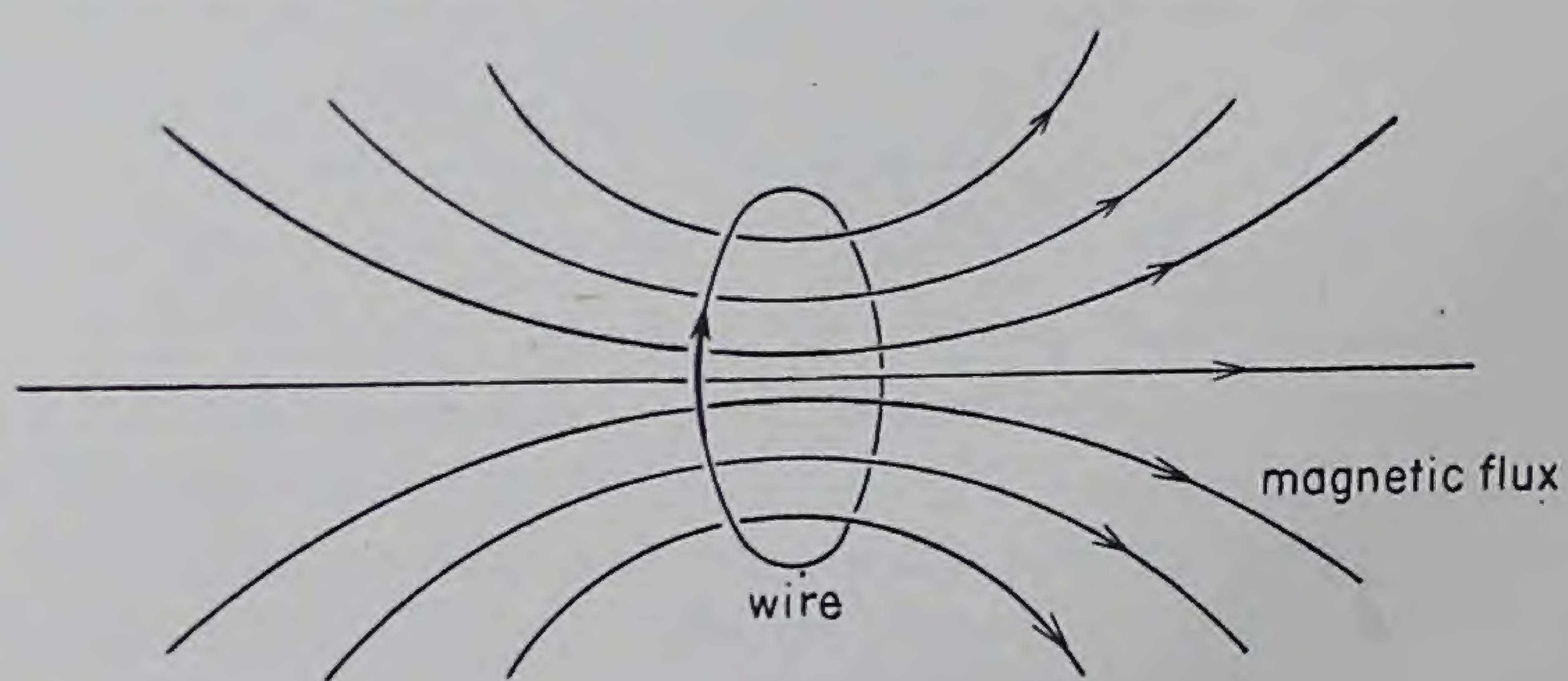


Fig. 39-1.

pose, however, that this magnetic flux is *increasing* with time. There will then be an emf induced around the wire producing an induced current. The magnitude of this emf will be directly proportional to the time rate of change of the magnetic flux. The direction of this induced emf and current must, by Lenz's law, be such as to oppose the increasing magnetic flux, *i.e.*, the induced emf and current must be around the loop in a counterclockwise direction as one views the coil from the rear looking in the direction of the magnetic flux. In this case the magnetic field set up by the induced current will be in a direction *opposite* to that of the *increase* of magnetic flux. On the other hand, if the magnetic flux linking the coil is *decreasing* with time, the

induced current will set up a field which opposes the *decrease* in this magnetic flux. This means that the induced current is in the direction opposite to that in the first case.

In this experiment the induced emf in the secondary coil will be detected by connecting it to a galvanometer and observing the deflection. The magnetic flux linking this secondary coil will be produced either by use of a current-carrying primary coil or by use of a permanent bar magnet.

Method: First determine the relation between the direction of the current in the galvanometer and the direction of its deflection. To do this connect the battery in series with the galvanometer and a very high resistance (20,000 ohms) as shown in Fig. 39-2. Since the direction of the current furnished by the battery is known, the direction of the current through the galvanometer and the corresponding deflection can be determined.

Then connect the secondary coil (wound with fine wire) to the galvanometer. Connect the primary coil (wound with coarse wire) to the battery through a switch. Keep the switch closed only while readings are being made. The direction of current in the primary is determined by the battery. The direction of any current in the secondary may be deduced from the observed direction of the galvanometer deflection.

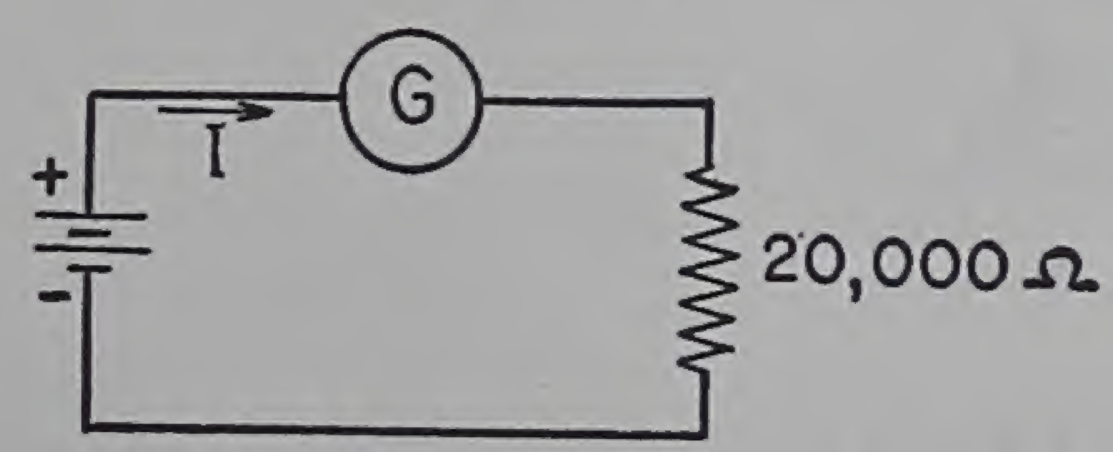


Fig. 39-2.

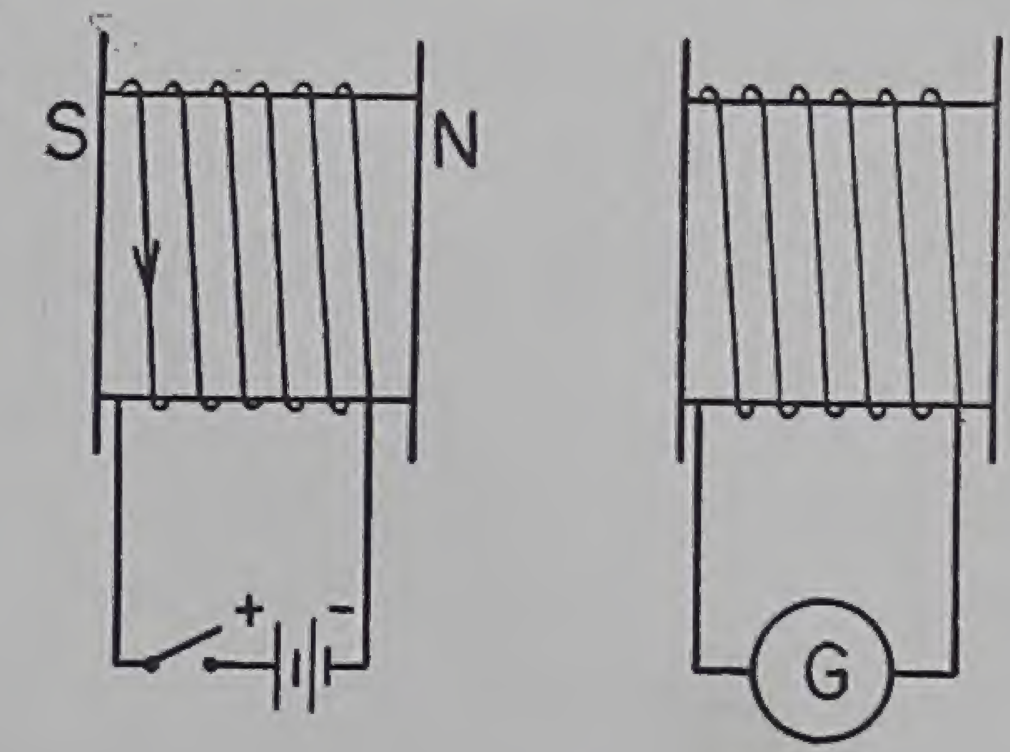


Fig. 39-3.

Step 1. Line the coils up as shown in Fig. 39-3 so that the windings in both coils are in the same direction (clockwise or counterclockwise). Close the battery switch. Move the secondary quickly away from the primary. Bring it quickly back. Record the directions of the currents in the primary and secondary coils under these two conditions. To do this, draw diagrams similar to Fig. 39-3 and put in arrows indicating directions of motion and currents. Repeat this process, moving the primary instead of the secondary. Again record directions of motion and currents in this case.

Step 2. Make and break the current in the primary and record the corresponding directions of the primary and secondary currents. Determine the galvanometer throws while the primary current is made and broken when the primary and secondary coils are separated by 0, 1, 2, 3, 4, 5, and 10 cm. Plot a curve of galvanometer throws against distance between coils.

Step 3. Open the primary circuit. Place the secondary as shown in Fig. 39-4. Thrust the N pole of the bar magnet into the secondary coil and note the direction of the galvanometer throw. Pull the N pole out and observe the throw. Perform the same operations using the S pole of the magnet. Record the direction of motion of the magnet and that of the induced current by means of a diagram similar to Fig. 39-4.

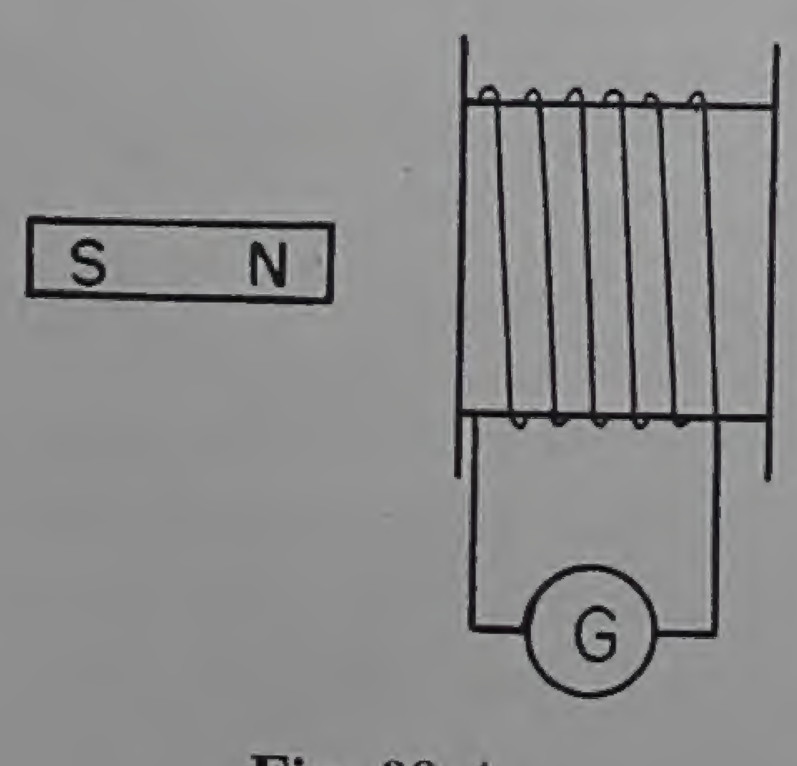


Fig. 39-4.

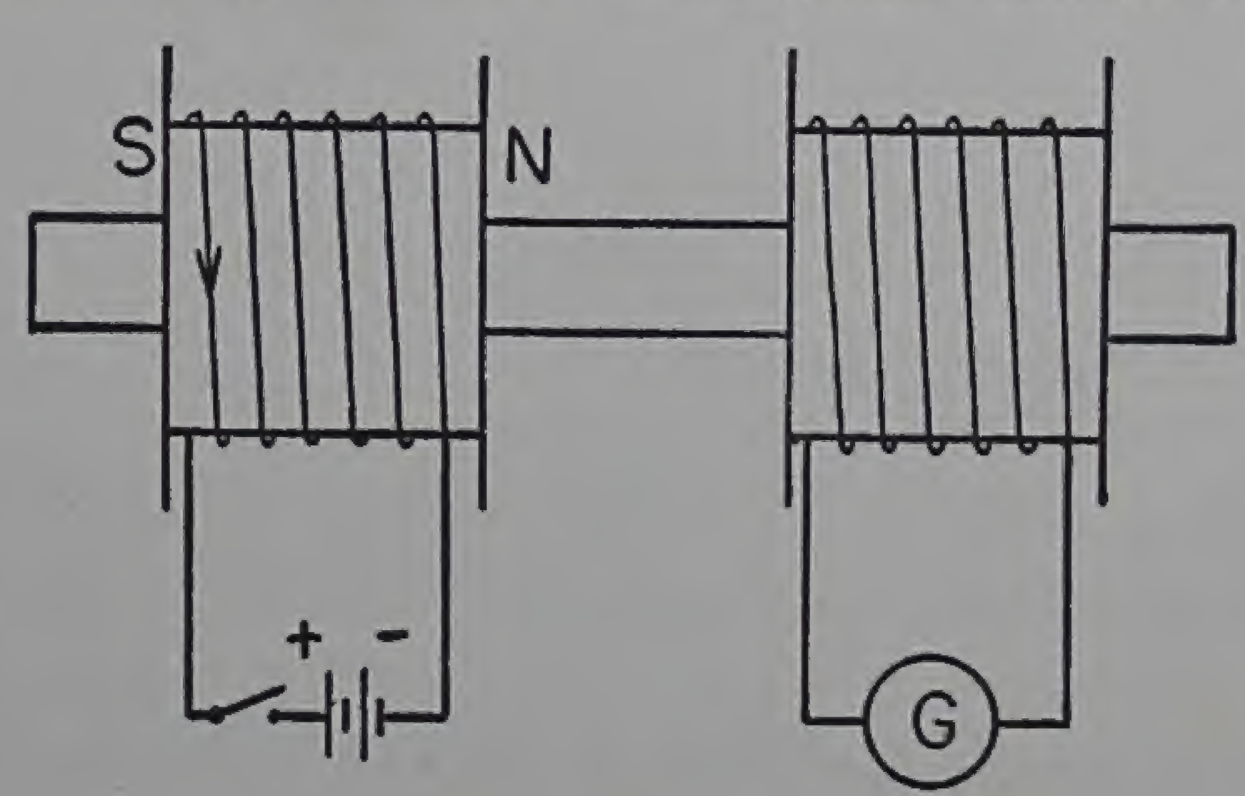


Fig. 39-5.

Step 4. Line the two coils up as in Fig. 39-5 with a distance of 2 cm between the ends. Place a brass rod through the cores of both coils and repeat the first part of Step 2. Record your results. Replace the

brass rod with an iron rod and repeat. Record your results. Be sure to include the sizes of the throws of the galvanometer in this step.

Step 5. Place the secondary on the primary as shown in Fig. 39-6. Determine whether or not there is any position for which the galvanometer throw is zero when the primary current is made and broken.

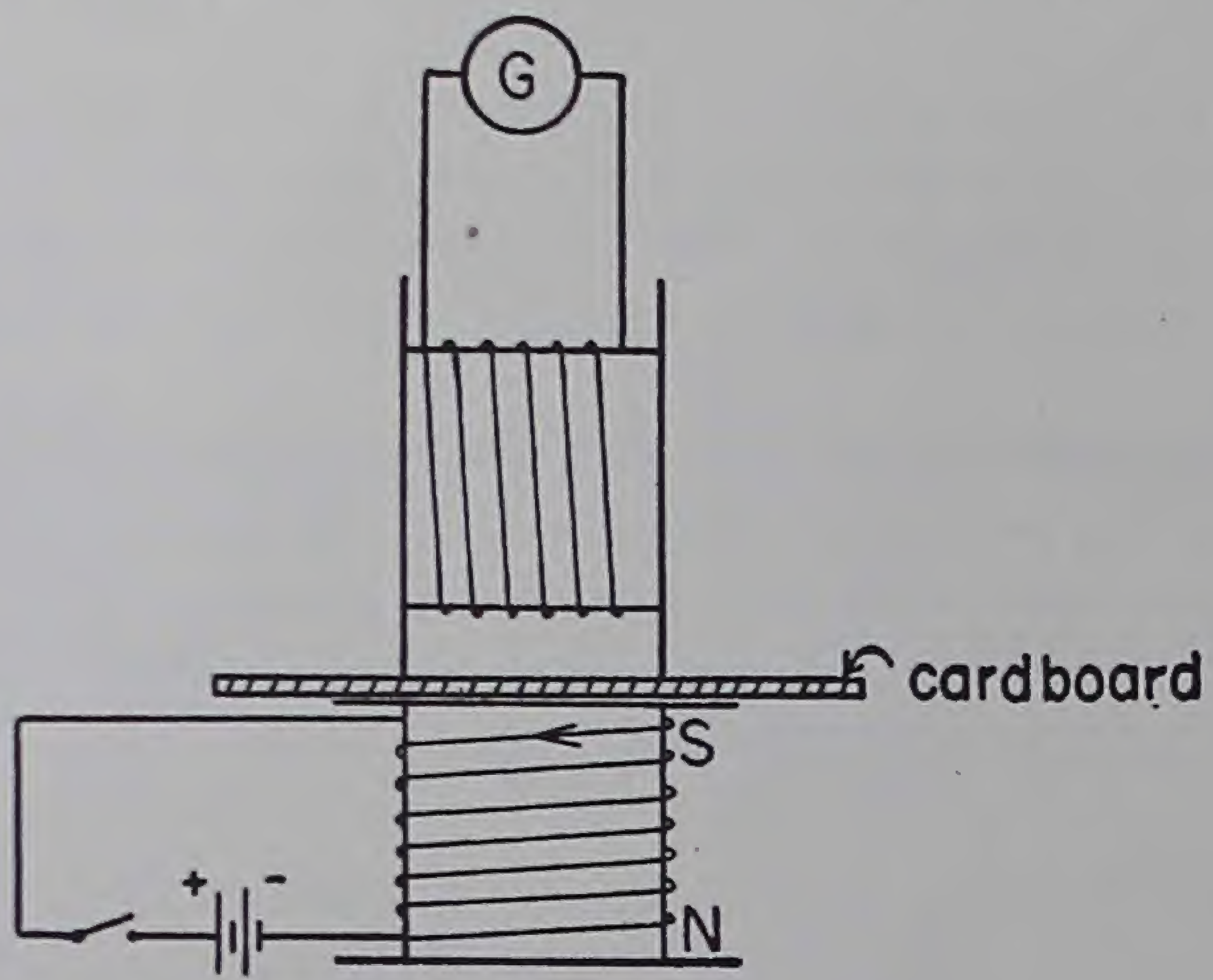


Fig. 39-6.

QUESTIONS

1. In Step 1 what is the source of the energy which appears in the secondary circuit?

2. In Step 2 what is the source of energy which appears in the secondary circuit when the primary circuit is broken? HINT: A magnetic field possesses energy. What is the source of energy in the secondary circuit when the primary circuit is made? Explain the observed difference in galvanometer deflections when the distance of separation is 2 and 10 cm.

3. In Step 4 explain the difference in galvanometer deflections for the brass and iron rods. Compare these deflections with the corresponding one obtained in Step 2.

4. In Step 5 explain in terms of magnetic-flux linkage why, for a certain orientation of the secondary coil, there is no appreciable induced current in the secondary. Use a sketch.

Experiment 40.

Earth Inductor

Object: To determine the horizontal and vertical components of the earth's magnetic field using an earth inductor.

Apparatus: Earth inductor, ballistic galvanometer, damping key, dial resistance box, ammeter, rheostat, battery, switch, and magnetic compass. See Appendix II, Sections H1, 3, 4.

Theory: The horizontal and vertical components of the earth's magnetic field may be measured by turning a coil (earth inductor) in the earth's field in such a manner that it cuts the horizontal or vertical component of the magnetic flux. If this coil is connected to a ballistic galvanometer, the charge flowing through the coil and galvanometer will be proportional to the magnetic flux cut.

The earth inductor (Cenco) consists of a frame supporting a coil. There are two separate windings on this coil—the current winding and the inductor winding. The instrument is arranged so that the coil may be rotated, by spring action, through 180° about an axis either in a horizontal or vertical plane. The binding posts for the current coil are mounted on the ring, and those for the inductor coil are mounted on the framework. A data plate giving the electrical and geometrical characteristics of the coil is located on the frame. See Fig. 40-1.

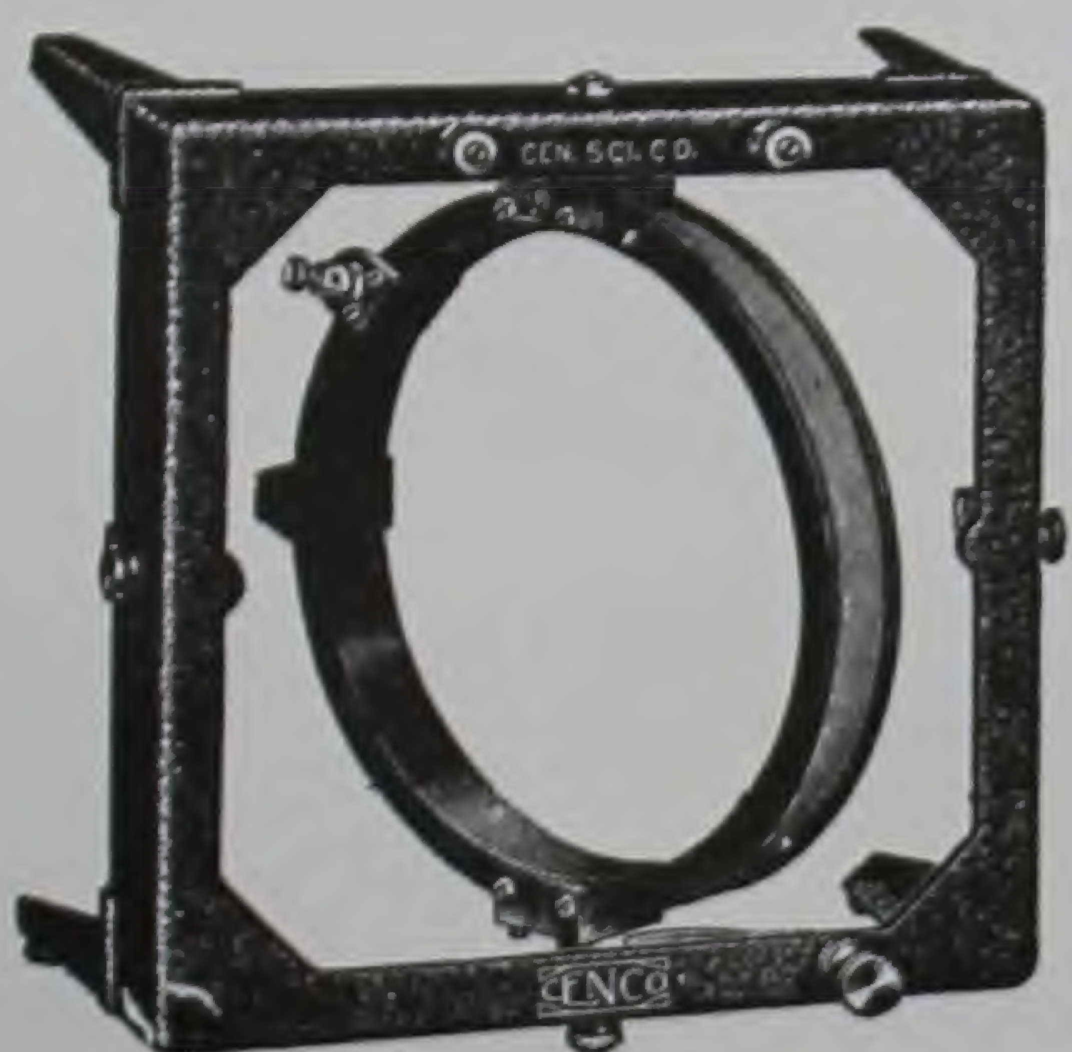


Fig. 40-1.

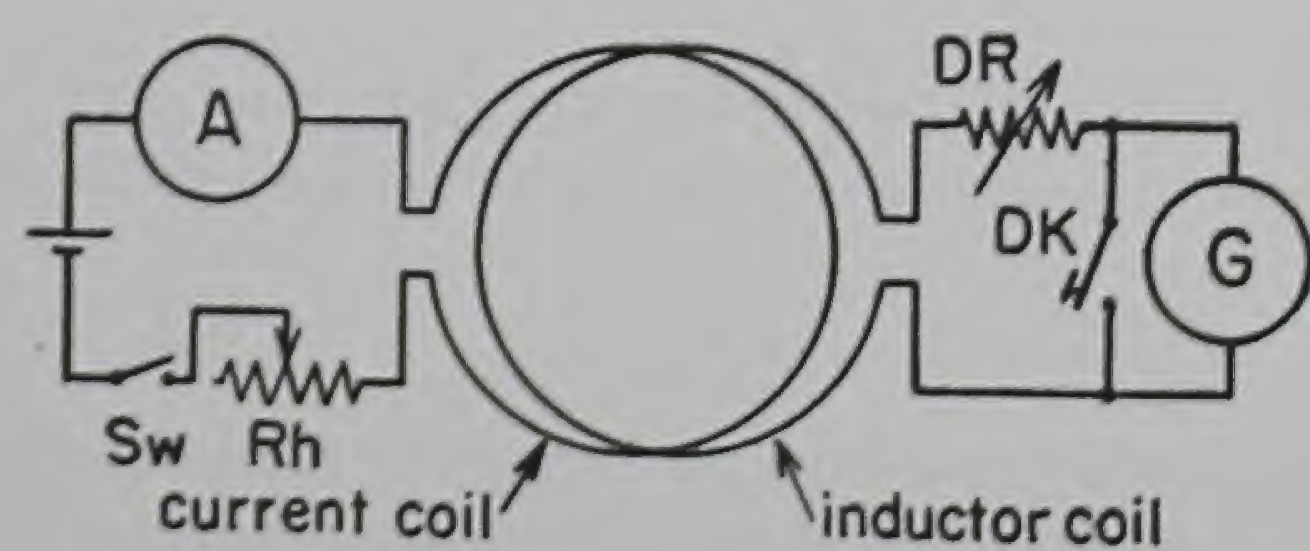


Fig. 40-2.

The connections to the earth inductor are shown in Fig. 40-2. The inductor winding is connected to a ballistic galvanometer G through the decade resistance box DR. A damping key DK is placed across the galvanometer to control its deflections.

The current winding is connected to a battery through an ammeter A, switch Sw, and rheostat Rh.

Suppose the earth inductor is placed in a horizontal plane with its axis of rotation parallel to the H field of the earth, *i.e.*, with its axis of rotation lying parallel to the direction of a compass needle. In this position it encompasses a maximum amount of the vertical flux of the earth's field, the V field. If now the coil is

suddenly rotated through 180°, all of the vertical magnetic flux will be cut *twice* without any disturbance caused by the H field. (In this process the current winding must be disconnected for mechanical reasons.) Let A represent the average area of the coil, V represent the vertical intensity of the earth's field, and N_i represent the number of turns in the inductor winding. Then ϕ_v, the total magnetic-flux change in this process, will be

$$\phi_v = 2AN_iV. \tag{1}$$

By Faraday's law of electromagnetic induction, the induced emf, E_i, in volts will be

$$E_i = -10^{-8}\frac{d\phi_v}{dt}. \tag{2}$$

If we substitute I_iR for E_i in Eq. (2) and integrate with respect to t over the time of the process, we get

$$RQ_v = -10^{-8}\phi_v = -2 \times 10^{-8}AN_iV, \tag{3}$$

where R is the resistance of the galvanometer circuit. The throw of the galvanometer d_v is proportional to the charge Q_v passing through it, *i.e.*, $Q_v = Kd_v$. Hence Eq. (3), after solving for V, becomes

$$V = \frac{-10^8RKd_v}{2AN_i} \qquad \text{(gauss)}. \tag{4}$$

In order to determine H, the horizontal component of the earth's field, we may place the earth inductor on its side so that its axis of rotation is vertical and so that its plane is perpendicular to the magnetic meridian. In this case the inductor encompasses the maximum amount of horizontal magnetic flux. When the coil is turned through 180°, this flux is cut twice without interference from the vertical flux. A throw d_H of the galvanometer results which is related to H by means of the equation

$$H = \frac{-10^8RKd_H}{2AN_i} \qquad \text{(gauss)}. \tag{5}$$

Equations (4) and (5) are sufficient to determine V and H provided K is known. This is generally not the case and it is necessary to carry out an auxiliary experiment in order to determine it. This may be done as follows: The current winding is now connected as shown in Fig. 40-2 and serves as a primary circuit. Any *change* in the primary current will induce an emf in the secondary circuit (inductor winding) thus producing a throw of the galvanometer. The emf, E_s, induced in the secondary because of a changing current I_p in the primary, is given by the relation

$$E_s = -M\frac{dI_p}{dt}, \tag{6}$$

where M is the mutual inductance between the two circuits. E_s may be replaced by RI_s in Eq. (6) and the equation integrated over the time of change in the customary manner. We get

$$RQ_s = RKd_s = -M\,\delta I_p, \tag{7}$$

where δI_p is the *change* in the primary current and RK is the same combination of secondary circuit resistance and galvanometer constant as appears in Eqs. (4) and (5).

If we solve Eq. (7) for RK and substitute in Eqs. (4) and (5), we finally get

$$V = \frac{10^8}{2}\frac{M\,\delta I_p}{AN_i}\frac{d_v}{d_s}, \tag{8}$$

and

$$H = \frac{10^8}{2}\frac{M\,\delta I_p}{AN_i}\frac{d_H}{d_s}. \tag{9}$$

These Eqs. (8) and (9) enable us to determine V and H in terms of the measured quantities d_v, d_H, d_s, δI_p, and the quantities M, A, and N_i obtainable from the data plate of the apparatus.

The approximate determinate-error equations for the experiment are

$$\frac{\Delta V}{V} = \frac{\Delta \delta I_p}{\delta I_p} + \frac{\Delta d_v}{d_v} - \frac{\Delta d_s}{d_s} \tag{8a}$$

and

$$\frac{\Delta H}{H} = \frac{\Delta \delta I_p}{\delta I_p} + \frac{\Delta d_H}{d_H} - \frac{\Delta d_s}{d_s}. \tag{9a}$$

The small errors in M, A, and N_i may be neglected.

Method: Determine the direction of the magnetic meridian at the place where the inductor is to be used. A large compass may be used for this purpose, and a chalk line should be drawn on the table to show this direction.

Connect the inductor winding (terminals on frame) to the galvanometer circuit as shown in Fig. 40-2. Do not make the connections as yet to the current windings (terminals on coil). Place the inductor so that its face is horizontal and so that its axis of rotation is in line with the magnetic meridian (chalk line). Set the dial box at about 1000 ohms resistance. Then release the spring catch allowing the coil to flip through 180°. Observe the throw d_v of the galvanometer. It should be about 125 mm. If it is much more or much less than this, adjust the decade resistance somewhat until the proper throw is obtained. The exact value of the decade resistance is immaterial since it does not enter the final equations. Make five determinations of the galvanometer throw d_v.

Then turn the inductor on its side with the face of the coil perpendicular to the magnetic meridian and with the axis of rotation in a vertical position. Place the inductor so that the d_H deflection is in the same direction as the d_v deflection. The inductor is now in position to "cut" the horizontal magnetic flux of the earth. Make five determinations of the throw of the galvanometer for this case, *i.e.*, d_H. *Do not change the decade resistance in this process* since the galvanometer constant K is a function of this resistance.

Finally connect the battery circuit to the terminals of the current winding on the ring. Set the rheostat *Rh* so that the ammeter reads about 0.50 amp when the battery switch is closed. The galvanometer will deflect in opposite directions when the switch is closed and when it is opened. Choose for your record that operation (close or open) which leads to a galvanometer throw in the *same* direction as for d_H and d_v. Make five determinations of the galvanometer throw d_s for this operation. Read the ammeter for each throw. This will equal the change δI_p in primary current.

Record the values of d_v, d_H, d_s, and I_p. Use their averages to compute V and H by means of Eqs. (8) and (9). The values of M and N_i are given on the data plate of the inductor. Also this plate gives the internal and external diameters of the coil. Use the average of these values in computing A. Note that the value of M is given in millihenrys (thousandths of a henry). It must be converted to henrys before being used in the equation.

In calculating the errors in V and H use the mean deviations in d_v, d_H, d_s; or $\frac{1}{2}$ mm; whichever is the larger. The error in the ammeter may be taken as $\pm 1\%$ of full-scale reading.

Determine the angle of dip θ by means of the equation

$$\tan \theta = \frac{V}{H}.$$

This is the angle which the total intensity of the earth's field makes with the horizontal.

Record:

App. No. Earth inductor__________
Galvanometer__________
Ammeter__________
Dial box__________

Dial-box setting (   ) ohms

Rdg	d_v, mm	d_H, mm	d_s, mm	I_p, amp
1	110.5	35.0	140.5	0.500
2				
3				
4				
5				
Average				

Inductor-coil Data:

N_i = 1000 turns
D (inside) = ( ) cm
D (outside) = ( ) cm
D (average) = ( ) cm
A (average) = ( ) cm^2
M = ( ) mh

Results:

V = ( ) gauss
ΔV = ( )
H = ( )
ΔH = ( )
θ = ( )
Total intensity = ( )

QUESTIONS

1. Under what condition is the throw of the galvanometer in this experiment independent of the time of flow of charge through it? Would you expect d_V or d_H to change materially if the inductor coil were rotated very slowly in this experiment? Explain.

2. Explain carefully how the earth inductor could be used to determine the magnetic north-south direction and hence act as a magnetic compass. (Principle of the earth inductor compass.)

Experiment 41.

Mutual Inductance

Object: To determine experimentally the mutual inductance of a primary-secondary air solenoid and to compare this value with the theoretical value computed from the geometry of the solenoid.

Apparatus: Reflecting galvanometer, Hibbert or Cenco magnetic-flux standard, primary-secondary air solenoid, damping key, single-pole switch, reversing switch, battery, ammeter, rheostat. See Appendix II, Notes H1, 3, 4; K.

Theory: When two coils are placed so that a current in one (the primary) produces a magnetic field such that some part of this field links the other (the secondary), the coils are said to be coupled magnetically and to possess mutual inductance.

The mutual inductance M between two coils is given by the equation

$$E_s = -M \frac{dI_p}{dt}, \tag{1}$$

which is essentially a form of Faraday's law of electromagnetic induction. In this equation E_s is the emf induced in the secondary coil and dI_p/dt is the time rate at which the current I_p in the primary coil is changing. If E_s is given in volts, I_p in amperes, and t in seconds, then M will be expressed in henrys. A pair of coils then will have a mutual inductance of 1 henry if the current in the primary, changing at the rate of 1 amp/sec, induces an emf of 1 volt in the secondary. This unit of mutual inductance, the henry, is a large one, and it is customary to express mutual inductance in a thousandth of a henry, the millihenry, or even in a millionth, the microhenry.

In laboratory work Eq. (1) is not convenient to use, involving, as it does, a rate of change of current. It may be put into a more convenient form by use of Ohm's law when the secondary coil is part of a closed circuit. In this case, the induced emf, E_s, in the secondary, produces a current I_s in the secondary circuit, which by Ohm's law is given by the equation

$$E_s = I_s R_s, \tag{2}$$

where R_s is the total resistance of the secondary circuit. If we eliminate E_s between Eqs. (1) and (2), multiply the resulting equation by dt, and integrate over the time during which the primary current is changing, we get

$$R_s Q_s = -M\ \delta I_p \tag{3}$$

where $Q_s(= \int I_s\, dt)$ is the total charge circulating in the secondary circuit, and δI_p is the total change in the primary current. Q_s may conveniently be measured by connecting a ballistic galvanometer in the secondary circuit; δI_p may be measured by using an ammeter in the primary circuit.

A circuit that enables us to carry out these operations, and hence determine M, is shown in Fig. 41-1. The chief element in this circuit is a primary-secondary air solenoid. It consists of a long primary coil with a short secondary coil wound around its center. The primary coil is connected to a battery through a reversing switch RS, an ammeter A, and a variable rheostat Rh. The secondary coil is connected to a ballistic galvanometer G through switch Sw and a Hibbert standard HS (or Cenco standard). A damping key DK is connected across the galvanometer.

In order to determine M, the mutual inductance between the primary and secondary coils, it is only necessary to cause an abrupt change in the primary current I_p and observe the corresponding deflection d of the ballistic galvanometer. This change in I_p may be made by closing, opening, or reversing the switch RS. δI_p will be the difference between the current in the primary just before the change has occurred and that just after the change has been made. The galvanometer deflection d will be proportional to Q_s, the quantity of charge which circulates in the secondary circuit as a result of this change. Equation (3) may then be written

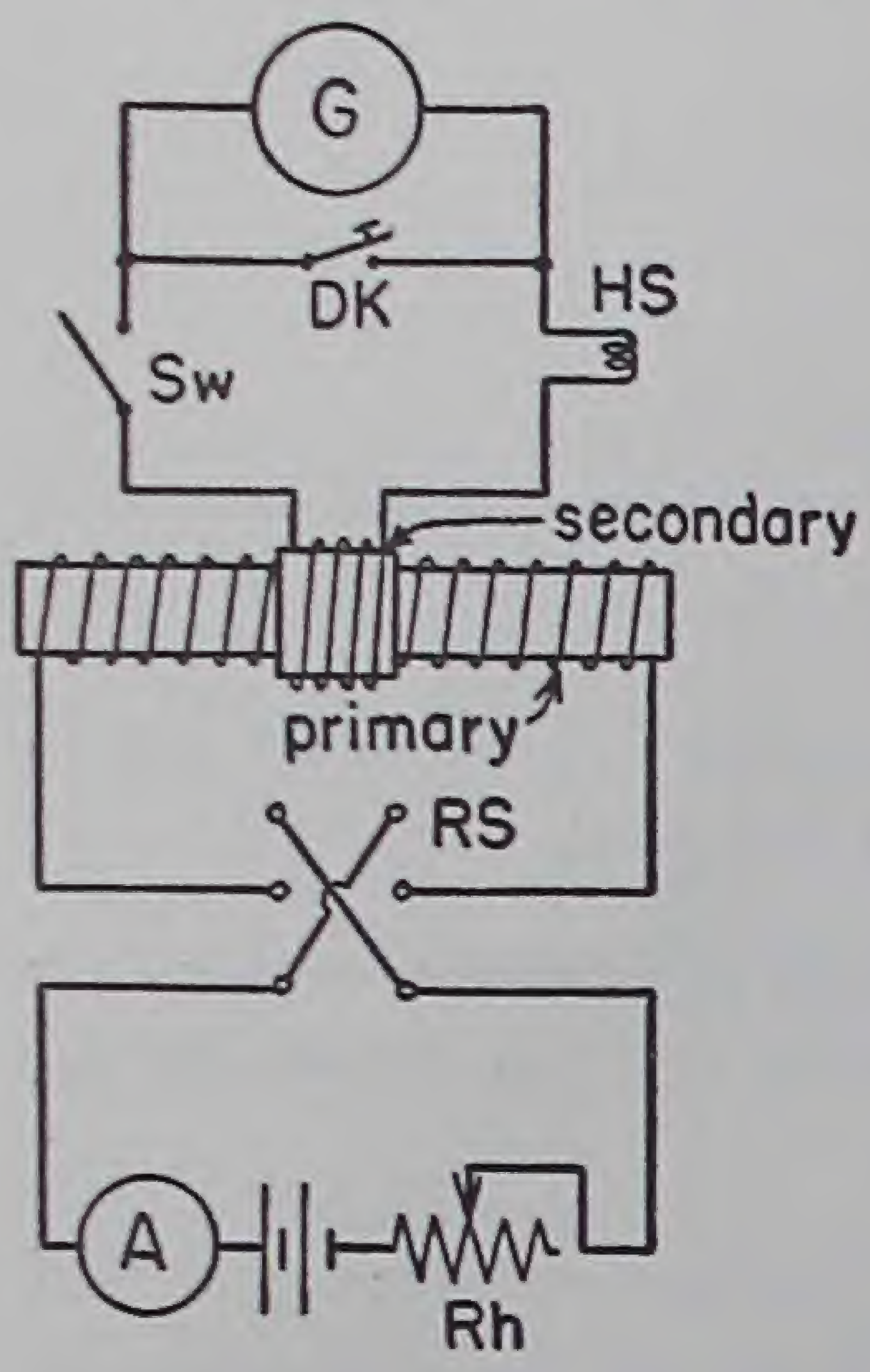


Fig. 41-1.

$$R_sKd = -M\ \delta I_p, \tag{4}$$

where K is the ballistic constant of the galvanometer, *i.e.*, the ratio of Q_s to d. Equation (4) is insufficient by itself to give us the value of M since R_sK is unknown. In order to determine R_sK, use is made of the flux standard.

Determination of K Using Flux Standard. Equation (6) of Appendix II, Note K may be written

$$E_H = -10^{-8}N_H\frac{d\phi_H}{dt}, \tag{5}$$

where E_H = induced emf, in volts,
N_H = number of turns in the coil of the flux standard,
ϕ_H = magnetic flux in Maxwells linking each turn, and
t = time in seconds.

Since the standard is connected in series with the galvanometer and secondary coil, the current I_H produced by the standard will be E_H/R_s. Hence Eq. (5) may be written in the form

$$R_sI_H\,dt = -10^{-8}N_H\,d\phi_H. \tag{6}$$

This equation may then be integrated to get

$$R_sQ_H = -10^{-8}N_H\phi_H, \tag{7}$$

where Q_H is the charge circulating in the secondary circuit because of the action of the standard. This charge causes a galvanometer deflection d_H just proportional to it and with the same galvanometer constant K as before. Hence Eq. (7) may be written as

$$R_sKd_H = -10^{-8}N_H\phi_H. \tag{8}$$

Determination of M. The mutual inductance M may now be determined explicitly by dividing Eq. (4) by Eq. (8) and solving for M. We get

$$M(\text{exp}) = 10^{-8}\frac{N_H\phi_H}{\delta I_p}\frac{d}{d_H} \qquad \text{(henrys)}. \tag{9}$$

All of the quantities on the right side of this equation are either known or may be measured. Hence Eq. (9) is the working equation for this experiment.

From the geometry of the coil it is possible to develop a formula for the mutual inductance of the solenoid used in this experiment:

$$M(\text{theo}) = \frac{\pi^2}{10^9}\frac{D_p{}^2N_sN_p}{L_p}\left[1 - \frac{1}{2}\left(\frac{D_p}{L_p}\right)^2\right] \qquad \text{(henrys)}, \tag{10}$$

where D_p = *average* diameter of the primary coil in centimeters (core diameter + wire diameter),
N_s = number of turns in the secondary,
N_p = the number of turns in the primary, and
L_p = length of primary in centimeters.

The bracket factor in Eq. (10) is the so-called end correction and approaches unity when $D_p \ll L_p$.

Error Equations. The *determinate*-error equation corresponding to Eq. (9) may be written

$$\frac{\Delta M}{M} = \frac{\Delta d}{d} - \frac{\Delta d_H}{d_H} - \frac{\Delta\delta I_p}{\delta I_p} + \frac{\Delta(N_H\phi_H)}{N_H\phi_H}. \tag{9a}$$

The determinate-error equation corresponding to Eq. (10) is very nearly

$$\frac{\Delta M}{M} = \frac{2\Delta D_p}{D_p} - \frac{\Delta L_p}{L_p} \tag{10a}$$

since N_s and N_p are known exactly and the error in the correction term may safely be ignored. Why?

Method: Connect the apparatus according to Fig. 41-1 but have all switches open during this process.

Adjust the galvanometer scale so that the initial reading on the scale is zero when the galvanometer coil is at rest. Since the determination of M (exp) depends primarily upon the ratio of two galvanometer throws—d when the *primary current* is *reversed* and d_H when the *flux standard* is *used*—it is highly advisable to have these two throws in the *same direction* and of *about the same magnitude*. Therefore, a preliminary trial should be made with the view of satisfying the above conditions either by adjustment of the flux standard or by adjustment of the primary current. If the Hibbert flux standard (fixed) is used, adjustment of the primary current should be made by use of the battery rheostat to meet these conditions. If the Cenco flux standard (variable) is used, it may be adjusted along with the primary current in order to meet these conditions.

After this preliminary trial has been run and adjustments made so that d and d_H are approximately equal in magnitude and occur in the same direction for a noted change of the reversing switch RS, the principal part of the experiment may be performed.

With the primary circuit open but with the galvanometer switch Sw closed, determine the throw (maximum deflection) d_H of the galvanometer when the Hibbert coil is dropped (or the Cenco coil released). Make five determinations of d_H. Use their average in the determination of M and use the mean deviation or $\frac{1}{2}$ mm as the error in d_H, whichever is the larger.

Next close the reversing switch RS choosing its closed position such that on *reversing it* the resulting galvanometer deflection is in the *same direction* as the deflection d_H. Take five readings of the galvanometer throw d and the corresponding reading of the ammeter. After each reading the reversing switch must be returned to its initial position and the galvanometer brought to rest at its zero position. The change of current δI_p in the primary circuit in this process will be $2I_p$ where I_p is the ammeter reading.

From these data (d_H, d, I_p) and the values of N_H and ϕ_H given on the Hibbert standard (or Cenco standard), compute, by use of Eq. (9), the value of the mutual inductance, M(exp). Determine also the error in M(exp). The error in the calibrated ammeter readings may be taken as $\pm 1\%$ of full-scale deflection. The error in the flux standard calibration may be taken as $\pm\frac{1}{2}\%$.

From the geometrical data given on the primary-secondary solenoid (D_p, L_p, N_s, N_p) compute, by use of Eq. (10), the theoretical value of the mutual inductance, M(theo). Also compute the error in M(theo) by use of Eq. (10a). The errors in both D_p and L_p may be taken as ± 0.02 cm.

Compare M(exp) with M(theo).

Record: Give the apparatus numbers of the galvanometer, flux standard, solenoid, and ammeter. Tabulate your data. Summarize your results.

QUESTIONS

1. Develop the error equations (9a) and (10a).
2. What percentage error would be introduced into your value of M(theo) if the end correction in Eq. (10) were neglected? Is this error significant?
3. Which of the two values, M(theo) or M(exp), is the more accurately determined in this experiment? Explain.
4. If it were desired to determine the numerical value of the ballistic galvanometer constant K in this experiment, what additional information would be required?
5. If a piece of soft iron happened to be near the end of the primary-secondary solenoid, what effect would this have on the results of this experiment?

Experiment 42.

Alternating-current Series Circuit

Object: To study current, potential difference, impedance, power consumption, and phase relationships in an R-L-C series a-c circuit.

Apparatus: A-c circuit board, a-c ammeter (0—1), a-c voltmeter (0—150), a-c wattmeter (0—150), rheostat (100 ohms). The circuit board consists of the following elements in series: switch, fuse, condenser bank (8 μf paper), choke (inductance, 0.7 henry), and 60-watt lamp bulb (resistance). Jacks are placed at the points M, N, O, P, as indicated in Fig. 42-1, so that measurements may be made across any or all of the elements without disconnecting the circuit.

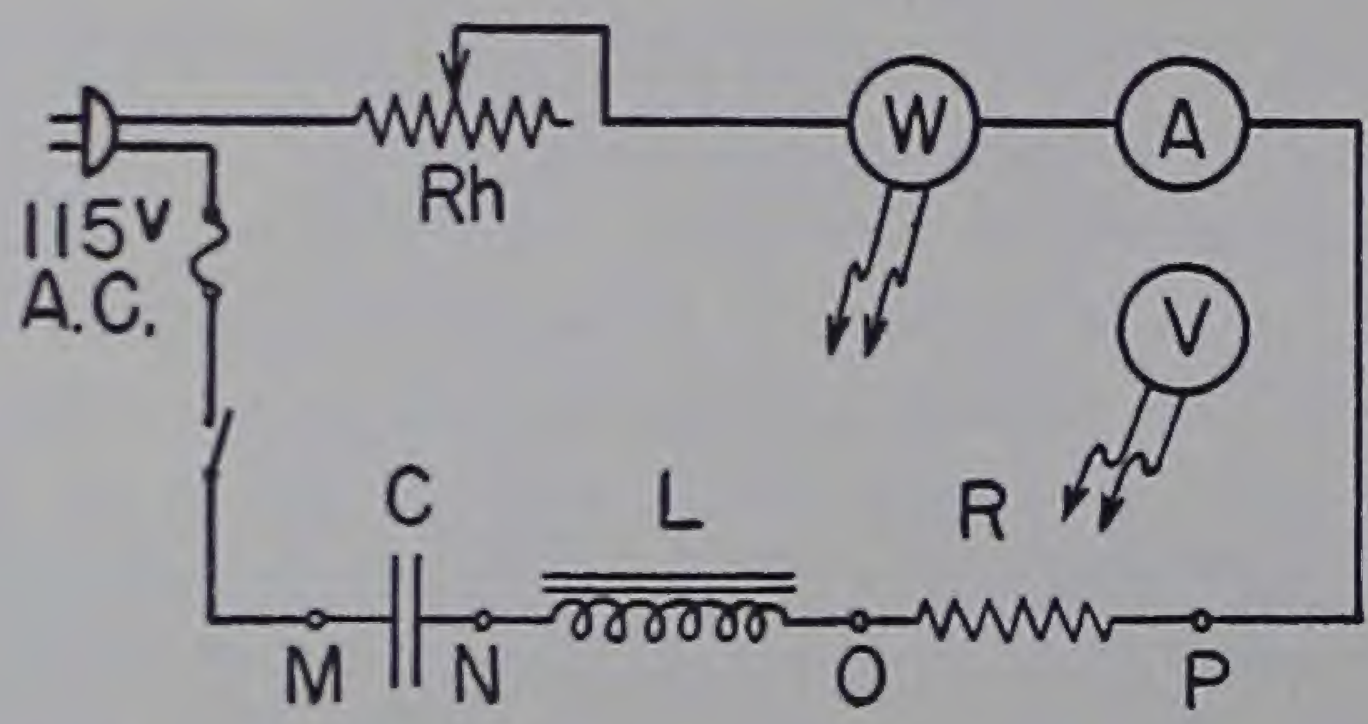


Fig. 42-1.

Theory: 1. *Phase Relationships.* When a pure resistance carries an alternating current, the resulting potential difference (P.D.) is also alternating, and *in phase* with the current. The same current in a pure inductance results in an alternating P.D. which *leads* the current by 90°. In a capacitance, the P.D. *lags* the current by 90°. The angle by which the P.D. leads or lags the current is called the *phase angle.* The fact that the currents and P.D.'s have, besides magnitudes, angular relationships, indicates that they may be treated as *vectors*, in the operations of addition and resolution into components.

2. *Power.* The calculations of power, using effective values of current and P.D. (the values indicated by most a-c electric meters) are identical for a *resistive* element with those in d-c theory; that is,

$$P_R = EI.$$

However, in elements which include inductance or capacitance, the phase angle between the current and the P.D. must be taken into account:

$$P = EI \cos \theta. \tag{1}$$

This, of course, is the general expression, since in a purely resistive element the P.D. and the current are "in phase," that is, there is no phase difference between them, and the term $\cos \theta$ becomes unity. On the other hand, in a perfect condenser or a perfect inductance the current respectively leads or lags the impressed voltage by 90°, and the power consumption as given by Eq. (1) is seen to be zero in each case. The term $\cos \theta$ is often called the *power factor*. See Appendix II Section J4 for a discussion of the wattmeter.

3. *Impedances.* A generalized form of Ohm's law may be applied to a-c circuits. Since all elements in an a-c circuit are not pure resistances, a new term is introduced: *impedance,* the symbol for which is Z. It is defined by means of the equation

$$Z = \frac{E}{I} \tag{2}$$

and is measured in ohms. With a little thought it may be seen that Z has associated with it a phase angle since E and I have such a relationship with each other. In fact, the impedance of an element determines the phase angle between E and I. We have seen that there are two basic types of impedance: the pure resistance which does not cause a phase difference between E and I, and the pure capacitance or the pure inductance which shifts the phase 90°. This latter type is called a *reactance* and is designated by the symbol X. It is measured in ohms, and has an algebraic sign associated with it. A *positive reactance* (a pure inductance) is one in which the *voltage leads* the current by 90°; a *negative reactance* (a pure capacitance) is one in which the *voltage lags* the current by 90°.

In general, an impedance consists of a combination of a resistance and a reactance. The impedance of the ordinary inductance coil combines a resistance with a positive reactance, and therefore has a positive phase angle greater than 0° but less than 90°. Since reactances and resistances are 90° out of phase, they are taken as the *components* of the impedance. From the sketch of Fig. 42-2 and Eqs. (1) and (2), it may be seen that power is consumed only by the resistive component of an impedance.

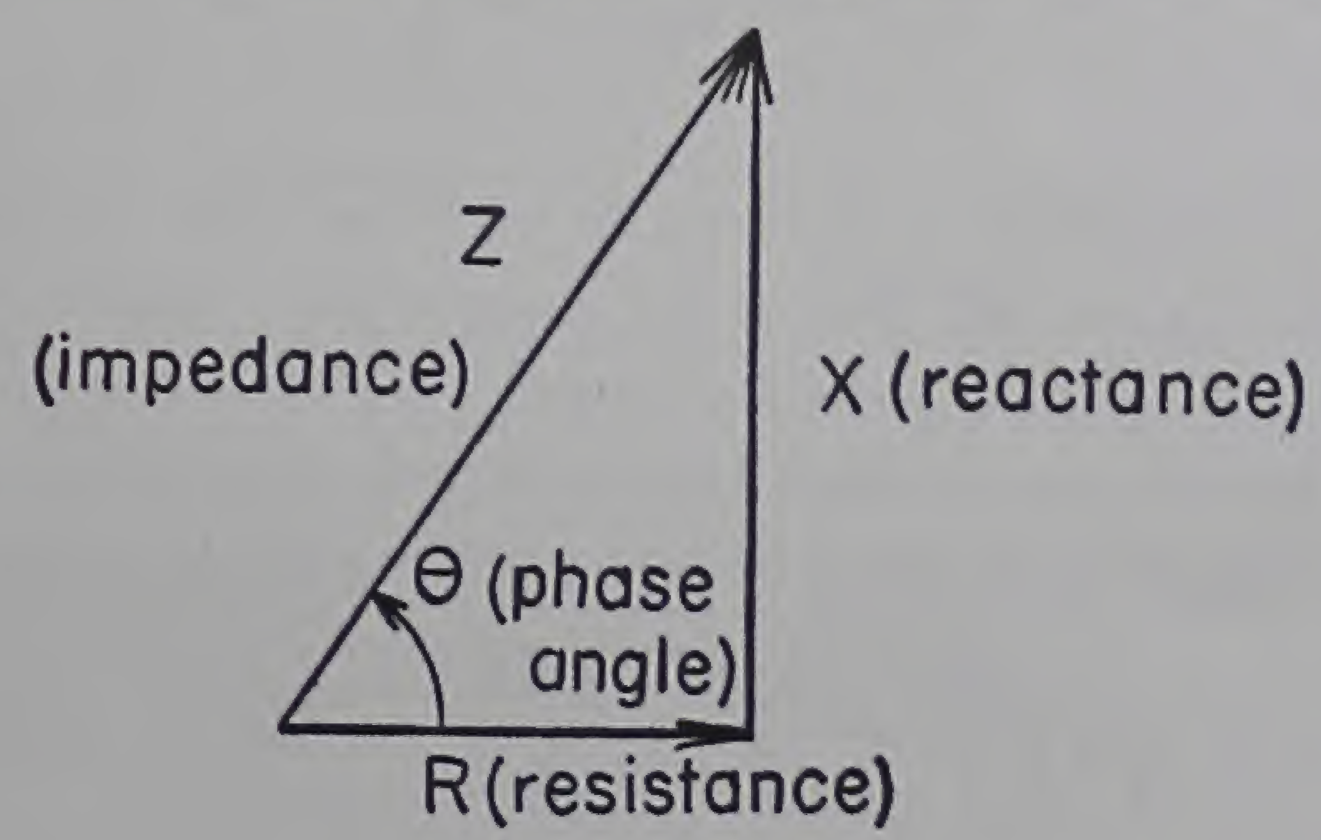


Fig. 42-2.

4. *Calculation of Reactances.* The reactance of an inductance depends on the size of the inductance and on the frequency of the current:

$$X_L = 2\pi f L, \tag{3}$$

where X_L is in ohms, L in henrys, and f in cycles per second.

The reactance of a capacitance is similarly given by

$$X_C = -\frac{1}{2\pi f C}, \tag{4}$$

where X_C is in ohms, C in farads, and f in cycles per second.

The reactive component of any impedance (see Fig. 42-2) is given by

$$X = Z \sin \theta, \tag{5}$$

whereas the resistive component is given by $R = Z \cos \theta$. (6)

5. *Series Circuit.* In a series circuit such as the one to be studied, the current in each element of the circuit must be identical with that in all the other elements. The current surging through the circuit and reversing its direction in the entire circuit 120 times per second (in the case of 60-cycle current) must be in the same phase in each circuit element. Clearly, then, if the phase relationships between current and voltage in the individual elements are to hold, the individual potential differences must differ in phase, each from the others.

It will be convenient, therefore, to use the common current as the reference of phase. It is obvious from the foregoing that the voltage across the resistor may be expected to be in phase with the current, the

b. Compute the power factor for each element of the circuit and for the total circuit, using Eq. (1). Assume that the power factor of the condenser is zero. It is too small to be measured by these methods.

c. Compute the phase angle θ for each element and for the total circuit. Its sign is known for each element but not for the total circuit.

d. Compute the impedance Z in ohms of each element and of the total circuit, using Eq. (2).

e. Compute the reactance X and the resistance R of each element and of the total. See Fig. 42-2. The sign of X for each element is known, but it is not known for the total.

f. Compute the size of the inductance L in henrys of the choke and the size of the capacitance C in farads of the condenser bank using Eqs. (3) and (4). The frequency is 60 cps.

g. Compare the corrected total power dissipated in the entire circuit with that of the corrected sum of those in the three elements.

h. Compare the total resistance with the sum of the separate resistances.

i. Compare the total reactance with the algebraic sum of the separate reactances. Choose the sign of the total reactance to correspond with the sign of the algebraic sum of the separate reactances.

j. Compare the total impressed voltage with the vector sum of the separate potential differences. To carry out this procedure, construct a vector diagram. See Fig. 42-3. Use a full sheet of graph paper for accurate work.

k. Compare the calculated values for the capacitance of the condenser and the resistance of the lamp bulb with the expected values. Account for differences in each case.

l. By use of Eq. (7) compute the expected value of Z for the entire circuit and compare this with the measured value.

m. By use of Eq. (8) compute the expected value of the phase angle θ of the entire circuit and compare this with the measured value and with the value obtained from the vector diagram of item j.

n. By use of Eq. (9) compute the expected P.F. for the entire circuit and compare this with the measured value.

Record: (*Partial sample*)

App. No. A.c. circuit board________

Wattmeter________

Voltmeter________

Ammeter________

Resistance of wattmeter 10,050 ohms

Size of condenser________ μf

Size of lamp bulb________watts,________volts

	Quantity / Element	I, ave read amps	E, ave read volts	P, ave read watts	P, actual watts	Power factor	θ, deg	Z, ohms	R comp of Z ohms	X comp of Z ohms	Size	Expected size
Measured	Lamp	0.400	79.7	32.6	32.0	1.00	0.0	199	199	0	199 ohm	
Measured	Choke	0.400	106.2	6.6	5.5	0.129	82.6	266	34	264	0.700 henry	⊗
Measured	Condenser	0.400	110.8	1.0	0.0	0.000	−90.0	277	0	−277	9.58×10^{-6} farad	
Measured	Total	0.400	95.3	38.8	37.9	0.994	±6.3	239	238	±26	⊗	⊗
	Calc Tot	(0.400)	94.2*	⊗	37.5	0.998	−3.2 −3.4*	236	233	−13	⊗	⊗
	% Diff in Total	⊗	1.2	⊗	1.1	0.4	⊗	1.2	2.1	⊗	⊗	⊗

* Indicates value obtained from the vector diagram.
⊗ Indicates a space which is not to be filled.

QUESTIONS

1. In this experiment a large percentage difference frequently occurs between X_{total} as measured and X_{total} as calculated. Explain. [HINT: See Eq. (1a).]

voltage across the inductance to lead the current, and the voltage across the condenser to lag the current. Further, in general, the voltage impressed across the entire circuit will not be in phase with the current in the circuit. It will, in fact, be the *vector sum* of the individual potential differences in the circuit. (Compare this with the d-c case; can it be said that the d-c circuit is a special case of the a-c circuit? In what way?) Since the voltages add vectorially it is possible that an individual P.D. may be larger in magnitude than the vector sum of all the potential differences (*i.e.*, the applied emf). See Fig. 42-3.

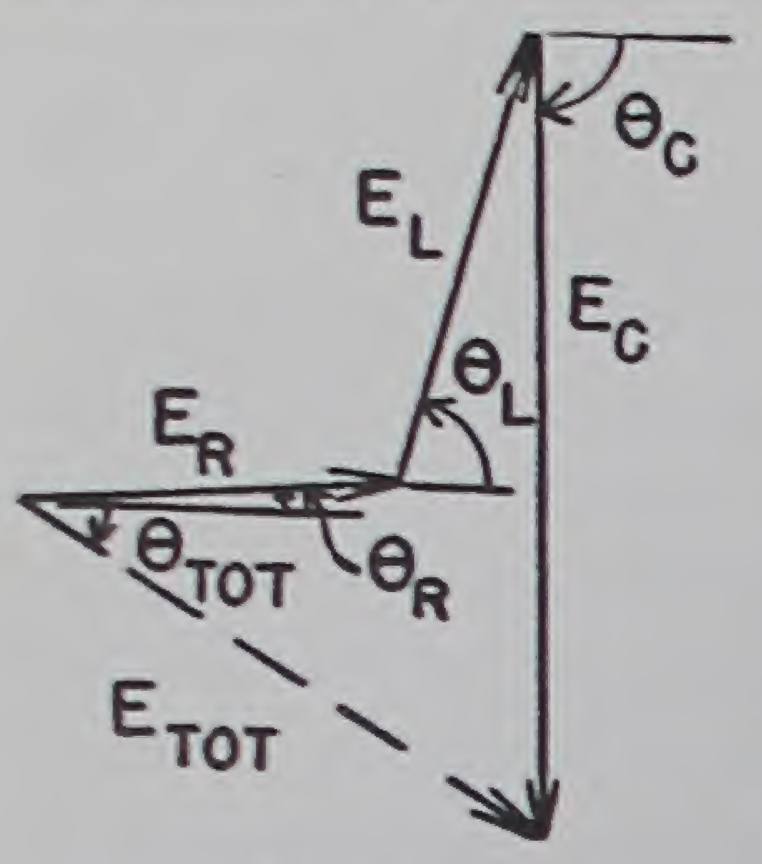


Fig. 42-3.

It is clear, from Sections 3 and 4, that the algebraic sum of the reactances in a series a-c circuit is the total reactance of the circuit. Similarly, the simple sum of all the resistive components in the circuit must add up to the resistive component of the entire circuit. The total impedance Z of the circuit will then be given by

$$Z = \sqrt{R^2 + X^2} \tag{7}$$

where R and X are respectively the total resistance and the total reactance in the circuit. Likewise the phase angle θ for the circuit is given by

$$\tan \theta = \frac{X}{R}. \tag{8}$$

Finally the power factor for the circuit is given by

$$\text{P.F.} = \cos \theta = \frac{R}{Z}. \tag{9}$$

See Fig. 42-2.

The error equations in this experiment are complicated by the presence of trigonometric functions; hence no attempt will be made to derive all the error equations. However, there is one place where the errors in P, E, and I have a pronounced effect on the result. This occurs when one attempts to determine the phase angle θ from the power factor $\cos \theta$ when this power factor is very nearly equal to one, *i.e.*, when θ is very nearly equal to zero. In this case it may be shown that $\Delta\theta$ is given approximately by the equation

$$\Delta\theta = \pm \sqrt{2\left(\frac{\Delta I}{I} + \frac{\Delta E}{E} + \frac{\Delta P}{P}\right)} \quad \text{for} \quad \theta \cong 0. \tag{1a}$$

Method: 1. Check the zero readings of all the instruments. They must lie flat on their backs. Avoid parallax (see Appendix II, Note F) by using the mirror behind the scale and lining up the needle with its image in the mirror before taking a reading.

2. Use the smaller capacitance available in the condenser bank if an adjustment is provided.

3. Adjust the current to 0.400 amp with the voltmeter and wattmeter potential leads *disconnected.* Later, when these are connected, the ammeter reading will change, but this may be ignored since the current in the *circuit element* will remain very close to 0.400 amp. (Why?)

4. Use the 150-volt scale on both the voltmeter and the wattmeter.

5. Connect the voltmeter leads across R, across L, across C, and across MP (whole circuit) in succession, recording each reading. The current should be checked before and after each reading. If the current has changed after removing the voltmeter leads, the reading should be discarded and another one taken after readjusting the current.

6. Repeat instruction 5 for the wattmeter potential leads.

7. Change the current slightly with the rheostat and then readjust it to its original value of 0.400 amp. Then repeat instructions 5 and 6.

8. Repeat instruction 7.

9. *Computations.* Average the three groups of readings on each instrument and enter the averages in columns 2, 3, and 4 of the record.

a. A determinate error is introduced into the data because the wattmeter measures not only the power dissipated in the element being measured, but also that consumed by the voltage winding of the wattmeter itself. This latter amount should be subtracted from the wattmeter reading to obtain the power actually dissipated in the measured element alone. The correction may be taken as E^2/R_{wm}, where the wattmeter resistance, R_{wm}, is printed on the dial of the instrument. Compute the actual power consumed in each element of the circuit and in the total circuit.

Experiment 43.

Vacuum Tube

Object: To obtain characteristic curves of a vacuum tube; to find its amplification factor.

Apparatus: Vacuum-tube test board with 6J5 tube, socket, filament transformer, plate voltmeter (0—150 dc.), plate milliammeter (0—15 dc.), and plate voltage potentiometer (5000 ohm, 50 watt); grid battery; source of plate voltage (about 300 volts dc.). See Appendix II, Section G1 for vacuum-tube circuit symbols.

Theory: Electronic vacuum tubes depend for their functioning on the fact that certain metals, when heated, emit electrons quite freely from their surfaces. If a filament made of, or coated by, such a metal is introduced into a tube which is then evacuated, and if a current is now passed through this filament, heating it, the space surrounding the filament will be occupied by a cloud of electrons which have been "boiled" off the filament. The process will not, however, go on indefinitely, since the cloud of electrons (the "space charge") quickly becomes so dense that it is able to repel subsequent electrons back into the filament. Equilibrium is reached when as many electrons fall back onto the filament per second as are boiled off. We have described what may be designated as a "monode," the simplest electron tube. See Fig. 43-1. The ordinary incandescent lamp bulb is a "monode."

Fig. 43-1.

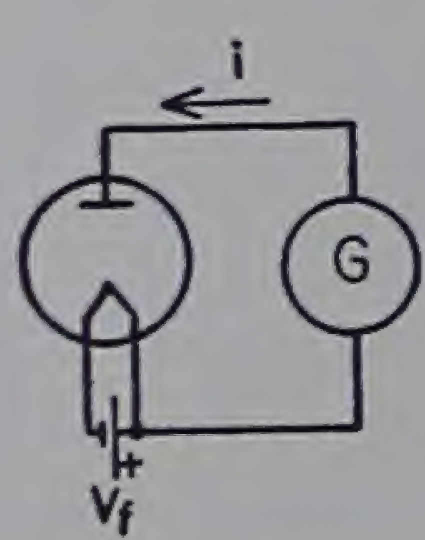


Fig. 43-2.

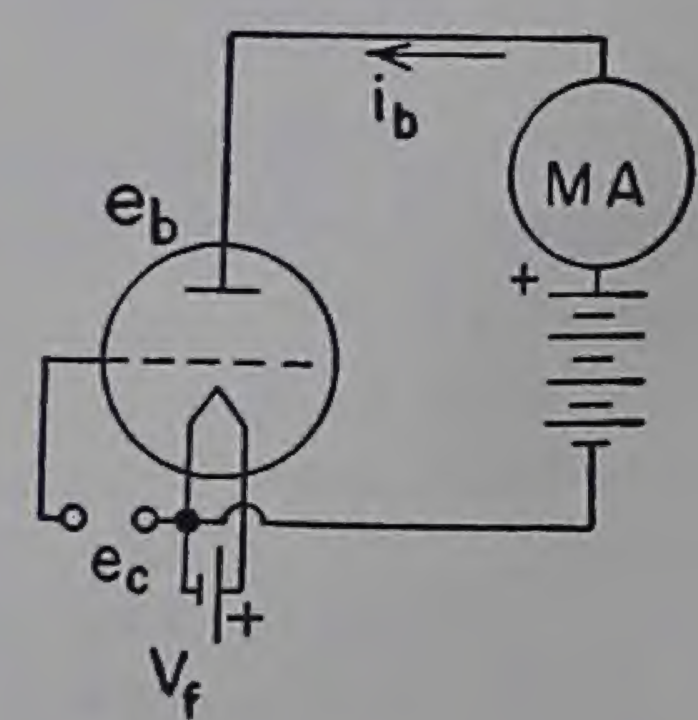


Fig. 43-3.

The Diode: Edison Effect. While developing the incandescent lamp, Edison noticed the gradual blackening of the inside of the bulb by evaporation emitted by the carbon filament. He inserted a plate of metal into the bulb near the filament, and upon connecting a galvanometer between this plate and the positive side of the filament a current was observed, whereas none occurred if the connection was to the negative side of the filament. See Fig. 43-2. This observation, made in 1883, and called the Edison effect, was unexplained until about 15 years later when the existence of electrons was discovered. Apparently some of the space charge reaches the plate, thus closing a circuit through the galvanometer.

If now a battery is connected in the plate circuit, as shown in Fig. 43-3, with the plate maintained at a positive potential e_b with respect to the filament, a much stronger flow of electrons will be obtained, since there is an electric field between the plate and the filament, attracting the electrons to the plate.

It should be noted that the conventional current is actually opposite in direction to the flow of electrons.

It should also be clear that this plate current can be only in the direction shown, since the plate is cold, and therefore does not emit electrons.

As e_b is increased, the plate current i_b also increases, but only up to a certain maximum. When all the electrons emitted by the filament are drawn over to the plate, further increase in plate voltage will not increase plate current. See Fig. 43-4. To increase i_b, it is necessary to raise the filament temperature, which results in an increased emission of electrons. However, vacuum tubes are almost always operated in such a manner that some space charge exists, so that the plate voltage affects the plate current.

The Triode. If between the filament and the plate of the diode already described a grid or mesh of wire is inserted in the region of maximum space charge, a triode ("three electrodes") has been formed. See Fig. 43-5. Because of its proximity to the space charge, the grid has a greater effect on the motion of the electrons in this region than does the plate. If the grid is negative with respect to the filament, it repels the electrons back toward the filament, forming a smaller but more compact charge nearer the filament: thus, fewer electrons get through to the plate, and i_b is decreased. By making the grid sufficiently negative, the plate current can be entirely *cut off*.

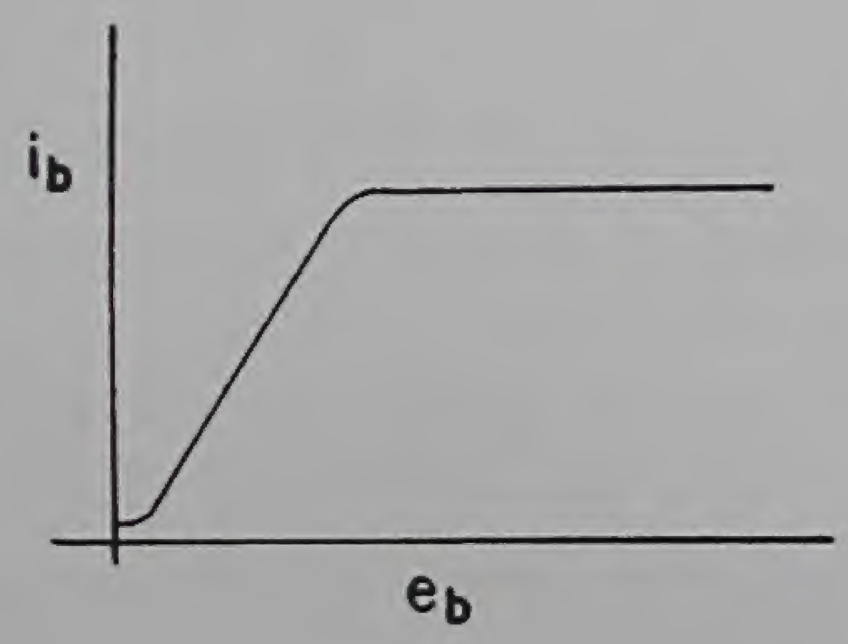


Fig. 43-4.

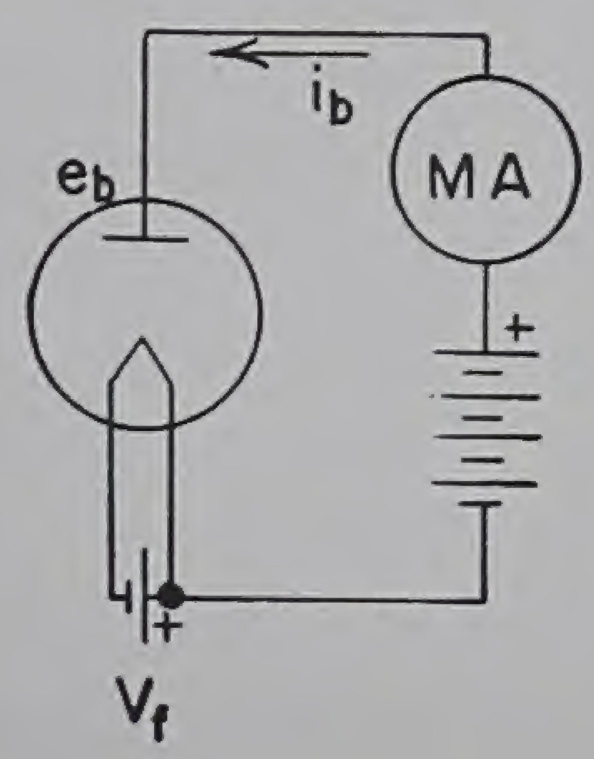


Fig. 43-5.

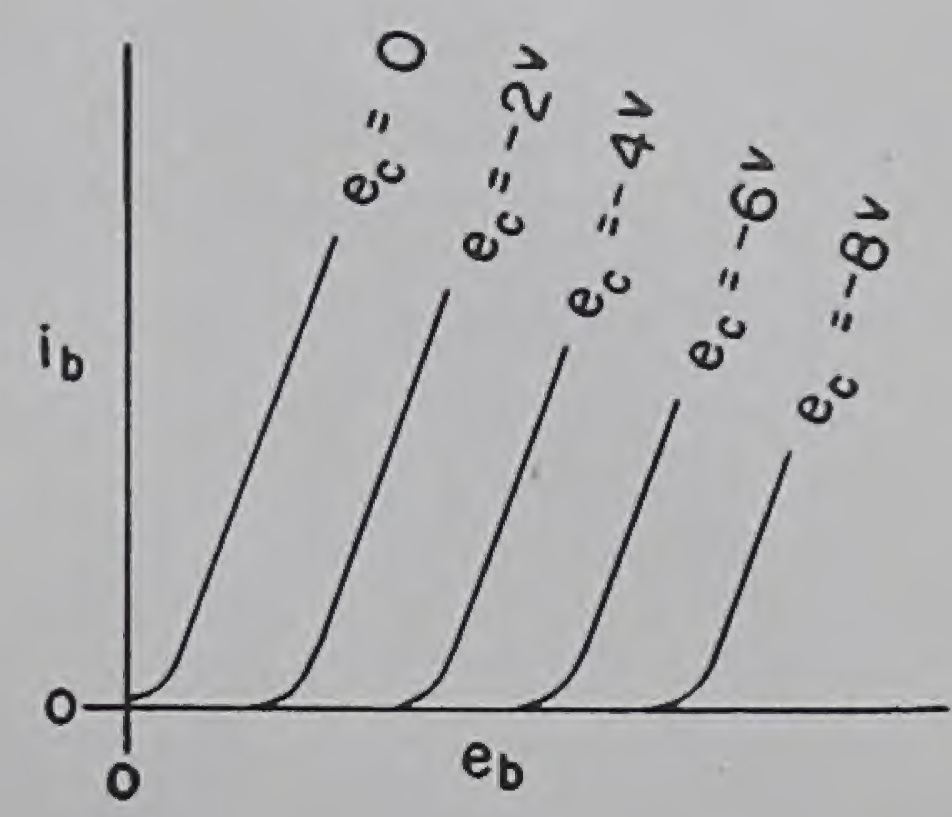


Fig. 43-6.

Since a small negative grid voltage e_c reduces the current to the plate, an entirely new curve like that of Fig. 43-4 will be obtained by varying the plate voltage. By plotting such curves for several values of grid voltage, a family of such curves will be obtained; a family of *plate characteristics*. See Fig. 43-6.

If, on the other hand, the plate voltage is held constant, a series of observations of plate current versus grid voltage will give a grid characteristic curve. A family of such characteristics for different values of e_b is called *transfer characteristics*. See Fig. 43-7. (They are so called because action on the grid is transferred to results in the plate circuit.)

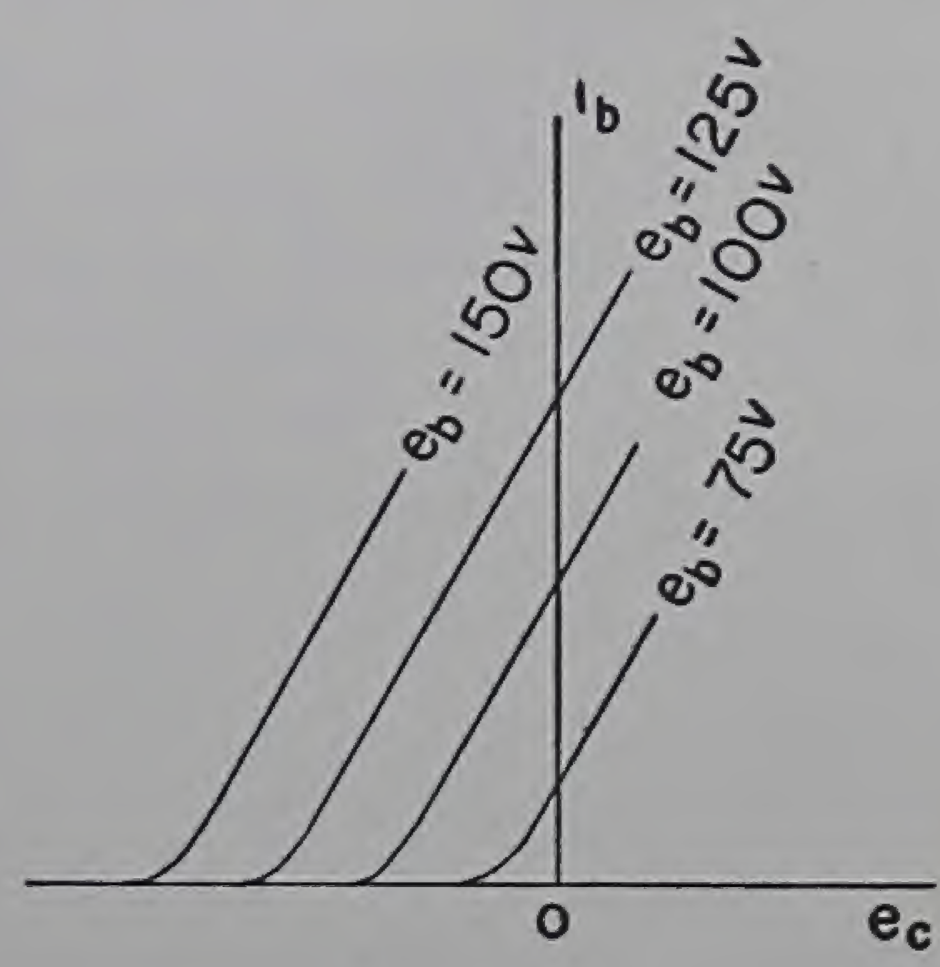


Fig. 43-7.

The slope of a typical curve of Fig. 43-6 at any definite point on the curve (definite i_b and e_b) shows the rate of change of the plate current with respect to the plate voltage for the value of the grid voltage e_c corresponding to that curve. Suppose for example, that this is 1 ma per 10 volts. The slope of the corresponding curve in Fig. 43-7 (same e_b) at the corresponding point (same i_b) shows the rate of change of the plate current with respect to the grid voltage. Suppose that this is 1 ma per 1 volt. In this example it is necessary to change the plate voltage 10 times as much as the grid voltage to produce the same change in the plate current. Thus, in this example, the grid is 10 times as effective as the plate in controlling the plate current. The quantity 10 is called the voltage *amplification factor* of the tube and is represented by the symbol μ. This factor may be obtained approximately from a single set of characteristics by noting that in order to hold i_b constant during a small increase of the grid voltage Δe_c, the plate voltage will have to be decreased by a larger amount Δe_b. The ratio of the latter to the former will equal the amplification factor as given by the equation

$$\mu = \frac{\Delta e_b}{\Delta e_c} \qquad \text{for constant } i_b. \tag{1}$$

It should be noted that this property of amplification is the basis of many electronic devices including the radio. A small voltage fed into the grid produces a change in the plate current; the changing current in an external resistor will cause a larger changing IR drop across the resistor, depending on the size of the amplification factor. The input voltage has, in effect, been amplified. If further amplification is desired, this new voltage may be fed into the grid of a second tube and again be amplified. By using several stages of amplification, the original signal voltage may be amplified by many millions.

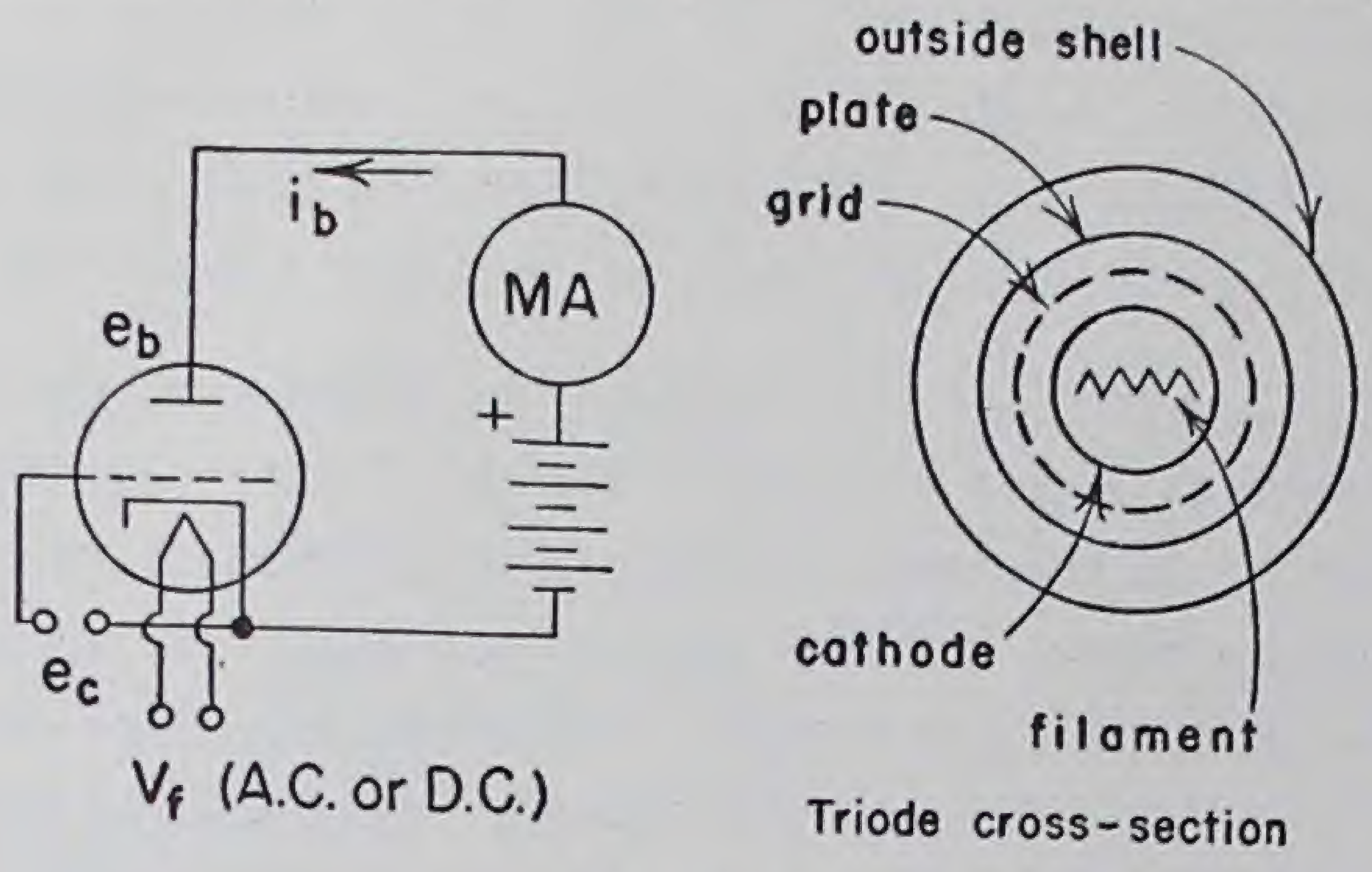


Fig. 43-8.

Method: The filament of the tube to be used (6J5) is surrounded by a metal cylinder called the *cathode*, which when heated by the filament emits electrons. See Fig. 43-8. The filament in this case is not a good emitter. The cathode is built to emit a maximum of a little over 0.01 amp of electrons.

Do not exceed 15 *ma.* Note that this limitation means that not all the spaces in the record will be filled. With zero or positive grid voltages, less plate voltage is required to attain the maximum allowable plate current. On the other hand with large negative grid voltages, even the maximum available plate voltage will not be enough to reach the maximum current.

1. Using the test voltmeter, measure the filament a-c voltage. It should be close to 6.3 volts.
2. Connect the grid battery so that $e_c = 1\frac{1}{2}$ volts (grid positive with respect to the cathode). Take a series of readings of the plate current for values of the plate voltage differing by 20 volts, being careful not to exceed the limit of 15 ma. Now connect the grid to the cathode so that $e_c = 0$ volt. (Note that merely disconnecting the grid is not sufficient, for then it "floats" in potential. It catches electrons which it cannot dispose of and rapidly becomes very negative.) Again take a series of plate currents versus plate voltages. Repeat for values of e_c in $1\frac{1}{2}$-volt steps to minus $7\frac{1}{2}$ volts.
3. Trace the circuit, paying particular attention to polarities.
4. Plot the family of plate characteristics. Plot the family of transfer characteristics. Connect the plotted points by smooth curves. See, for example, Fig. 43-9.
5. Determine the amplification factor for i_b constant at 5 ma, from the second set of curves, in the neighborhood of $e_c = -3$ volts. Mark on the graph the voltage changes measured.
6. Draw the circuit diagram in the clearest and most easily followed manner.

Record:

e_b \ e_c	$1\frac{1}{2}$ v	0	$-1\frac{1}{2}$	-3	$-4\frac{1}{2}$	-6	$-7\frac{1}{2}$
0 v							
20							
40							
60							
80							
100							
120							
140							

Type of tube__________
Filament voltage__________
From graph:
Δe_c__________
Δe_b__________
μ__________

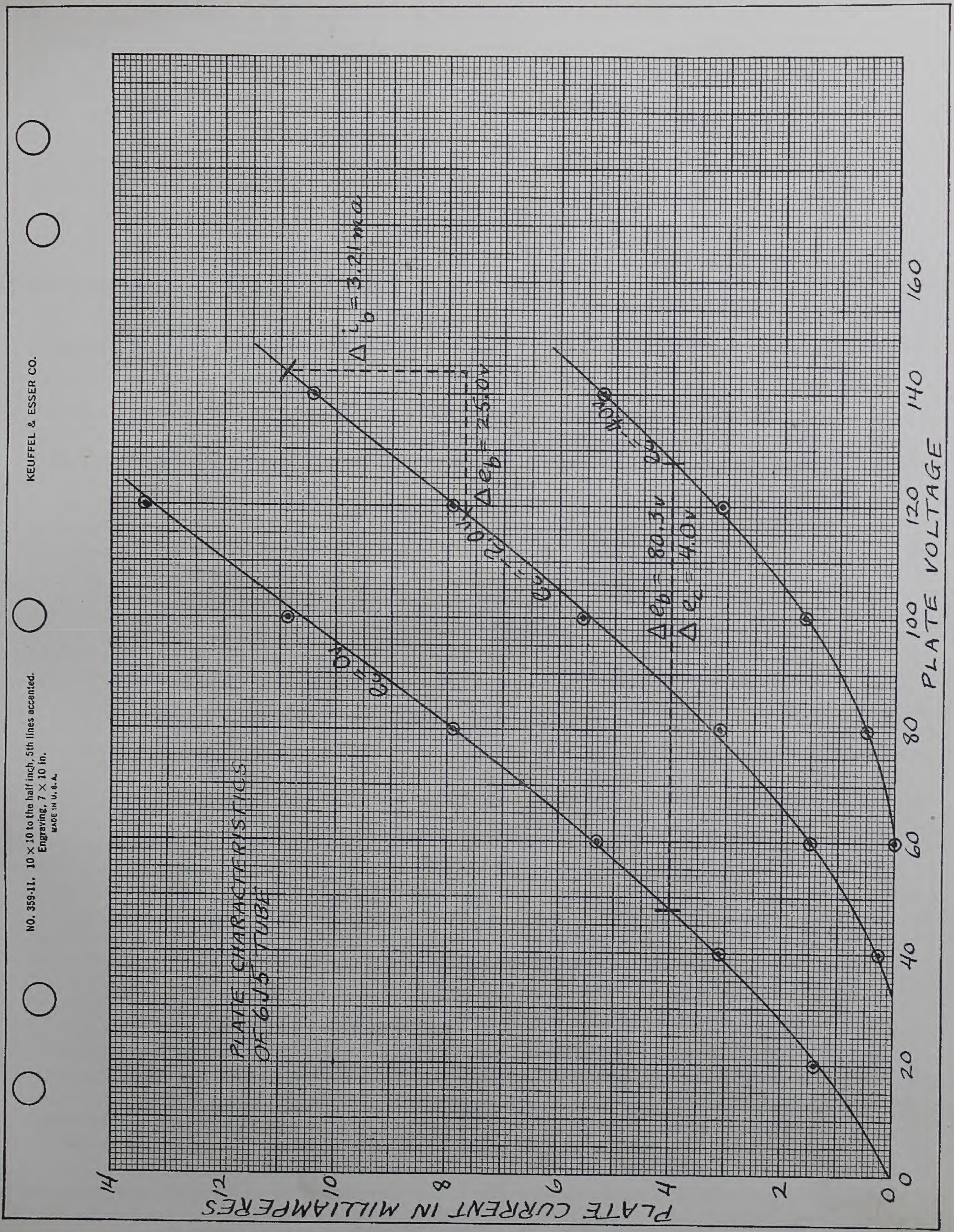

KEUFFEL & ESSER CO.
NO. 359-11. 10 × 10 to the half inch, 5th lines accented. Engraving, 7 × 10 in. MADE IN U.S.A.
PLATE CHARACTERISTICS OF 6J5 TUBE
PLATE CURRENT IN MILLIAMPERES
PLATE VOLTAGE
0 2 4 6 8 10 12 14
0 20 40 60 80 100 120 140 160
$e_c = 0$
$e_c = -2.0$
$e_c = -4.0$
$\Delta i_b = 3.21\ ma$
$\Delta e_b = 25.0v$
$\Delta e_b = 80.3v$
$\Delta e_c = 4.0v$

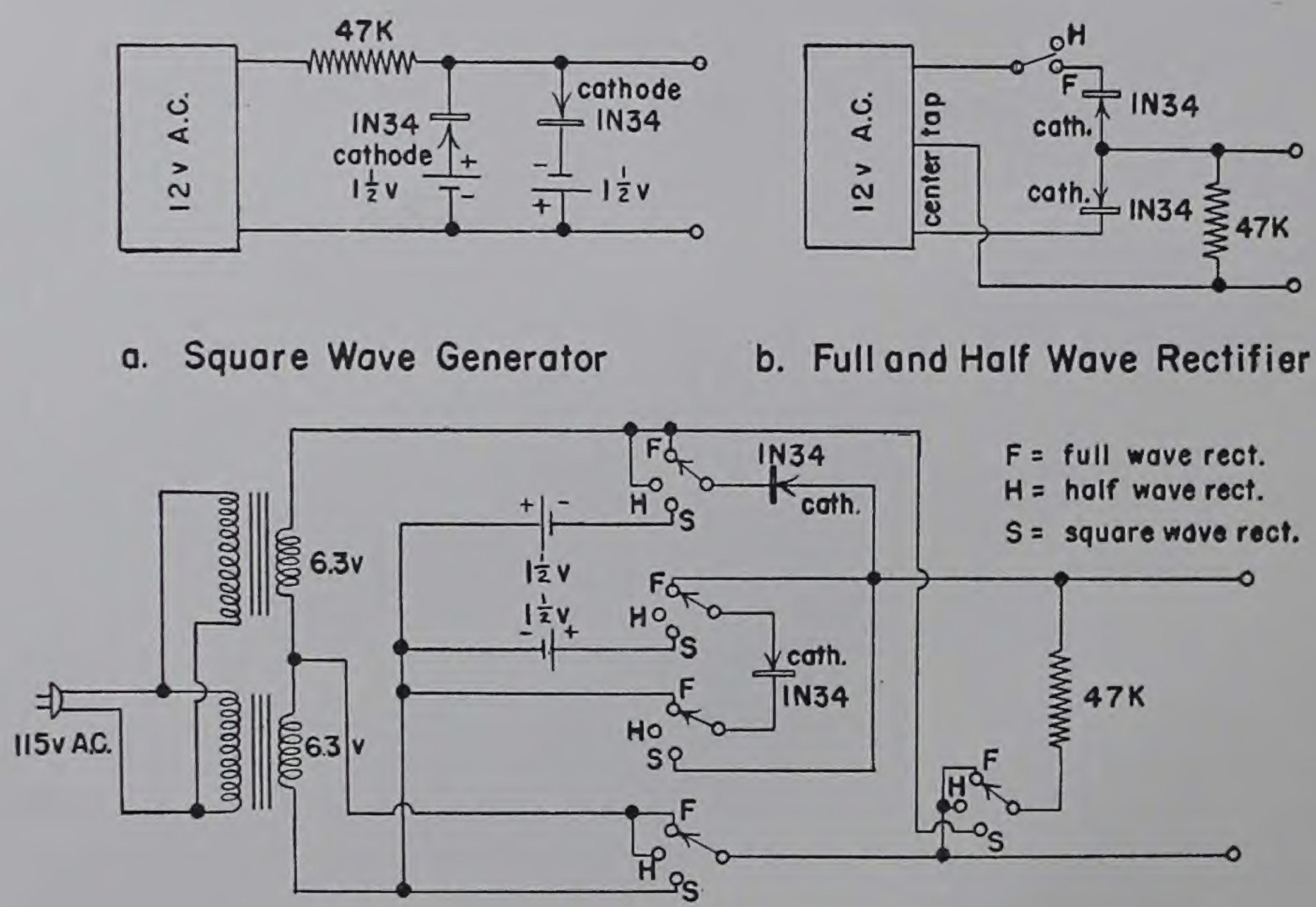
Fig. 43-9.

Experiment 44.

Cathode-ray Oscilloscope

Object: To become familiar with the operation and some of the uses of a cathode-ray oscilloscope; to study Lissajous figures.

Apparatus: Cathode-ray oscilloscope, audio-frequency oscillator, standard-frequency oscillator, permanent-magnet dynamic loud-speaker with output transformer (10,000 ohms to speaker coil), 6-volt storage battery, 6-volt radio vibrator, rectifying and square-wave generating circuit (see Fig. 44-1), 0.004-μf condenser, 470,000-ohm resistor.



a. Square Wave Generator

b. Full and Half Wave Rectifier

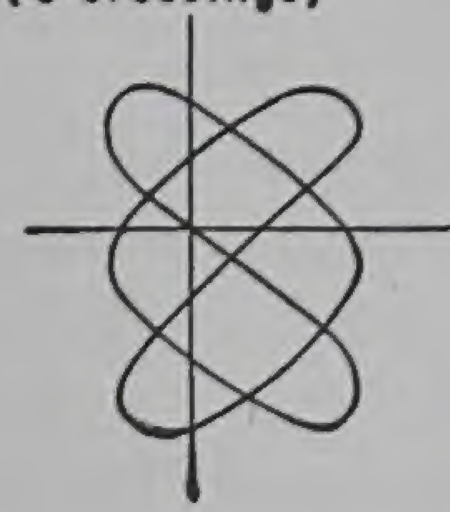
c. Rectifier and Square Wave Generator (combination of a. and b.)

Fig. 44-1.

Theory: The student should study carefully Note N, Appendix II, before attempting to perform this experiment. The construction and operation of the cathode-ray oscilloscope and some of its more common uses are discussed in that note.

When two simple harmonic motions are plotted against each other at right angles, the resulting pattern is called a *Lissajous figure.* The time plot of a simple harmonic motion is sinusoidal. Thus, two sinusoidal electrical voltages plotted against each other by means of a cathode-ray oscilloscope result in a Lissajous figure. Stationary patterns are formed only when the frequencies of the two alternating voltages are the same or bear a rational relationship to each other (*i.e.*, the ratio of the frequencies must be a rational number

such as 2/3, 51/45, etc.). Each such frequency ratio results in a characteristic pattern, which itself takes on different forms depending on the phase relationship of the two components. See a physics textbook such as Lemon and Ference or Hausmann and Slack for a discussion on the formation of Lissajous figures. To determine the frequency ratio corresponding to any Lissajous figure, it is only necessary to imagine straight lines passing through the figure in directions parallel to the axes of formation and to count the number of times each line is crossed by the figure. The ratio of these crossings is the inverse ratio of the frequencies in the directions of the lines. See, for example, Fig. 44-2. The student should decide for himself why this is true.



$$\frac{f_{vert.}}{f_{horiz.}} = \frac{4}{6} = 2:3$$

Fig. 44-2.

Method: Referring to Section 3 of Note N, Appendix II, examine the cathode-ray oscilloscope, locating each control and input terminal. Turn on the power, and reduce the horizontal and vertical gain controls to zero. Also set the "Synch. Amplitude" control to zero. When the beam appears, or after about a minute, adjust the intensity and focusing controls until the beam is a single sharply defined spot of faint intensity. Move the spot to the center of the scope face by means of the centering controls. Turn up the horizontal gain control until, with the sweep circuit connected, the beam traces a horizontal path nearly the full width of the face. Adjust the sweep frequency to the lowest possible value. On some instruments this is as low as one or two sweeps per second.

Part I. Wave Forms as Functions of Time. Connect the output of the audio-frequency oscillator to the vertical input terminals of the cathode-ray oscilloscope (CRO). Note that each instrument has a terminal labeled "ground"; these should be connected to each other. Set the oscillator frequency to about 1000 cps and adjust the oscillator amplitude control and the vertical gain control of the CRO until the beam deflects vertically about two-thirds of the face diameter. Readjust intensity and focusing controls if necessary. Adjust the frequency controls of the oscilloscope sweep circuit until a sinusoidal pattern is visible. It will be necessary to adjust the vernier frequency control carefully to "stop" the pattern. When the hand is taken off the vernier control, the pattern will probably soon begin to "walk" along the sweep. When it does so, set the scope synchronization control to "Internal," and adjust the "Synch. Amplitude" control to "stop" the pattern. Study the interaction of the "Synch. Amplitude" and the frequency vernier controls; that is, determine how much the basic sweep frequency may be moved before the pattern "jumps out of synch" as a function of how much synchronizing signal is used. With a moderately large synchronizing signal, adjust the frequency controls to obtain different numbers of cycles of the sinusoidal pattern on the scope. It should be possible to "synch in" at one, two, three, etc., up to perhaps 15 cycles of the pattern being presented on the scope. Change the frequency of the oscillator, and repeat the previous process. With a large number of cycles crowded onto the scope face, increase the horizontal gain until two or three of the center cycles now occupy the whole scope face. Move the horizontal centering control to examine in detail other cycles of the wave. Return centering and gain controls to their original position.

Connect the radio vibrator in series with a resistor of about 4 ohms to the 6-volt battery, and connect the oscilloscope leads across the resistor. Examine the wave form of the vibrator current. (NOTE: This is a standard technique for presenting *current* wave forms rather than voltage wave forms on an oscilloscope; the element being checked is placed in series with a resistance of a size which will not appreciably change its characteristics, and the voltage across the resistor is examined. It is, of course, proportional to the current in the tested element.) As in the previous paragraph, expand and examine details of the wave form.

Connect the standard audio-frequency oscillator to the vertical terminals of the oscilloscope, and to a storage battery if necessary. Examine its output.

Examine the output of the rectifying and square-wave generating circuit. Study the full-wave rectifica-

tion, the half-wave rectification, and the square-wave output. Connect the 0.004-μf condenser and the 470,000-ohm resistor in series across the output of the square-wave generator, and examine the voltage across the resistor by means of the oscilloscope. This voltage is similar in wave form to the current in the condenser. *Why?* Note that the condenser charges rapidly during the rising portion of the square wave and then discharges through the resistor exponentially; note the reverse charge and discharge during and following the falling portion of the square wave. If a decade condenser box is available, substitute this for the fixed condenser, and note the wave form for different capacity values. See Eq. (6) of Experiment 31, and read the paragraph on Time of Charging. *How does the rate of discharge depend on the* time constant *of the circuit?*

Sketch a typical wave form from each of the previous experiments.

Connect the input of the scope to the high-impedance winding of the output transformer of the loudspeaker. Adjust the vertical gain of the scope until a good deflection is obtained when speaking in a normal voice into the speaker. Adjust the frequency of the sweep to get a good presentation of the wave forms. The synchronization controls will not be of use here. *Why?* Hum a steady tone into the speaker, and examine the wave form. The synchronization control will be of assistance here. *Why?* Sing a note in low register into the speaker and examine this wave form. Sing a clear falsetto note and again examine. *What is the difference in appearance?* Whistle a clear tone and again examine the wave form. *How does this compare with the sine waves previously viewed?* Sketch a typical pattern from each of the preceding procedures.

Part II. Frequency Comparison. Connect the standard-frequency oscillator (1000 cps) to the vertical input, and feed the output of the variable audio-oscillator into the horizontal input. Switch the horizontal amplifier so that it accepts the audio signal rather than the sweep. Adjust the audio-oscillator to 1000 cps by watching the oscilloscope for the appearance of an ellipse. If possible, "stop" the ellipse. Record the oscillator dial reading at this point. Allow the ellipse to change phase, and note the various shapes it takes on. Adjust the oscillator to 500 cps to get the 1:2 Lissajous figure. This is a "figure 8" on its side. Obtain the 2:1 pattern at 2000 cycles. This is also a "figure 8," but it is upright. Obtain various integral Lissajous figures down to 1:10 and up to 10:1. Note that since it is difficult to count line crossings above 4:1, the vernier control should be moved gradually until the next integral figure is formed. In this way it is possible to obtain each integral figure without actually counting line crossings. Next obtain other patterns such as 3:2 ($1\frac{1}{2}$:1) at 1500 cycles, 4:3 ($1\frac{1}{3}$:1), 5:4, and also 2:3, 3:4, and 4:5. Record the oscillator dial reading at each setting. Tabulate the dial readings and the actual frequencies at those points.

Replace the standard 1000-cycle oscillator by the radio vibrator at the vertical CRO terminals. Adjust the audio-oscillator until the 1:1 pattern corresponding to an ellipse is obtained. Read this frequency. Adjust to the 2:1 and the 1:2 patterns to examine their shapes.

Replace the radio vibrator with the rectifying and square-wave generator, and determine the repetition frequency of each of the three wave forms available. *Why is it different for full and half-wave forms?*

Part III. Amplitude Comparison. As a standard, the output of the square-wave generator may be used, since it is limited to 3 volts peak-to-peak by its design. Using the sweep, present the square wave on the scope. With the vertical gain control, adjust the size of the pattern so that the peak-to-peak amplitude occupies a convenient space on the scope, say two scale divisions per volt, or six divisions in all. Do not change the setting of the vertical gain control after this point. Then present the full- and half-wave rectified patterns, and count the number of scale divisions of amplitude of each, inserting these figures in a sketch of each wave. Similarly measure the amplitudes of each of the salient features of the output of the radio vibrator in volts. It may appear during the work that a different scale would have been more convenient. In this case, readjust the size of the standard figure, and remeasure all the other figures in the new terms.

Record: Include in the record all wave form sketches and measurements made. Answer the italicized questions in the text of the *Method*.

Experiment 45.

Vacuum-tube D-C Amplifier

Object: To study the use of a vacuum tube as a d-c voltage amplifier; to construct a vacuum-tube voltmeter.

Apparatus: Vacuum-tube test board, with 6J5 tube, socket, filament transformer, voltmeter (0 to 150 volts dc, 1000 ohms per volt or better), and 5000-ohm 50-watt potentiometer; grid-bias battery (3 volts); 0 to 3 volt d-c voltmeter; 100-ohm potentiometer; two dry cells; dial resistance box; source of high-voltage plate current (about 300 volts); 0 to 150 volt mirror-scale d-c voltmeter (100 ohms per volt or better); 47,000-ohm 2-watt plate load resistor; milliammeter (0 to 1 ma); 10,000 + 10,000 ohm resistor; 56,000-ohm resistor.

Theory: As mentioned in Experiment 43, a vacuum tube is capable of acting as a voltage amplifier, in the sense that a small signal voltage applied to its grid results in a larger output voltage at its plate, given the proper circuit connections. Such a tube with its associated circuits is called a *stage of amplification*. A *vacuum-tube amplifier* consists of one or more stages of amplification. In any given application, enough stages of amplification are used to amplify the incoming signal to the point where the final output voltage is capable of actuating a suitable device, such as a voltmeter, loud-speaker, impulse counter, etc. The over-all amplification obtained in such an amplifier is equal, of course, to the product of the individual *stage gains*, and may range from factors of many billions down to values of less than unity, depending on the application. The present experiment concerns a single-stage amplifier, and some of the concepts and characteristics associated with it.

A principal matter of interest is the *gain* of the stage. The gain may, of course, be measured. Calculation or prediction of its value is often an involved procedure if exact values are desired, but good approximate values can be predicted rather easily under normal circumstances. To do this, we use an "equivalent circuit," which depicts the tube as a source of emf with an internal resistance and places this source in the remainder of the circuit as it actually is, except that the fixed supply voltages are eliminated. The equivalent circuit resulting from this treatment is solved by use of Kirchhoff's laws and in this way the behavior of the actual circuit is usually quite well predicted.

In Fig. 45-1 is shown a simple vacuum-tube amplifier stage and its equivalent circuit. The actual circuit (a) consists of the tube, a load resistor, R_L, a plate supply voltage, V_p, and a grid bias voltage, e_{co}. A signal voltage, e_g, to be amplified, is inserted as shown, and the output voltage, e_{out}, is taken between plate and cathode. The action of the circuit is as follows: with no signal voltage, $e_g = 0$; the d-c value of the plate voltage is determined by the value of the grid bias, the plate supply voltage, and the load resistor. If there is any plate current, i_b, then the plate will be at a lower potential than the supply voltage because of the "IR drop" in R_L. Thus e_b ($= e_{DB}$) will have a no-signal, or quiescent, value, e_{bo}. We are interested, however, only in the *change* in voltage across the terminals B and D in response to an incoming signal, e_g. It is this *change* in e_b that we define as e_{out}. If a positive signal, e_g, is applied, then the grid of the tube is driven more positive (less negative) with respect to the cathode. This causes an increased plate current. The increased current in the load resistor, R_L, causes an increased IR drop across it, and thus the plate of the tube goes to a lower potential. The difference between the quiescent value, e_{bo}, and the new value of e_b is e_{out}. Since

the plate voltage *decreases* for a *positive* input signal, we see that the circuit reverses the sign of a signal as well as amplifying it. It will be instructive for the student to verify this by applying the above reasoning to a negative input signal.

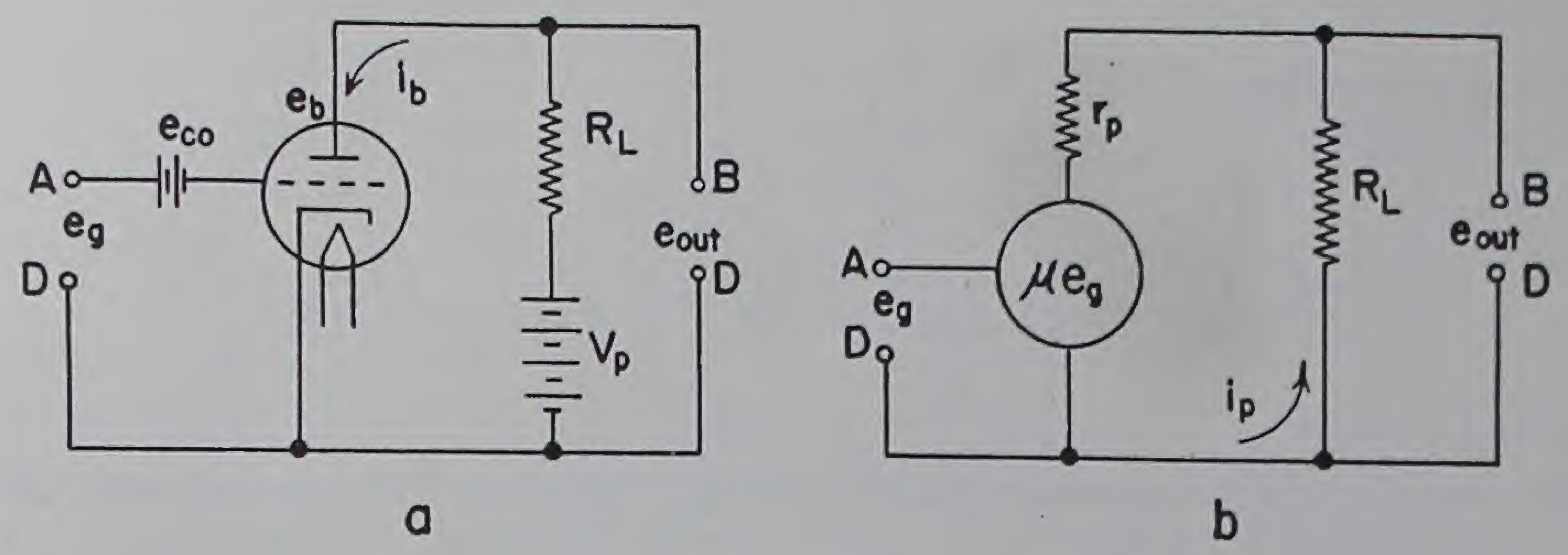


Fig. 45-1.

In the equivalent circuit (b) the tube has been replaced by a voltage source, μe_g, and a resistance, r_p. μ is the amplification factor of the tube; r_p is the plate resistance. Since the equivalent circuit concerns only *changes* resulting from an input signal, the plate and grid batteries have been left out of the circuit. A positive input signal, e_g, causes an output current, i_p, in the direction shown. As it is drawn, the equivalent circuit may now be analyzed by ordinary Kirchhoff methods. This is a single-loop circuit, and its equation for voltages around the loop may be written as follows:

$$\mu e_g - i_p R_L - i_p r_p = 0. \tag{1}$$

The output voltage, e_{out}, is simply the IR drop of i_p across R_L, and, since the voltage is measured with respect to point D, convention states that i_p, as shown, is in a negative direction.

Thus
$$e_{\text{out}} = -i_p R_L. \tag{2}$$

Solving Eq. (1) for i_p we get

$$i_p = \frac{\mu e_g}{r_p + R_L}.$$

We substitute this in Eq. (2) and find that

$$e_{\text{out}} = -\frac{\mu R_L}{r_p + R_L} e_g \tag{3}$$

The quantity $-\mu R_L/(r_p + R_L)$ is called the gain of the stage, since it is the ratio of the output voltage to the input voltage. It is directly proportional to the amplification factor of the tube, and depends also on the plate resistance of the tube and on the load resistance. If the load resistance is high compared to the plate resistance, then the gain is seen to reach the value $-\mu$, in the limiting case. In all other cases, the gain of the stage is less than the amplification factor of the tube.

The analysis thus far assumes that the output voltage is fed into an infinite resistance; *i.e.*, the external resistance across points B to D is infinite. If the external circuit has a finite resistance, this resistance must be inserted in the equivalent circuit between B and D. It is then simply in parallel with resistance R_L, and the formula of Eq. (3) must be modified to

$$e_{\text{out}} = -\frac{\mu R_b}{r_p + R_b} e_g \tag{4}$$

where R_b is the net load resistance, and is equal to the parallel resistance of R_L and the following circuit.

Vacuum-tube Voltmeter. A very useful voltage-measuring instrument can be made from a vacuum-tube amplifier. Such a "vacuum-tube voltmeter" possesses two principal advantages over electromagnetic measuring instruments: in the first place, it can be constructed so as to draw only an insignificant amount of current from the voltage source being measured; and in the second place, it can be constructed to have very high sensitivity, while costing considerably less than a direct type of instrument of comparable sensitivity. A very simple type of vacuum-tube voltmeter is one using the circuit of Fig. 45-1a, with an ordinary volt-

meter between points B and D. For any given value of the grid input signal, e_g, there exists a corresponding value of the plate voltage, e_b. Once calibrated by use of a circuit such as that of Fig. 45-2, the amplifier could be used to measure voltages across points AD. Since the grid circuit contains no loading resistance, and since the grid itself draws no current when it is negative with respect to the cathode, such a vacuum-

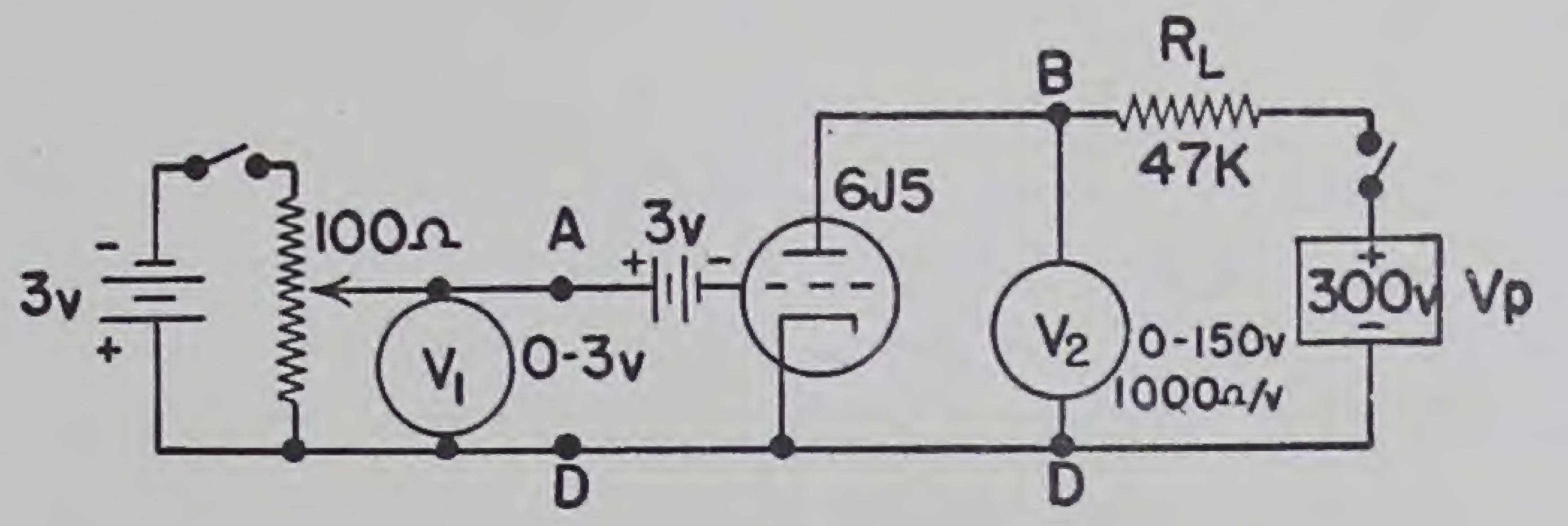


Fig. 45-2.

tube voltmeter does not affect the (d-c) circuit to which it is connected. This simple vacuum-tube voltmeter has the disadvantage that its scale does not read zero for zero input signal, but reads instead the quiescent value of the plate voltage. To eliminate this difficulty, the voltmeter may be connected between the plate and a source of voltage whose value is just equal to the quiescent value of the plate voltage. Then, when there is no input signal, the potential difference between the terminals of the voltmeter is zero, and the meter reads zero for zero input signal. Any input signal will then cause a deflection on the voltmeter proportional to the e_{out} of Eq. (4).

***Method:** Part I. Determination of Stage Gain with Load.* Familiarize yourself with the connections of the vacuum-tube test board, and then set up the circuit of Fig. 45-2. Using 3 volts of grid bias (grid connected to negative end of bias battery), take a set of plate voltage readings, e_b, for values of e_g from zero to -3 volts. Record the resistance of the plate voltmeter, V_2, since this resistance in parallel with R_L constitutes the net load resistance. (NOTE: Most voltmeters are rated in "ohms per volt." The resistance of the voltmeter is the number of ohms per volt times the full-scale value of voltage.) Subtract the quiescent value from each reading of the total plate voltage to obtain the value of e_{out} for each signal voltage. Plot the output voltage against the input signal voltage. Determine the slope of the resulting line, and calculate the actual stage gain. By use of Eq. (4) calculate the theoretical stage gain, using the values $\mu = 20$ and $r_p = 7700$ ohms for the 6J5 tube.

Part II. Determination of Stage Gain without Load. Modify the circuit of Fig. 45-2 to be like that of Fig. 45-3. Use a mirror-scale voltmeter in the high-voltage circuit. The 5000-ohm potentiometer is

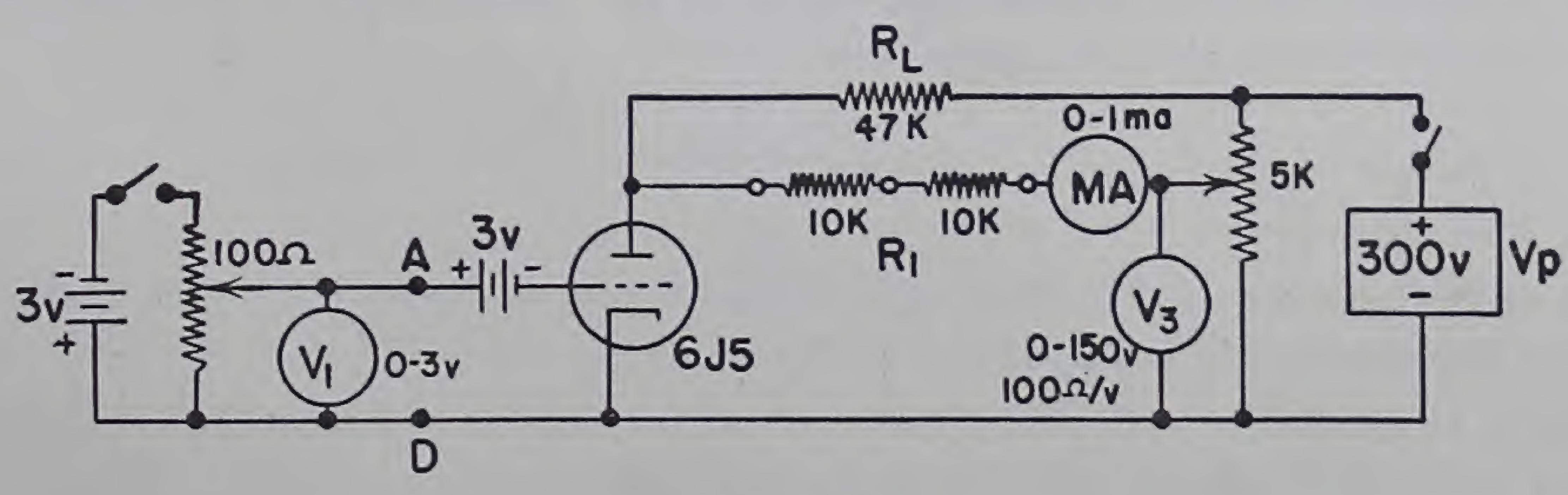


Fig. 45-3.

mounted on the vacuum-tube test board. With the grid signal voltage, e_g, set at zero, adjust the 5000-ohm potentiometer to get a zero reading on the milliammeter. The high-voltage voltmeter, V_3, now reads the quiescent plate voltage. *Why?* Take a set of readings of plate voltage for values of signal voltage from zero to -3 volts, resetting the potentiometer so that the milliammeter reads zero each time. From each reading of the total plate voltage, subtract the quiescent value to obtain the value of e_{out} for each signal voltage. (It should be noted that this value of e_{out} is the same as would be obtained under no-load conditions. Adjusting the potentiometer to a zero reading on the milliammeter each time ensures this—a circuit drawing no current has effectively an infinite resistance.) Plot the output voltage against the input signal voltage

on the same graph as used in Part I. Determine the slope of this line and the actual stage gain. Compute the theoretical stage gain using $\mu = 20$ and $r_p = 7700$ ohms, and compare with the actual stage gain. Calculate the value of μ which would make the theoretical stage gain equal to the actual gain. Assume that this value of μ, rather than 20, is the actual value for your tube, and recalculate the theoretical gain of the stage for Part I of this experiment. Compare this recalculated theoretical value with the actual value obtained in Part I.

Part III. Vacuum-tube Voltmeter. Modify the previous circuit to be like that of Fig. 45-4. Make R_1 20,000 ohms, and set dial box R_2 at maximum value. Turn on the high voltage, and with zero input signal, set the 5000-ohm potentiometer to give zero current in the milliammeter. (In an actual vacuum-tube voltmeter, this adjustment is labeled "zero set.") Without changing the zero-set potentiometer, adjust the

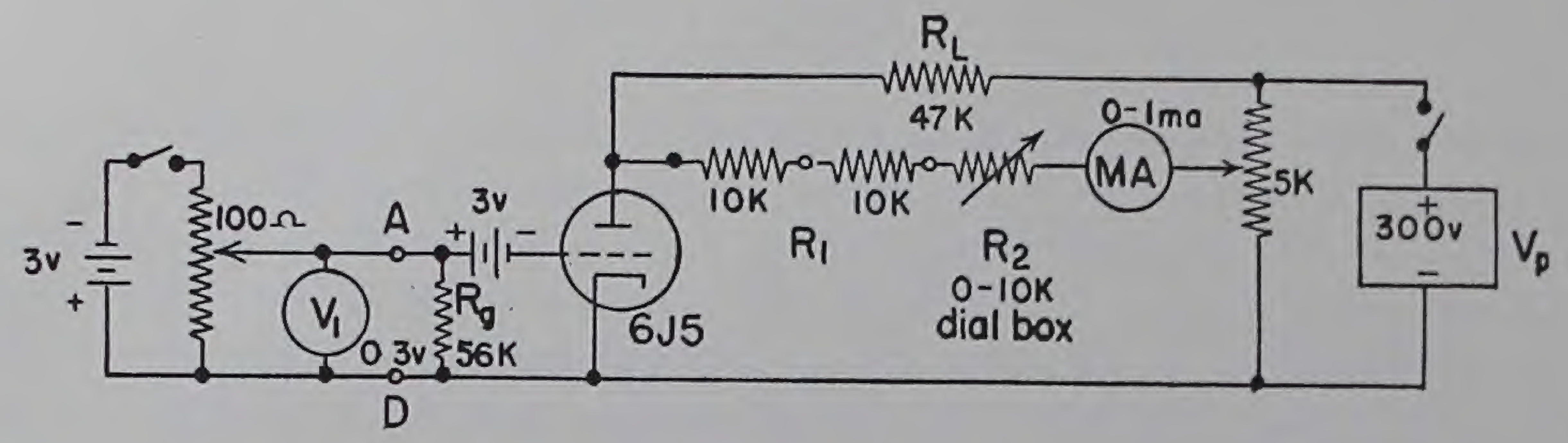


Fig. 45-4.

input signal to -2.0 volts. Adjust the dial box R_2 to make the milliammeter read 1.0 ma (full scale). If the minimum setting on the dial box does not bring the current up to full scale, turn off the high voltage, make R_1 10,000 ohms, set dial box to maximum, and reconnect the high voltage. Readjust the zero set with zero input signal, and reset the input to -2.0 volts. Adjust the dial box to obtain full-scale deflection on the milliammeter. Take milliammeter readings for a series of input signals from 0 to -2.0 volts. Record the sizes of R_1 and R_2. Disconnect the input signal circuit at the points labeled A and D, and substitute a test cell furnished by the instructor. This consists of a dry cell with a resistance in series to simulate an internal resistance. Record the voltage of this cell as measured by your VTVM. Also measure the voltage of the test cell with the grid voltmeter, V_1. Record the resistance of the grid voltmeter. The input impedance of the vacuum-tube voltmeter is 56,000 ohms ($= R_g$). Plot the calibration curve for the VTVM with milliammeter readings versus input volts.

Record: Record apparatus numbers and the values of all components used in each circuit. List tables of inputs and outputs and values plotted on graphs.

QUESTIONS

1. Discuss the output circuit of Fig. 45-3 as a Wheatstone bridge, pointing out which elements in this circuit correspond to the elements of the circuit of Fig. 35-1.

2. Draw the equivalent circuit of the vacuum-tube voltmeter, considering the resistance of the 5000-ohm potentiometer to be negligible compared to that in the milliammeter circuit. (Hint: The milliammeter circuit is now simply in parallel with the plate resistor.)

3. Calculate the stage gain of the vacuum-tube amplifier used as a vacuum-tube voltmeter, using the corrected value of μ obtained in Part II. As the resistance in the milliameter circuit use $R_1 + R_2$ as set for full-scale deflection for input of -2 volts. Calculate the current to be expected in the milliammeter for an input to the grid of -1.5 volts. (Hint: Calculate the change in plate voltage for such an input using the value of stage gain just obtained, and then calculate the current which this voltage will cause in the milliammeter circuit.) Does this current agree with the calibration curve obtained in the experiment?

4. Calculate the emf and the "internal" resistance of the test cell measured in Part III, as determined by use of the grid voltmeter and the vacuum-tube voltmeter. Compute the errors encountered in the read-

ings of these two instruments, under the circumstances (*i.e.*, by what factor does each instrument fail to measure the true emf of the cell?).

5. If an amplifier were constructed so as to consist of two stages like that of Fig. 45-1a with 56,000-ohm grid resistors, calculate the over-all gain, assuming that $\mu = 20$ and $r_p = 7700$ ohms, and that the last stage "looks into" a 56,000-ohm load. (Assume that a "d-c restorer" is used between stages so that the difference in potential between the first plate and the second grid may be ignored. This could be simply a battery of voltage equal to the quiescent plate potential.)

6. What would be the theoretical effect on the gain of the single-stage amplifier of Fig. 45-1 of the following? (Assume that $\mu = 30$, $r_p = 10{,}000$ ohms, and $R_L = 50{,}000$ ohms.)

(a) Placing a 50,000 ohm grid resistor between the terminals marked A and D.
(b) Doubling the plate resistor.
(c) Placing a load of 200,000 ohms across the terminals marked B and D.
(d) Changing the plate supply voltage from 300 to 310 volts.
(e) Replacing the tube with one having twice the amplification factor.

7. The vacuum-tube amplifier as used in the vacuum-tube voltmeter may be thought of as a current amplifier. That is, it draws a certain current from the circuit being measured, and furnishes current to deflect the milliammeter. What is the current-amplification factor in the circuit of the experiment? Theoretically, how does this factor depend on the μ of the tube?

Experiment 46.

Vacuum-tube A-C Amplifier

Object: To study the behavior of a vacuum-tube amplifier with a-c signals.

Apparatus: Vacuum-tube test board with 6J5 tube, socket, and filament transformer; 15-volt grid bias battery; source of plate voltage (about +300 volts dc.); 47,000-ohm 2-watt plate resistor; output load board with 100,000-ohm calibrated resistor and 300-$\mu\mu$f condenser; input circuit board with 56,000-ohm grid resistor and 0.05-μf coupling condenser; 10,000-ohm General Radio dial box; audio-frequency oscillator with range from 20 to 200,000 cps; cathode-ray oscilloscope; single-pole double-throw switch. A-c vacuum-tube voltmeter optional, for use in Part I-B.

Theory: In addition to the factors affecting the vacuum-tube amplifier under d-c conditions, as discussed in Experiment 45, we must consider others which occur when the signals to be amplified are alternating in character. These additional factors are principally those of *frequency response.* The gain of a stage of vacuum-tube amplification is usually independent of the frequency over a certain limited range of frequencies. This range includes the "mid-frequency" of the amplifier, which is that frequency at which no decrease of gain is experienced because of the reactances in the circuit. At low frequencies the gain usually decreases because of insufficient coupling capacity, and at high frequencies the gain falls off because of the loading effect of certain circuit capacitances. If gain is plotted against frequency, the resulting curve is horizontal over a certain range of frequencies, and falls off at each end. The gain of the amplifier is said to be "flat" between the frequencies at which the curve begins to deviate appreciably from the horizontal portion. The "low-frequency half-power point" is that lower frequency at which the gain is down to 70.7% of the mid-frequency gain. The "high-frequency half-power point" is similarly that higher frequency at which the gain is down to 70.7% of the mid-frequency gain. ("Half-power" implies a resistive load, in which case the power output is directly proportional to the square of the voltage output. Thus when the voltage is 0.707 times as large as the reference value—the mid-frequency output voltage—the power is $0.707 \times 0.707 = 0.500$ times the reference power.)

For a better understanding of the following discussion, the student is advised to review Experiment 42, A-c Circuits. It will be recalled that the reactance of a condenser varies inversely as the frequency of the impressed voltage. Thus, the series coupling condenser, C_c, of Fig. 46-1 passes high-frequency signals very well, but at lower frequencies, its reactance becomes comparable in size to the resistance of the grid resistor, R_g, and the signal is voltage-divided between C_c and R_g. In addition, there is an accompanying phase shift, the voltage at the grid leading that impressed on the input terminals. It is seen by applying the principles of Experiment 42 that when the frequency is such that the reactance of the coupling condenser is equal to the resistance of the grid resistor, the phase shift is 45° and the signal at the grid is 0.707 of the input signal. Thus this frequency is the lower frequency half-power point.

By referring to the equivalent circuit, Fig. 46-1b, it is seen that a capacitance between plate and cathode circuits has the effect of a condenser in parallel with the load and plate resistors. At high frequencies, therefore, this capacitance shunts the output circuit with a low reactance, thus reducing the stage gain with an accompanying shift of phase. An analysis somewhat beyond the scope of this discussion shows that the

half-power point is reached when the reactance of the condenser is equal to the parallel resistance of the load resistance R, the plate resistor R_L, and the plate resistance r_p. In the experimental setup, a physical capacitor is placed in the circuit. In actual amplifiers, this capacitance is kept to as low a value as possible to improve the high-frequency response of the amplifier. In this case the capacitance, C_L, consists of the distributed capacity of the wiring between the plate and the load, the capacitance of the load itself, and the internal capacities of the elements of the tube and of the socket—the sum, in fact, of all the capacitances between the plate and the cathode of the tube, both internal and external.

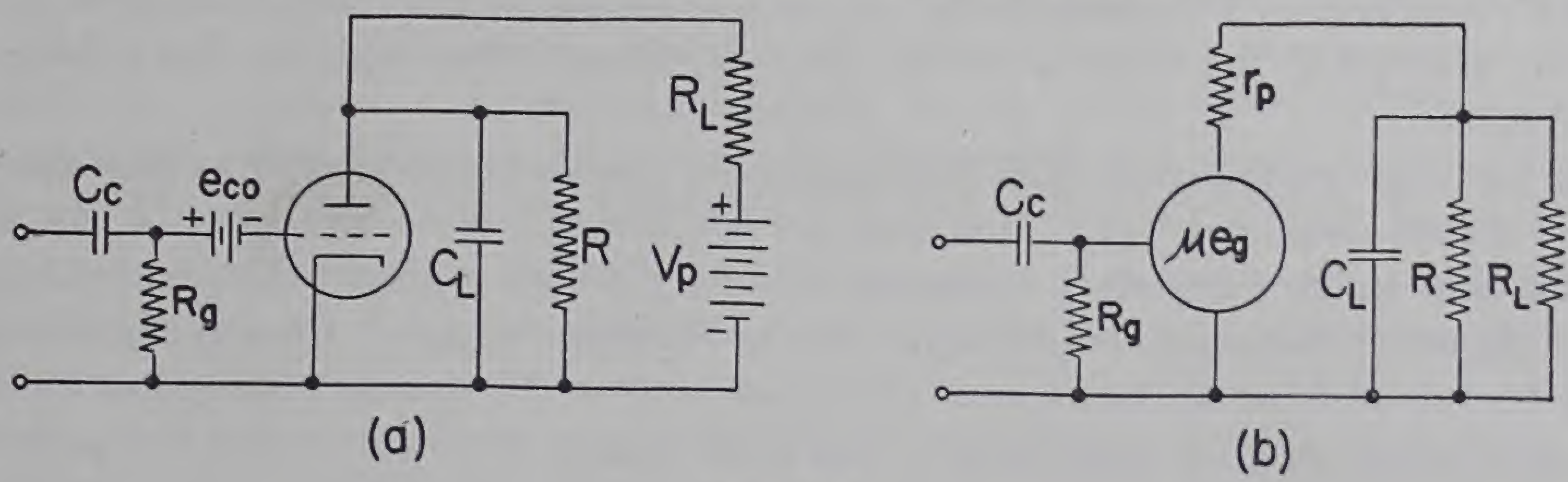


Fig. 46-1.

Another aspect of vacuum-tube amplifiers which it will be of interest to examine is the amplitude-range of operation. The previous analysis has assumed that the characteristics of the amplifier are linear, *i.e.*, that the output is always exactly proportional to the input signal. However, the tube characteristics have a limited range, and with large input signals, the operation is no longer linear. The nonlinearity becomes particularly striking in two cases: that of grid cutoff and that of plate saturation. If the grid is driven sufficiently negative, the plate current is reduced to zero. This condition is known as "cutoff." If the grid is driven to zero or to a positive value, the plate current increases, causing the IR drop across R_L to become larger and the plate voltage smaller. When the plate voltage has dropped to a low value (determined by the tube characteristics) increasing the grid voltage will not increase the plate current. This condition is known as "plate saturation."

***Method:** Part I. Determination of Frequency Characteristics.*

A. Using Cathode-ray Oscilloscope. (See Appendix II, Note N on the cathode-ray oscilloscope.) Connect the circuit as shown in Fig. 46-2, with the oscilloscope sampling the voltages at points C or A, as

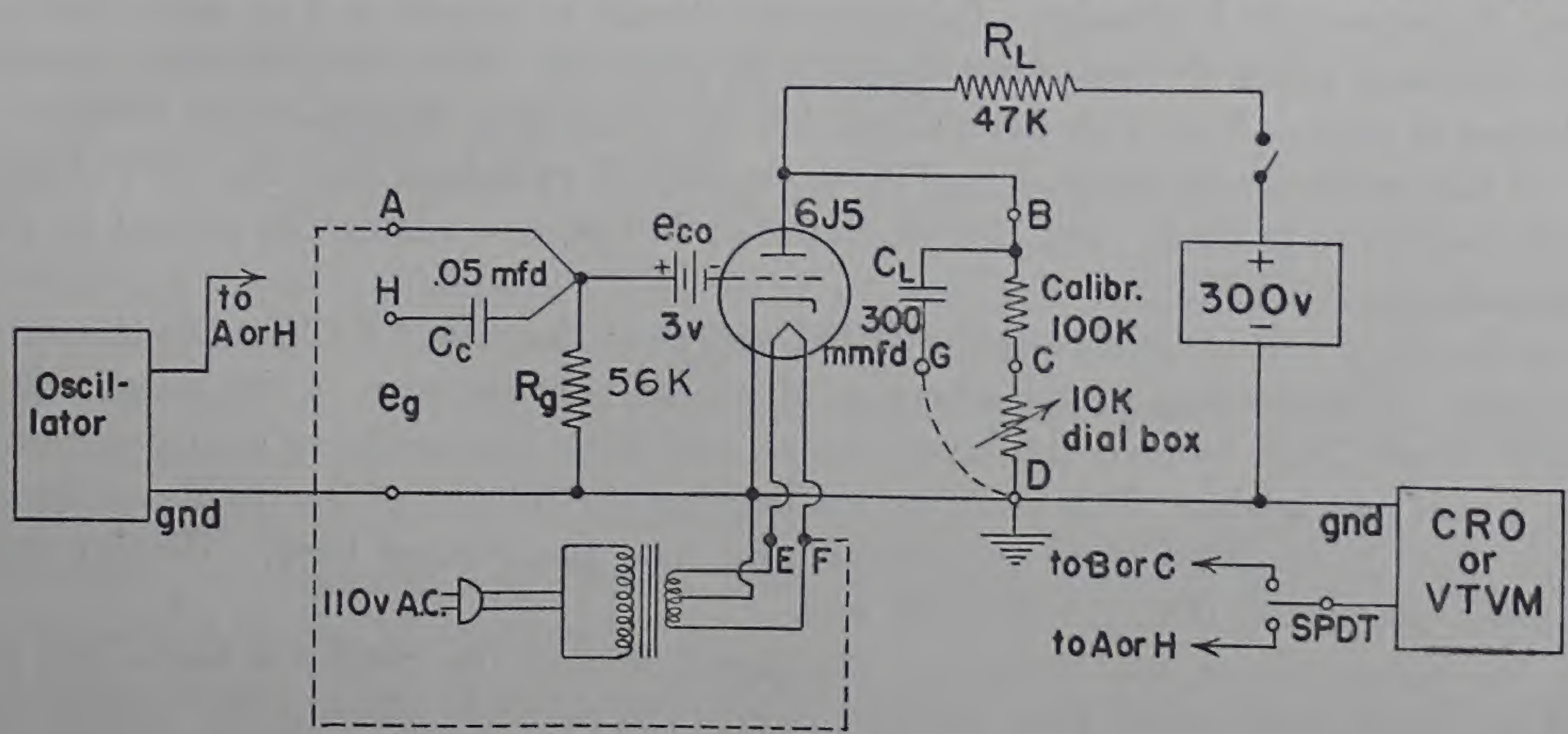


Fig. 46-2.

determined by the single-pole double-throw switch. Connect points A and F together, as shown by the dotted line. This places an alternating voltage on the grid equal to half of the filament voltage, *i.e.*, 3.1 volts. In this experiment, we wish to use a signal of approximately 1 volt, rms, and the 3.1-volt signal is used to calibrate the oscilloscope. Connect the scope to the grid input (point A) by means of the switch,

and adjust the sensitivity of the scope until the 3.1-volt signal occupies about three-quarters of the scope face. Count the number of divisions from peak to peak of this wave, using the scope synchronization to halt the pattern at two or three complete cycles. Disconnect the wire between A and F, and connect the oscillator as shown to feed its signal to point A. Adjust the output of the oscillator (running at about 60 cps) to give a deflection on the scope of about one-third that obtained using half the filament voltage. The oscillator is now adjusted for an output of about 1 volt. Readjust the sensitivity of the oscilloscope until the 1-volt signal fills about two-thirds of the face of the tube, and has its peaks *accurately* on two of the major lines on the scope face. This amplitude will be a reference or standard from now on. Use both the centering and gain controls of the scope to set up this condition. Once this has been done, *do not alter the scope gain control further.*

Now turn on the high voltage and shift the single-pole double-throw switch so that the scope reads the output across the 10,000-ohm dial box. It should be noted that the combination of the dial box and the calibrated 100,000-ohm resistor is simply a voltage divider connected across the plate output. Adjust the dial box until the signal on the scope is just equal to the reference signal. Record the resistance set on the dial box. Knowing this resistance, the ratio of the signal at B to the signal measured at C can be computed. Since the signal at C is just equal in amplitude to the input signal, this ratio is just the gain of the stage.

Set the oscillator to 20 cps. Switch the oscilloscope to read the input signal, and with the output control *of the oscillator* adjust the input signal to the reference value. Switch the scope to the output, and adjust the dial box to give a standard deflection on the scope. Record this value. Repeat this procedure at 200, 600, 2000, 6000, 20,000, 60,000, and 200,000 cps.

Disconnect the plate voltage and reconnect the circuit so that the oscillator feeds into point H, and so that the oscilloscope measures that input; also, connect point G to point D, placing the loading capacitor C_L in the circuit. Reconnect the plate voltage. Repeat the frequency run, starting at 20 cps, and measuring at the following additional frequencies: 40, 60, 90, 140, 200, 600, 2000, 6000, 10,000, 20,000, 40,000, 60,000, 90,000, 140,000, 200,000. In situations where the dial box is not large enough to bring the output signal up to reference amplitude, set it at 10,000 ohms and count the number of divisions on the scope. Assuming the scope response to be linear, compute the gain on the basis of the deflection on the scope and the resistances used in the voltage divider. On logarithmic paper, plot in each case the gain of the amplifier versus the frequency. Find the half-power points on each curve if possible. Calculate, from the circuit constants, the theoretical gain at mid-frequency, using $\mu = 20$ and $r_p = 7700$ ohms, and remembering that R_L is shunted by the voltage divider network.

B. Using a-c Vacuum-tube Voltmeter. Connect the circuit as shown in Fig. 46-2, except that the dial box may be left out and point C connected directly to point D. The vacuum-tube voltmeter (VTVM) samples the voltages at points B or A as determined by the single-pole double-throw (SPDT) switch. Connect the output of the oscillator to point A, and close the SPDT switch so that the VTVM reads the voltage at this point with respect to ground. Set the oscillator at 20 cps, and adjust the output to give a reading of exactly 1 volt rms on the VTVM.

Now turn on the plate voltage and shift the SPDT switch so that the VTVM reads the output at point B, the plate of the tube. The reading in volts here is, of course, just the gain of the stage. Set the oscillator to 60 cps. Switch the VTVM to read the input signal, and with the oscillator's output control, adjust the signal to exactly 1 volt rms again. This is necessary because the output of the oscillator depends somewhat on its frequency. Switch the VTVM to the output and read the voltage here. Repeat this procedure at 200, 600, 2000, 6000, 20,000, 60,000, and 200,000 cps.

Disconnect the plate voltage, and reconnect the circuit so that the oscillator feeds into point H, and so that the VTVM measures that input; also, connect point G to point D, placing the loading capacitor in the circuit. Reconnect the plate voltage. Repeat the frequency run, starting at 20 cps and measuring at the following other frequencies: 40, 60, 90, 140, 200, 600, 2000, 6000, 20,000, 40,000, 60,000, 90,000, 140,000, 200,000 cps. On logarithmic paper, plot in each case the gain of the amplifier versus the frequency. Find the half-power points on each curve if possible. Calculate from the circuit constants the theoretical gain at mid-frequency, using $\mu = 20$ and $r_p = 7700$ ohms, and remembering that R_L is shunted by the 100,000-ohm resistor.

Part II. Phase Relationships. With the circuit connected as in Fig. 46-2, connect point A to point F as indicated by the dotted line. Wire the SPDT switch so that the cathode-ray oscilloscope samples the voltages at either point A or point B. Connect the plate voltage, and synchronize the oscilloscope on 60 cycles (line). Adjust the oscilloscope frequency controls so that two cycles of the input wave are displayed. Note the phase of this input (whether it is rising or falling at the left-hand end). Switch the scope to read the output voltage. After adjusting the amplitude control, note the phase of this wave form. What is its relationship to the input wave form?

Disconnect the plate voltage, and connect the oscilloscope so that the input of the amplifier (point H) is connected to the vertical-deflection terminal of the oscilloscope, and the output (point B) is connected to the horizontal-deflection terminal. Set the oscilloscope's horizontal amplifier switch so that the amplifier output is substituted for the ordinary sweep. Connect the amplifier so that the input from the oscillator is capacitively coupled (*i.e.*, the oscillator is connected to point H) and so that the loading capacitor, C_L, is in the circuit. Set the oscillator at about 600 cps with an output of about 1 volt rms, and connect the plate voltage. Adjust the gain controls of the oscilloscope so that the pattern is tilted at about 45° and has a maximum dimension equal to about three-quarters of the scope diameter. Make a sketch of the type of pattern presented. If it is not a straight line in the second and fourth quadrants, call the instructor.

Now reduce the frequency until the pattern becomes an ellipse with the major axis two to three times as large as the minor axis. Using the centering controls on the scope, move the pattern until the center of the ellipse is at the center of the scope. Count and record the number of divisions on the scope between the places where the ellipse crosses the vertical axis. Count and record the divisions between the maximum vertical limits of the ellipse. Record the frequency. See Appendix II, Note N, for the method of determining the phase angle these measurements indicate. As mentioned in the Theory, this is a leading angle, *i.e.*, the output voltage leads the input voltage by this angle. Examine the amplitude-frequency curve previously obtained. How do phase shift and amplitude loss correlate at a given frequency?

Part III. Overdriven Amplifier. With the plate voltage off, connect the circuit as in Fig. 46-2, with the scope measuring the output voltage at point B. Connect point F to point A so that the input signal is about 3 volts rms. Connect the grid bias battery for a bias of $-4\frac{1}{2}$ volts. Turn on the plate voltage and examine the output on the scope. Note whether it seems to be a good sine wave. Now examine the output with the different biases available, namely -15 volts, $-13\frac{1}{2}$, -12, $-10\frac{1}{2}$, -9, $-7\frac{1}{2}$, -6, -3, $-1\frac{1}{2}$, and zero (point A connected directly to the grid). Sketch the output wave form in each case. Describe what is happening in the circuit to produce the observed wave forms. Remember that the output is inverted compared to the input.

Record: Record apparatus numbers and the values of all components used in each circuit. List tables of inputs and outputs, and other data called for in the Method.

QUESTIONS

1. By use of the applicable portions of Experiment 42, calculate the half-power frequencies of the circuit of Fig. 46-2 with the coupling and loading condensers used in the circuit.

2. Calculate the value of the tube amplification factor which would yield the same mid-frequency stage gain which you measured in Part I.

3. The oscilloscope or vacuum-tube voltmeter used in Part I does not have an infinite input impedance. If its input impedance is 1 megohm (10^6 ohms), what percentage error in the stage gain is introduced by ignoring its loading effect on the plate circuit?

4. It was tacitly assumed that the response of the oscilloscope or the vacuum-tube voltmeter used in Part I was independent of frequency. However, both instruments use vacuum-tube amplifiers similar to the one under investigation. Discuss briefly the errors introduced into Part I of the experiment by a frequency response in these instruments similar to (but a good deal better than) that of the amplifier stage studied.

5. Over what range of frequencies is the experimental vacuum-tube amplifier "flat"? For the purposes of this question, assume that a drop of 3% or less is satisfactorily flat. Answer this for each gain-frequency curve plotted.

6. What would be the oscilloscope pattern in Part II indicating a 90° phase shift? Prove your answer.

7. In most oscilloscopes, the amplifier for the horizontal deflection plates is a different type from that for the vertical deflection plates, and thus the two amplifiers will have different frequency characteristics. Suppose that the horizontal amplifier were poorer in quality than the vertical amplifier. What effect would this have on the results in Part II? Describe how, using the *same* input signal to both, the oscilloscope amplifiers could be "calibrated" at some frequency, and then used to get an accurate measure of the phase shift in the experimental amplifier.

8. Assuming that the input signal used in Part III were known to be 3.15 volts rms, and that the indicated values of bias voltage used were the true ones, estimate the approximate limits of linear operation of the experimental amplifier in terms of the maximum and minimum total instantaneous grid voltage allowable. (HINT: The total instantaneous grid voltage is the algebraic sum of the grid bias and the instantaneous value of the input signal; the peak-to-peak amplitude of a sine wave is 2.83 times the rms value.)

9. In the capacitively coupled vacuum-tube amplifier, consisting of two or more stages of amplification such as the one studied, no "d-c restorer" is necessary between stages. (See Question 5 of Experiment 45.) Explain.

Experiment 47

Thermionic Emission: Richardson's Equation

Object: To study the relation between the saturation current and the filament temperature of a diode vacuum tube—Richardson's equation. To determine the work function of tungsten.

Apparatus: Mounted FP400 tube, filament d-c ammeter (0—2.5 amp), plate d-c milliammeter (0—25 ma), plate d-c milliammeter (0—1 ma), plate d-c voltmeter (0—150 volts), traveling microscope, storage battery, rheostats, switches.

The FP400 tube (General Electric Company) is a diode well adapted for the study of thermionic emission in a high vacuum. It consists primarily of a pure tungsten filament axially located in a cylindrical zirconium-coated-nickel anode. There is a small hole in the anode through which one may view the central portion of the filament for the purpose of temperature determination. The use of pure tungsten for the filament is advantageous since its properties are well known over a wide range of temperatures. The electrical operating values of this tube are convenient. A heavy-duty 6-volt storage battery may be used to supply the filament current and the plate voltage may be supplied from a d-c 110-volt power supply. For a more complete description of this tube see Appendix II, Note P.

Theory: A metal is characterized by the existence in its structure of a large number of free electrons (about one per atom) which are not permanently attached to the atoms but are virtually free to move about in the metal. A potential barrier at the metal surface tends to prevent these free electrons from escaping at normal temperatures. However, when the metal is heated to a sufficiently high temperature, some of these free electrons acquire sufficient additional energy to carry them over the potential barrier. Thus at high temperatures the metal emits an appreciable number of these electrons which may be drawn away from the heated metal by a suitable electric field.

In the case of the ordinary diode this field is produced by impressing a suitable potential difference between the filament (heated metal) and the plate. If the plate is held at a large enough positive potential with respect to the filament so that all of the electrons emitted by the filament are drawn to the plate, then the plate current is controlled primarily by the filament temperature and is practically independent of the plate potential. Under these conditions the plate current is said to be saturated and is purely a function of the temperature of the filament and its surface area.

The relation between the saturated plate current i_s and the absolute temperature T of the filament is given by the equation

$$i_s = AT^2e^{-w_o/kT} \qquad \text{(Richardson's equation),} \tag{1}$$

where A is a constant directly proportional to the surface area of the filament, k is Boltzmann's constant, and w_o is the work function of the metal filament. The work function w_o is the minimum energy required to transfer an electron from an interior to an exterior point of the filament. It is possible to verify Richardson's equation and, at the same time, determine w_o, by observing the saturation currents i_s for different filament temperatures T.

The theoretical derivation of Richardson's equation is a difficult matter. It may be deduced either by use of the laws of thermodynamics or by use of Fermi-Dirac statistics applied to electrons within a metal.

A rigorous development on either of these bases lies outside the province of this discussion. However, a suggestive thermodynamical argument for the equation is given in the following section (fine print).

The emission of electrons by a hot metal corresponds in many respects to the sublimation of a solid. In both cases particles (electrons or molecules) are transferred from the solid to the vapor state until, under the proper conditions, equilibrium between the solid and vapor states is realized. When this occurs the relation between the vapor pressure P and the absolute temperature T of the system is given by Clapeyron's equation

$$\frac{dP}{dT} = \frac{L}{T\,\Delta V}. \tag{2}$$

Here L is the molar latent heat of sublimation of the solid and ΔV is the difference between the molar volume of the vapor and that of the solid. If it is assumed that ΔV is essentially the volume of the vapor because of the low density of the vapor as compared to that of the solid, and also that the vapor approximates an ideal gas, then

$$\Delta V = \frac{RT}{P}. \tag{3}$$

The molar latent heat of sublimation L is the amount of energy required to transfer 1 mol of the particles from the solid to the vapor state. L may be regarded as the sum of three energies: the energy L_o required by 1 mol of the particles to surmount the potential barrier at the surface of the solid; the kinetic energy $\frac{3}{2}RT$ of 1 mol of the particles in the vapor state; and the work RT done by 1 mol of the particles against the prevailing vapor pressure. Thus L may be written

$$L = L_o + \tfrac{5}{2}RT. \tag{4}$$

In the case of ordinary sublimation L_o is a function of T since the specific heat of a solid is a function of T. However, in the case of electron sublimation it may be shown that L_o is practically independent of T up to extremely high temperatures as evidenced by the fact that the free electrons in a metal do not contribute appreciably to its specific heat. The behavior of free electrons in a metal is described by Fermi-Dirac statistics which give these electrons an enormous energy even at absolute zero but which prevent most of them from absorbing any additional thermal energy. Hence the free electrons *inside* a metal do not act at all like an ideal gas.

If we substitute the values of ΔV [Eq. (3)] and L [Eq. (4)] in Eq. (2), and then integrate this equation, we get

$$P = P_o T^{5/2} e^{-L_o/RT}, \tag{5}$$

where P_o is an undetermined constant of integration. Equation (5) gives the vapor pressure P of the electron gas in equilibrium with the metal at any temperature T.

Since this electron gas is in equilibrium with the metal, the number of electrons leaving the metal during any time must just equal the number of electrons returning to the metal from the vapor in this same time. This latter quantity may be calculated from elementary kinetic theory and turns out to be proportional to $P/T^{1/2}$. If the electron vapor is not allowed to accumulate in the neighborhood of the metal but is pulled away by an electric field, this in no way affects the emission of electrons from the metal, *i.e.*, the same number per unit time are emitted. The saturated plate current i_s is proportional to this number. Hence

$$i_s \propto \frac{P}{T^{1/2}}. \tag{6}$$

The value of P is given by Eq. (5) which value when substituted in (6) gives

$$i_s \propto T^2 e^{-L_o/RT}. \tag{7}$$

Relation (7) is essentially equivalent to Richardson's equation (1). It is only necessary to introduce the proportionality constant A and to set $L_o = N_o w_o$ and $R = N_o k$ where N_o is Avogadro's number.

Thermodynamics alone can determine neither the integration constant P_o in Eq. (5) nor the proportionality constant A in Eq. (1). Their evaluations require a use of statistical mechanics and kinetic theory.

Temperature of Filament. It is necessary to determine the absolute temperature of the emitter (tungsten filament) in order to verify Richardson's equation. This is a difficult task, even though several different methods are available, because the temperature of the filament is not uniform throughout its entire length.

An optical pyrometer or a radiation pyrometer may be used to determine the mid-point filament temperature. Both of these instruments operate on the basis of radiation principles. The first instrument

matches a band of radiation from the hot body against a similar band from a calibrated lamp filament. The second instrument measures the total radiation from the hot body and hence "deduces" the temperature. Both of these instruments are difficult to use in this experiment because of the small size of the "filament viewing hole" in the anode of the FP400 tube.

Another method which is frequently used in determining the temperature of a pure tungsten filament is the resistance method. The ratio of the "hot" to the "cold" resistance of the filament may be correlated with the "hot" temperature of the filament. This method gives an average temperature of the hot filament which may be much lower than its maximum temperature. Because of the exponential form of Richardson's equation, however, it is better to use the maximum temperature of the filament (at its mid-point) rather than its average temperature.

A third method—the one used in this experiment—determines the mid-point temperature of the filament by utilizing a relation between this temperature, the filament current, and the diameter of the heated filament.

Consider a small mid-point section δl of the filament as shown in Fig. 47-1. Let its absolute temperature be T, its diameter be d, and its resistance be δR (ohms). In the steady state we may assume that the total

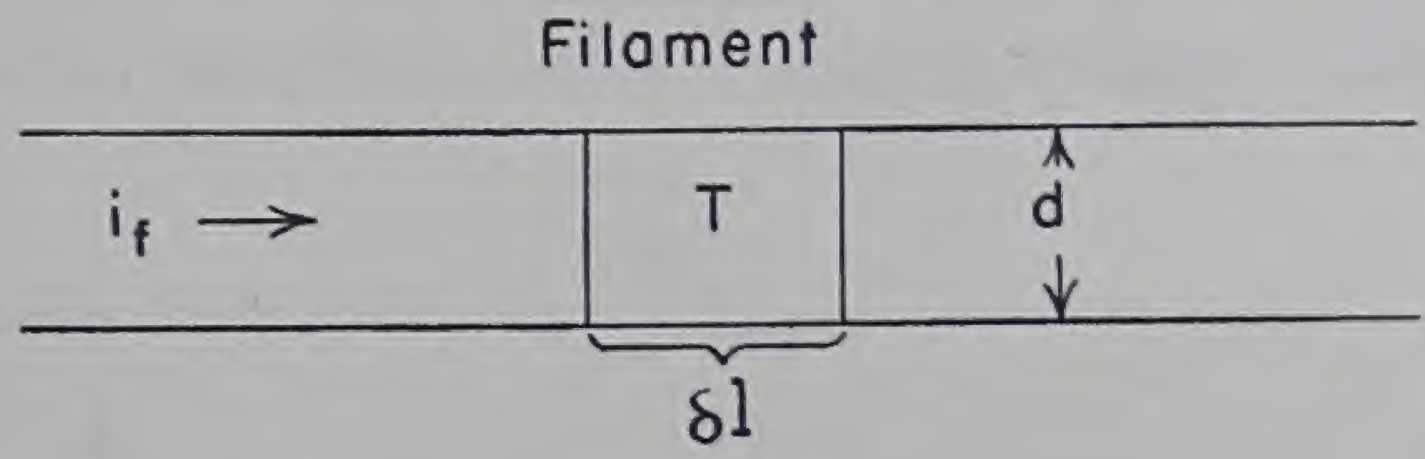


Fig. 47-1.

electrical energy per unit time fed into this section is converted into heat and then radiated out through the exterior surface of the section. The electrical energy input per unit time is $i_f^2\ \delta R \times 10^7$ ergs/sec, while the total energy radiated per unit time is, by the Stefan-Boltzmann law, $\epsilon(T)\sigma T^4 \pi d\ \delta l$ ergs/sec, where $\epsilon(T)$ is the total emissivity of the tungsten at temperature T and σ is the Stefan-Boltzmann constant. Hence

$$i_f^2\ \delta R \times 10^7 = \epsilon(T)\sigma T^4 \pi d\ \delta l. \tag{8}$$

In this equation δR may be replaced by $\rho(T)\,\dfrac{4\delta l}{\pi d^2}$, where $\rho(T)$ is the resistivity of tungsten at temperature T. Making this substitution for δR in Eq. (8) and rearranging terms, we get

$$\frac{i_f^2}{d^3} = \frac{\pi^2}{4 \times 10^7}\,\sigma\,\frac{\epsilon(T)}{\rho(T)}\,T^4. \tag{9}$$

The right side of Eq. (9) is a function of T alone and, in the case of tungsten, a known function. Hence it is possible to correlate T with the value of $i_f/d^{3/2}$ over a wide range of temperatures by means of a table of values.

Errors: The important errors in this experiment mostly arise from the difficulty of determining the effective temperature of the filament. As has been pointed out before, the filament temperature is not uniform. The filament is much cooler at the ends than in the middle. It is better to use the mid-point temperature rather than the average temperature in this experiment. The reason for this has already been given.

One may estimate the effect of temperature errors on the value of the work function w_o in the following manner. Equation (1) involves two unknowns, A and w_o. If these are to be determined, then at least two sets of values of i_s and T must be measured. Let these be i_{s1}, T_1 and i_{s2}, T_2, and let Eq. (1) be written for each of these sets. By eliminating A between these two equations, we get

$$\frac{i_{s1}}{i_{s2}} = \left(\frac{T_1}{T_2}\right)^2 e^{-\frac{w_o}{k}\left(\frac{1}{T_1}-\frac{1}{T_2}\right)}. \tag{10}$$

Solving Eq. (10) for w_o, we get

$$w_o = k\frac{T_1T_2}{T_2 - T_1}\ln\left[\frac{i_{s2}}{i_{s1}}\left(\frac{T_1}{T_2}\right)^2\right]. \tag{11}$$

The approximate determinate-error equation corresponding to Eq. (11) is

$$\frac{\Delta w_o}{w_o} = \frac{\Delta T_1}{T_1} + \frac{\Delta T_2}{T_2} - \frac{\Delta(T_2 - T_1)}{T_2 - T_1}. \tag{12}$$

In arriving at this equation we have omitted terms involving the natural logarithm since these are generally negligible. Why?

Equation (12) clearly indicates the effect of temperature errors on the value of w_o. If the temperature errors are determinate, then the first two terms of the right member of Eq. (12) become important; but if the errors are indeterminate, then the third term becomes predominant. The fact that errors in i_s do not appear in Eq. (12) simply indicates that these errors do not contribute much to the error in w_o. Hence it is much more important to measure T accurately than to measure i_s accurately.

Method: The circuit is shown in Fig. 47-2. A heavy-duty 6-volt storage battery supplies the filament current i_f; this *current should never exceed 2.25 amp.* A low-resistance rheostat Rh_1 and a high-resistance

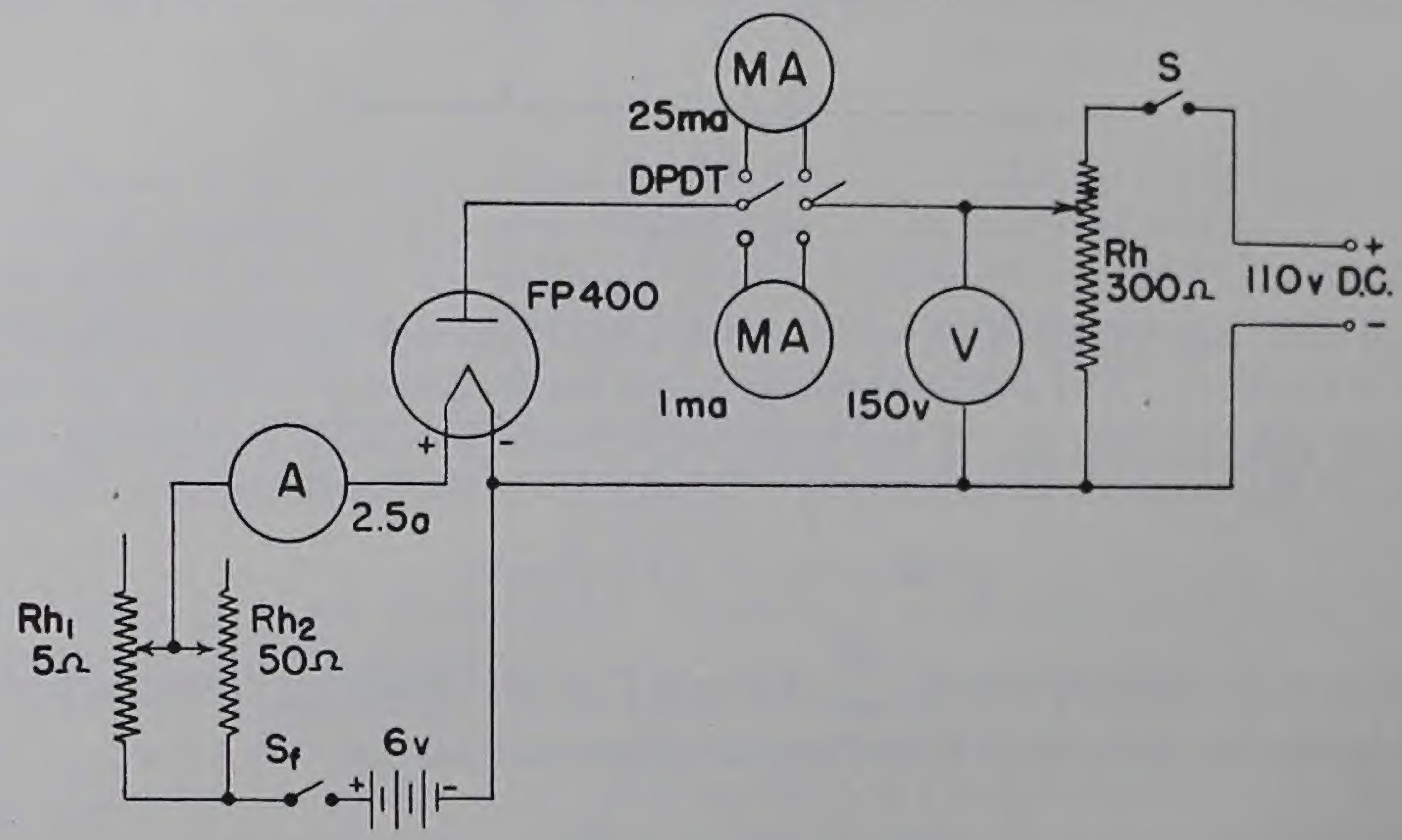


Fig. 47-2.

rheostat Rh_2, connected as shown, control the filament current. The rheostat Rh_1 gives coarse adjustment of the filament current, and the rheostat Rh_2 gives fine adjustment of this current. An ammeter A gives the filament current. This ammeter must be one of good quality with an error of less than 1% if a reliable determination of the filament temperature is to be obtained.

The plate voltage e_b is controlled by a 300-ohm potentiometer connected across a 110-volt d-c power line. Its value is given by voltmeter V. The plate current i_s is given by one of two milliammeters connected into the plate circuit by a double-pole double-throw swith.

Make the connections as shown in Fig. 47-2 (all switches open). Adjust rheostats Rh_1 and Rh_2 for *maximum resistance* in the filament circuit. Adjust the potential divider Rh in the plate circuit for *minimum* plate voltage. Connect the high-range milliammeter into the plate circuit. Then close the switches s_f and s, and note the readings of the meters.

Adjust the rheostats Rh_1 (coarse), then Rh_2 (fine) so that i_f is 1.50 amp. Increase the plate potential until the plate current is saturated, *i.e.*, no longer increases appreciably with increasing plate potential. Record the values of the *saturated* plate current and plate voltage for the filament current of 1.50 amp. Use the low-range milliammeter if $i_s < 1$ ma. Keeping the filament current at 1.50 amp, take five measurements of the diameter of the heated filament with the traveling microscope. Estimate to 0.0001 cm if possible.

Repeat the foregoing procedures for filament currents of 1.70, 1.90, and 2.10 amp. Under no circumstances allow the filament current to exceed 2.25 amp since, even though the filament may not burn out, its properties are likely to change.

Computations: Compute the absolute temperature T of the mid-point of the filament for each of the four filament currents by use of Table R, Appendix III. Use the average filament diameter for each value of the filament current in making the computations. Interpolate in Table R to get the temperature to the nearest 10°K. Calculate the error in each of these temperature values due to the estimated errors in i_f and d. Also calculate the errors in the values of $1/T$.

If Richardson's equation is valid, the graph of $\log_{10} (T^2/i_s)$ plotted against $1/T$ should give a straight line whose slope is $w_o/2.30k$. Verify this statement by use of Eq. (1). Make such a graph using your values of i_s and T. Indicate the error intervals in $1/T$ on this graph by drawing a small horizontal line segment of length $2\,\Delta(1/T)$ through each plotted point. Draw the best possible straight line for these plotted points. This line should cut all of the small line segments representing the error intervals in $1/T$. Determine the slope of this line and hence compute the work function w_o of tungsten in ergs and in electron volts. The value of k is given in Table L, Appendix III. Compare your value of w_o with that given in Table N, Appendix, III, for tungsten.

Determine the error in w_o by use of Eq. (12). Use your lowest and highest filament temperatures for this calculation.

Record: List apparatus and apparatus numbers. Tabulate i_f, d, i_s, $i_f/d^{\frac{3}{2}}$, T, ΔT, $1/T$, $\Delta(1/T)$, and $\log_{10} (T^2/i_s)$. Give your value of w_o, its error Δw_o, and the accepted value of w_o.

Experiment 48.

Thermionic Emission: Child-Langmuir Relation

Object: To obtain the relation between the plate current and plate voltage in a diode when the plate current is limited by space charge. To verify the Child-Langmuir relation. To estimate the value of e/m for electrons.

Apparatus: Mounted FP400 tube, filament d-c ammeter (0 to 2.5 amp), filament d-c voltmeter (0 to 5 volts), two-range d-c plate voltmeter (0 to 30 volts and 0 to 150 volts), plate milliammeter (0 to 5 ma), plate milliammeter (0 to 25 ma), rheostats, 6-volt heavy-duty battery, double-pole double-throw switch, two single-pole switches. For a description of the FP400 tube, see Experiment 47 and Appendix II, Note P.

Theory: When the filament of a diode tube is heated, it emits electrons which may be drawn over to the plate by a suitable potential difference between filament and plate. The flow of electrons from filament to plate constitutes an electric current (the plate current). In general this plate current i_b is a function of the plate potential e_b and of the filament temperature T. If the filament temperature is held constant, then i_b is a function of e_b. The form of this function may be determined experimentally; its complete theoretical derivation is one of great complexity.

The general form of the relation between i_b and e_b for constant T is illustrated in Fig. 48-1. The filament potential is assumed to be zero. For purposes of explanation the curve may be broken into three sections—AB, BC, and CD.

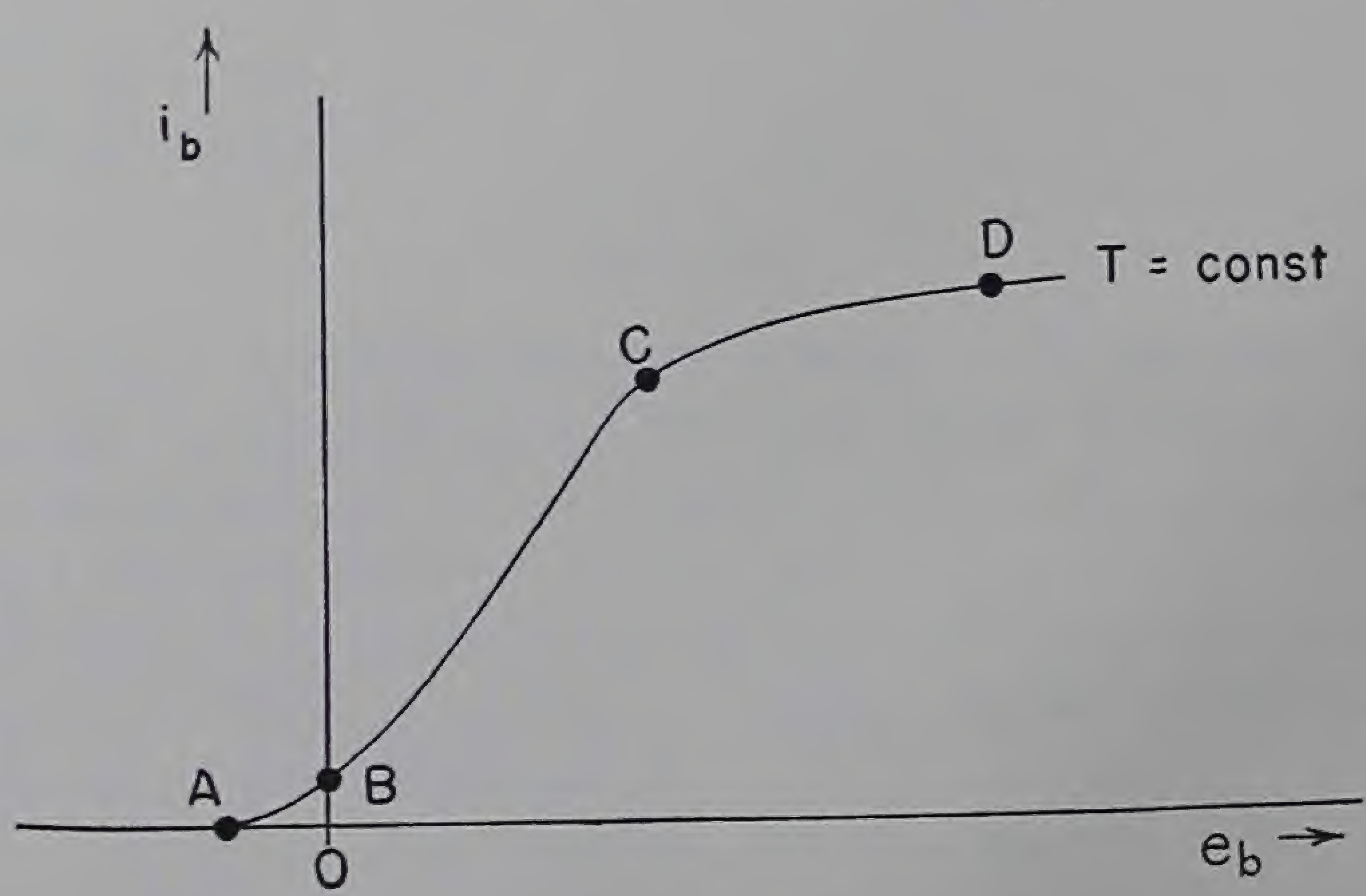


Fig. 48-1.

The first section AB of the curve indicates a flow of electrons from filament to plate even when the plate potential is zero or slightly negative. This arises because some electrons leave the filament with sufficient kinetic energy to carry them to the plate even against a small retarding potential (fraction of a volt).

The second section BC of the curve is explained in terms of the space charge between filament and plate. This region is occupied by a cloud of electrons (negative space charge) in transit to the plate. This

space charge is very dense in the immediate neighborhood of the filament and effectively sets up a small potential barrier there which forces some of the electrons to return to the filament. In a sense this space charge potential barrier near the filament acts like an increment in the work function of the filament that reduces emission. See Richardson's equation in Experiment 47.

It is possible to show for this section of the curve that the plate current i_b is proportional to the three-halves power of the plate potential e_b, *i.e.*, $i_b \propto e_b^{3/2}$. This is known as the Child-Langmuir relation.

The third section CD of the curve indicates saturation. The plate current in this section is practically independent of the plate voltage and depends only upon the filament temperature. The barrier due to space charge has disappeared and hence all electrons emitted by the filament are drawn to the plate. This section of the curve gives a constant value of the plate current, the value given by Richardson's equation.

In this experiment we are primarily interested in the section BC of the curve where space charge plays an important role.

Child-Langmuir Relation: Cylindrical Plate with Axial Filament

It may be shown for the case of a cylindrical plate with an axial filament that the Child-Langmuir relation takes the form

$$i_b = \frac{2}{9}\sqrt{\frac{2e/m}{3 \times 10^9}}\,\frac{l}{b\beta^2}\left(\frac{e_b}{300}\right)^{\frac{3}{2}}, \tag{1}$$

where i_b = plate current in amperes,
e_b = plate potential in volts,
l = effective length of filament in centimeters,
b = radius of plate in centimeters,
e = charge of electron in coulombs (absolute value),
m = mass of electron in grams, and
β^2 = function of ratio of plate radius to filament radius which function approaches unity for large values of the ratio.

Equation (1) shows not only that $i_b \propto e_b^{3/2}$ but also that the proportionality constant involves the value of e/m for electrons. Hence it is possible to estimate the value of e/m by determining this proportionality constant.

The complete derivation of Eq. (1) as given by Langmuir is quite complex. A fairly simple but incomplete solution is given in the following section in which the esu system of units is used exclusively.

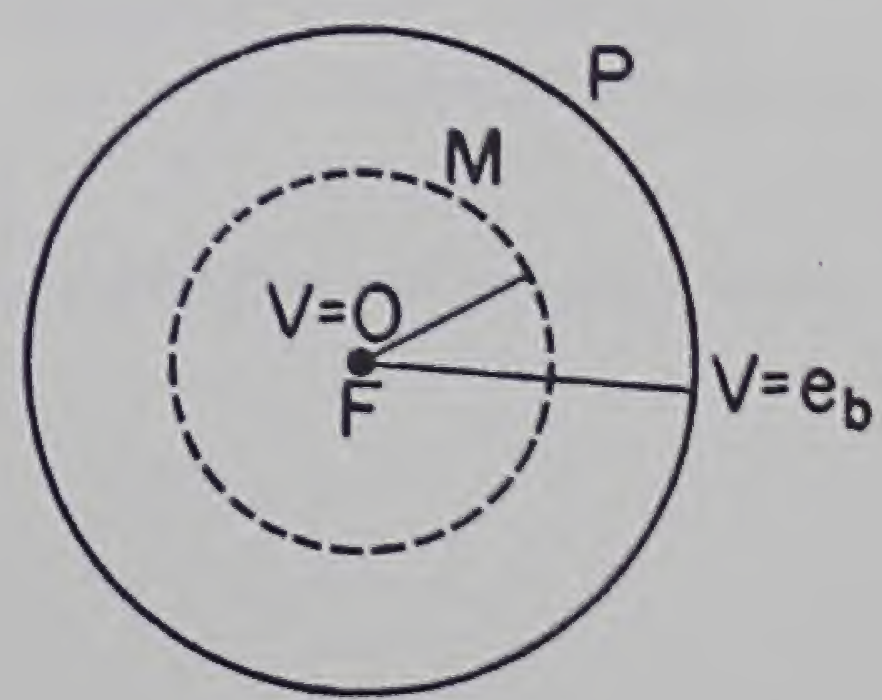


Fig. 48-2.

Consider the uniform emission of electrons from a line filament at zero potential lying along the axis of a cylindrical plate of radius b held at a positive potential e_b. A cross section of this setup is shown in Fig. 48-2. The electric field between F and P draws the emitted electrons to the plate thus producing the plate current. If end effects are neglected, and if it is assumed that the electrons are uniformly emitted at the filament with negligible initial velocities, then it is clear that the properties of the electric field and the motion of the electrons at any point M between F and P depend upon the single variable r, the radial distance from F to M. It is assumed of course that a steady state exists.

In order to arrive at an expression for the plate current under these conditions, consider the flow of electrons through any cylindrical surface having F as an axis, *e.g.*, the cylinder through the point M. At all points on this cylindrical surface electrons have the same radial velocity v, and the electron density n is the same. Hence the total current i through a length l of this cylinder is

$$i = ne_s v 2\pi r l, \tag{2}$$

where e_s is the absolute value of the electronic charge in statcoulombs. In the steady state this current i must be the plate current and must be independent of the value of r. Unfortunately this expression for the plate current involves both the electron density n and the electron velocity v, neither of which is known. It is thus necessary to obtain two additional relations involving n and v.

The energy equation gives one of these relations. Let V be the potential of the field at any point on the cylindrical

surface through M. Then an electron arriving at the surface of this cylinder must have kinetic energy given by the relation

$$\tfrac{1}{2}mv^2 = e_sV, \tag{3}$$

since its initial kinetic energy is assumed to be zero. This must be true for all other electrons arriving at the surface of the cylinder. If we eliminate v between Eqs. (2) and (3), we get

$$i = ne_s2\pi r\sqrt{2\frac{e_s}{m}V}. \tag{4}$$

One additional relation is required to eliminate n from Eq. (4). This may be obtained in the following way. Consider the strength of the electric field E at the point M on the cylinder. By the theorem of Gauss it is just $2Q/lr$ where Q is the total charge enclosed in length l of the cylinder of radius r. Hence

$$lEr = 2Q. \tag{5}$$

Now consider a second cylindrical surface of radius $r + dr$ described about the filament. Let the field at the surface of this cylinder be $E + dE$ and the total charge enclosed in length l of it be $Q + dQ$. Then, as in the first case, the field is $2(Q + dQ)/l(r + dr)$, or

$$l(E + dE)(r + dr) = 2(Q + dQ). \tag{6}$$

If we subtract Eq. (5) from Eq. (6) and neglect products of differentials, we get

$$E\,dr + r\,dE = d(Er) = \frac{2dQ}{l}. \tag{7}$$

But dQ is just the charge between the two cylindrical surfaces of length l. Hence it is equal to $-ne_s2\pi rl\,dr$. Thus

$$\frac{d(Er)}{dr} = -4\pi rne_s. \tag{8}$$

Equation (8) is a special form of a general differential equation known as Poisson's equation, an equation of fundamental importance in all field theory. By use of this equation the factor $2\pi rne_s$ in Eq. (4) may be replaced by $-\frac{1}{2}\frac{d(Er)}{dr}$, thus giving

$$i = \frac{1}{2}\sqrt{2\frac{e_s}{m}V}\left[-\frac{d(Er)}{dr}\right]l. \tag{9}$$

Equation (9), a nonlinear differential equation, may be solved in the following manner. Assume that V may be written in the form

$$V = cr^m \tag{10}$$

where c and m are constants to be determined. Then

$$E = -\frac{dV}{dr} = -cmr^{m-1},$$

$$Er = -cmr^m;$$

and

$$\frac{d(Er)}{dr} = -cm^2r^{m-1}. \tag{11}$$

If we substitute these values of V and $\frac{d(Er)}{dr}$ in Eq. (9) and simplify, we get

$$i = \frac{1}{2}\sqrt{2\frac{e_s}{m}}\,c^{3/2}m^2r^{(3/2m-1)}l. \tag{12}$$

Since i must be independent of r, the exponent of r in Eq. (12) must be zero. Therefore $m = \frac{2}{3}$. Also the constant c in Eq. (10) is

$$c = \frac{V}{r^{2/3}} = \frac{V_b}{b^{2/3}},$$

since at $r = b$, $V = V_b$. Thus Eq. (12) becomes

$$i = \frac{2}{9}\sqrt{2\frac{e_s}{m}}\frac{(V_b)^{3/2}}{b}l \qquad \text{(esu)}; \tag{13}$$

or in practical units

$$i_b = \frac{2}{9}\sqrt{\frac{2e/m}{3 \times 10^9}}\,\frac{l}{b}\left(\frac{e_b}{300}\right)^{3/2}. \tag{13a}$$

In the foregoing analysis it was assumed for purposes of simplicity that the radius of the filament is zero. The problem becomes much more difficult for a filament of finite radius $a < b$. The solution of this problem has been given by Langmuir and leads to Eq. (1) instead of Eq. (13a). The chief difference between the two solutions is the presence of an additional function β^2 in Eq. (1). In general β^2 is a function of r/a and approaches unity as r/a gets large, *e.g.*, $\beta^2 = 1.08$ for $r/a = 100$. This means that Eq. (13a) gives satisfactory results (within a few per cent) for $r = b \gg a$.

On the other hand the simple theory as given cannot be used to determine V, E, and n in the immediate neighborhood of a filament of finite diameter for, in this case, r/a is no longer large compared to unity and the function β^2 must be taken into account. In fact it is just the presence of β^2 in the general solution that prevents E and n from going to infinity at the filament as they obviously do in the simple theory. Also the corrected potential function $V(r)$ may be shown to exhibit a minimum value very close to the filament surface.

This is essential if one is to explain why the plate current is ever less than the saturation current. This potential minimum V_m acts as a potential barrier to electrons emitted by the filament and allows only those with thermal energy greater than eV_m to surmount this barrier. The others are returned to the filament. For this reason it is really necessary in the general theory to assume not only that the electrons have initial velocities at the filament surface but also that there is a distribution of these velocities.

The values of V_m (min potential) and r_m (its position) depend upon the filament temperature and the plate potential. For a constant filament temperature and an increasing plate potential, r_m tends toward the value a and V_m tends toward zero. At the onset of saturation V_m vanishes and $r_m = a$. At this point both the potential and the field at the filament surface vanish, the barrier is reduced to zero, and any further increase in the plate potential produces no further appreciable increase in the plate current.

Method: The circuit for this experiment is very similar to that of Experiment 47 and is shown in Fig. 48-3. A 6-volt storage battery (heavy duty) supplies the filament current i_f; this *current should never exceed 2.25 amp.* It is controlled by rheostats Rh_1 (coarse) and Rh_2 (fine) and its value is given by the filament ammeter A_f. The potential drop across the filament is given by the filament voltmeter V_f.

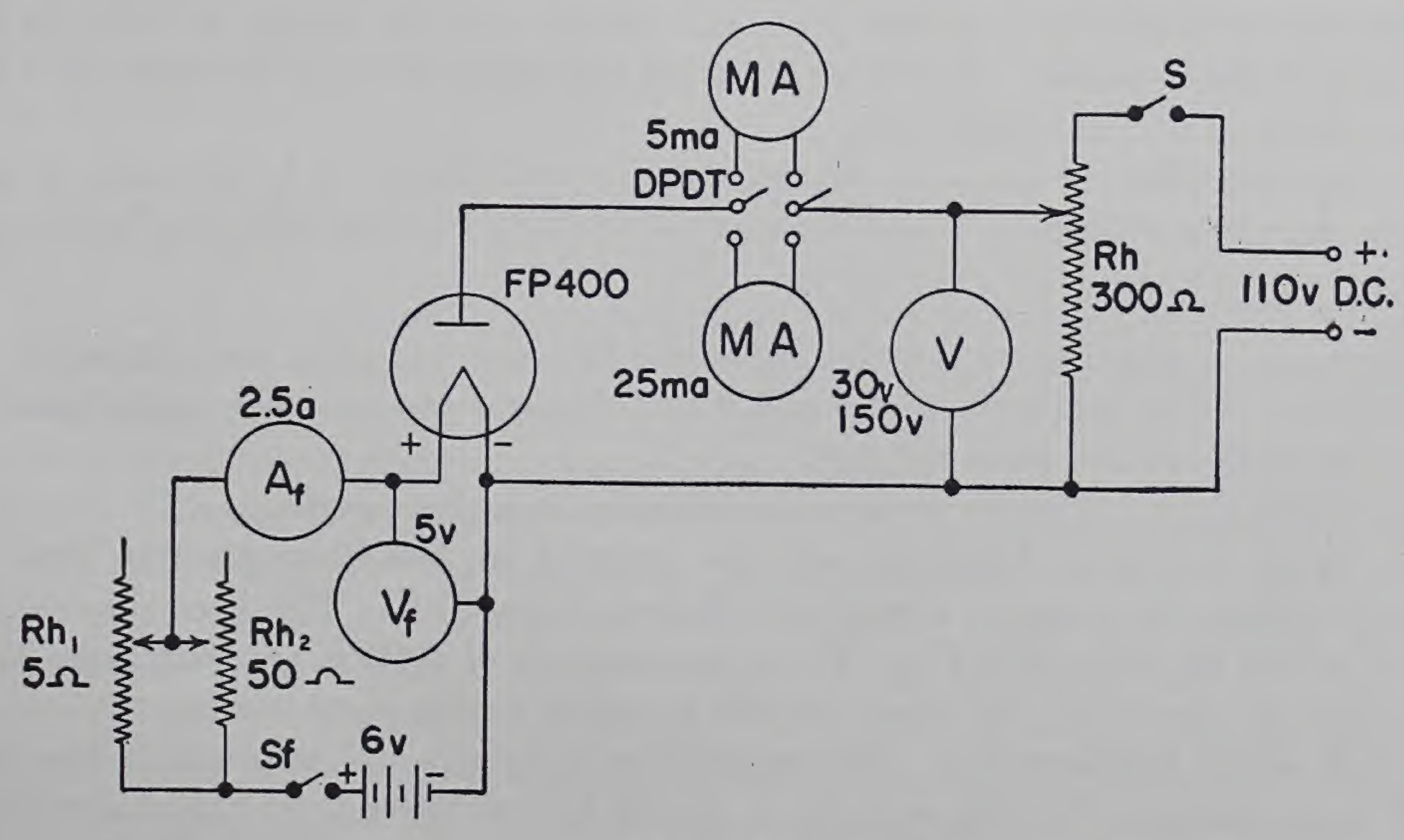


Fig. 48-3.

The plate voltage e_b is controlled by the potential divider Rh. The uncorrected value of the plate voltage is given by the plate voltmeter V. This voltmeter should be of good quality and should have two ranges, 0 to 30 volts and 0 to 150 volts. The plate current is given by either one of two milliammeters which may be connected into the plate circuit by use of a double-pole double-throw switch. These milliammeters should be of good quality.

Connect the circuit as shown in Fig. 48-3 with maximum resistance in the filament circuit and with the

potential divider set for minimum plate voltage. Use the high range on the voltmeter and the high-range milliammeter for an initial check on the circuit. If the circuit appears to be operating satisfactorily, proceed with the experiment.

Take data for three i_b vs. e_b curves corresponding to three fixed values of the filament voltage—3.20, 3.50, and 3.70 volts. Record the filament current for each of the three values of the filament voltage. In each case start with $e_b = 6$ volts, increase it by 3.0-volt steps to 30 volts (low-range voltmeter), then by 5-volt steps to 50 volts (high-range voltmeter), and finally by 10-volt steps to 80 volts. Record the plate currents i_b corresponding to these plate voltages. Use the low-range milliammeter for $i_b < 5$ ma and then switch to the high-range milliammeter. Take special pains in reading the plate current (low-range milliammeter) at $e_b = 15$ volts. This value will be used in computing a value of e/m.

Correct each plate voltage reading for filament potential drop by subtracting one-half the filament voltage. Why?

Plot the three i_b vs. e_b (corrected) curves on the same graph using i_b as the ordinate.

In order to check the validity of the Child-Langmuir relation, compute $[e_b \text{ (corrected)}]^{3/2}$ and plot these values against the corresponding values of i_b for the filament voltage of 3.70 volts. Estimate the indeterminate errors in each i_b value and in each e_b value. Compute the indeterminate error in each value of $[e_b \text{ (corrected)}]^{3/2}$. In order to indicate these errors on the graph, draw through each point a short vertical line of length $2\,\Delta i_b$ and a short horizontal line of length $2\,\Delta[e_b \text{ (corrected)}]^{3/2}$. Note that the points on the graph corresponding to unsaturated currents lie on a straight line within reasonable error limits. Draw this straight line and determine its slope. Compare with the accepted value of $14.6 \times 10^{-6} \dfrac{l}{b} \dfrac{\text{amp}}{(\text{volt})^{3/2}}$. l may be taken as 1.00 in. and b as 0.310 in. for the FP400 tube.

Determination of e/m. Equation (1) may be used to determine the value of e/m for electrons but it is difficult to get an accurate value by this method. Small errors in the values of i_b, e_b, b, and l lead to a large error in the value of e/m, *e.g.*, a 1% error in each of the former quantities leads to 9% error in the latter quantity. Perhaps the most doubtful of the quantities which must be known in order to calculate e/m is l, the effective length of the filament. In this experiment the value of l may be taken as 1 in., a value suggested by the manufacturer of the FP400 tube.

In order to reduce the effect of errors in this part of the experiment, it is advisable to make additional small corrections in the plate voltage. These corrections are listed in the following paragraph.

Plate Potential Corrections.

1. *Contact potential.* A small contact potential exists between tungsten and zirconium which the voltmeter does not indicate. This amounts to 0.4 *volt*, the difference between the work functions of the two metals, and should be *added* to the plate voltage.

2. *Filament potential drop.* This correction has already been described.

3. *Initial velocity of electrons.* Electrons are not emitted by the filament with zero initial velocity. They have thermal energy, the average energy per electron being $\frac{3}{2}kT$. This is equivalent to an additional potential difference across the tube of $\frac{3}{2}kT/e$. For a temperature of 2000°K (approximate filament temperature) this is equivalent to about 0.3 *volt* which should be *added* to the plate voltage.

4. *Potential drop across milliammeter.* This correction is simply i_bR where R is the resistance of the milliammeter. If it amounts to 0.1 volt or more, it should be *subtracted* from the observed plate potential.

Calculate the value of e/m by use of Eq. (1). Use the value of i_b at the *observed* plate potential of 15 volts (uncorrected) with the filament voltage of 3.70. Correct the observed plate potential for items 1, 2, 3, and 4. Use this corrected plate potential in the calculation. l may be taken as 1 in., b as 0.310 in., and β^2 as 1.07. Compare your value of e/m with the value given in Table L, Appendix III. Calculate the error in e/m due to your estimated errors in i_b and e_b.

Record: List the apparatus and apparatus numbers. Tabulate e_b, i_b, e_b (corrected), $[e_b \text{ (corrected)}]^{3/2}$ for each of the three filament voltages. Include the estimated and calculated errors in these tables.

List the other data and results (for e/m calculation) pertinent to this experiment.

Experiment 49.

Measurement of e/m: Magnetron Method

Object: To determine the value of e/m for electrons by the magnetron method.

Apparatus: Mounted FP400 tube, d-c ammeter (0 to 5 amp), filament d-c voltmeter (0 to 5 volts), plate d-c milliammeter (0 to 25 ma), plate d-c voltmeter (0 to 150 volts), heavy-duty 6-volt storage battery, five rheostats (one 5-ohm, one 300-ohm, one 50-ohm, two 100-ohm), lamp bank (one 200-watt bulb, one 100-watt bulb, one 50-watt bulb), solenoid, switches, search coil, ballistic galvanometer, magnetic-flux standard.

The FP400 tube in this experiment is used as a magnetron by placing it in a suitable magnetic field. This field is produced by a solenoid with a hollow core large enough to accommodate the FP400 tube in a longitudinal position. The tube is placed at the center of the solenoid with its filament in alignment with the magnetic field of the solenoid.

The solenoid should be capable of producing a maximum magnetic field of about 150 gauss with a magnetizing current of about 3 amp. Such a solenoid can be made by winding No. 20 insulated (cotton or formvar) copper wire on a stiff cardboard mailing tube about $2\frac{1}{2}$ in. in diameter and 12 in. in length. Several layers of windings are necessary.

Theory: If a diode consisting of a hot filament lying along the axis of a cylindrical plate is placed in a magnetic field so that the filament is parallel to the field, it is found that the plate current of the diode is a function of the strength of the magnetic field as well as of the plate potential and filament temperature. As the field is increased from zero, the plate current remains practically constant until a critical point is reached beyond which the plate current rapidly diminishes to zero.

The reason for this action is as follows. The electrons emitted by the filament are acted upon by crossed electric and magnetic fields which cause the electrons to move in curved paths closely approximating circular orbits. These orbits starting at the filament lie in planes perpendicular to the direction of the magnetic field, and intersect the plate for small magnetic field strengths. Thus all electrons leaving the filament reach the plate under these conditions. As the strength of the magnetic field is increased, however, the radius of curvature of the electron orbits decreases until the orbits just barely reach the plate, *i.e.*, are tangent to it. Any further increase in the magnetic field produces orbits that fail to reach the plate. In this latter case, electrons leaving the filament fail to reach the plate and return to the filament. This, of course, reduces the plate current to zero. Figure 49-1a shows the crossed electric and magnetic fields in the diode. Figure 49-1b illustrates the change in a typical electron orbit as the magnetic field is increased. Figure 49-1c shows the corresponding change in the plate current of the tube.

In actual practice it is found that the plate current does not drop off to zero as sharply as Fig. 49-1c indicates. This is probably due in part to a lack of alignment between filament and field and between filament and plate, and in part to the drop in potential along the filament and to the nonuniformity of the magnetic field.

The critical value of the magnetic field H_c for which the plate current drops to zero may be derived in the following manner. An electron leaving the filament with zero kinetic energy acquires kinetic energy because of the electric field between filament and plate. Because of the configuration of this field, almost

the entire potential difference between filament and plate occurs very close to the filament. Hence the electron acquires practically its entire kinetic energy before it has moved very far from the filament. Its kinetic energy $\frac{1}{2}mv^2$ is given by the equation

$$\tfrac{1}{2}mv^2 = eV_b \qquad \text{(emu)}, \tag{1}$$

where V_b is the plate potential (filament potential assumed to be zero). The magnetic field *does no work* on the electron and hence cannot contribute to its kinetic energy.

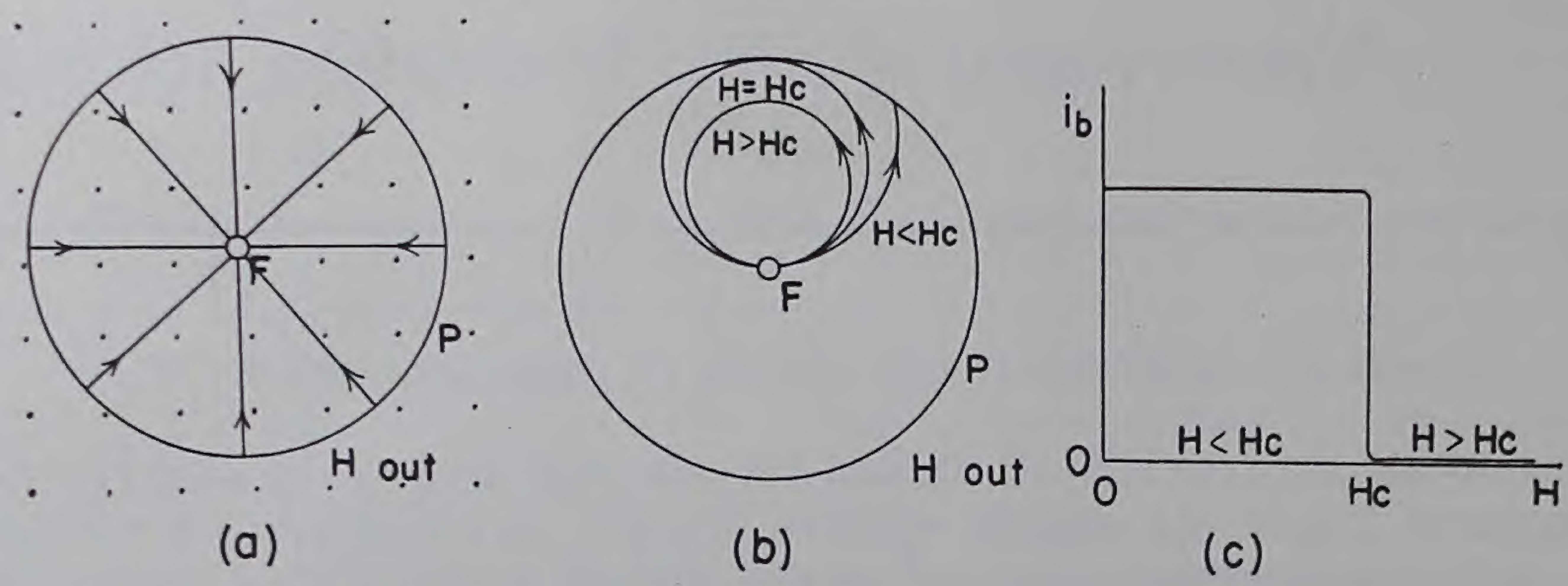


Fig. 49-1.

The magnetic field, however, does exert a centripetal force on the electron causing it to move in a circular orbit (we neglect the effect of the electric field except near the filament). This orbit has a radius of curvature R given by the well-known equation

$$Hev = m\frac{v^2}{R} \qquad \text{(emu)}, \tag{2}$$

provided the field H is perpendicular to v, as it is in this case. If this electron just fails to reach the plate, then it is clear that $R = b/2$ where b is the radius of the plate. The electron orbit is a circle similar to that shown in Fig. 49-1b for $H = H_c$. Hence

$$H_c ev = m\frac{v^2}{b/2}. \tag{3}$$

If v is eliminated between Eqs. (1) and (3) and the resulting equation is solved for e/m, we get

$$\frac{e}{m} = \frac{8V_b}{H_c^2 b^2} \qquad \text{(emu)}. \tag{4}$$

If the plate potential is expressed in volts e_b, then V_b must be replaced by $e_b \times 10^8$.

Equation (4) has been derived by assuming that the electric field and the magnetic field act consecutively instead of simultaneously upon an electron coming from the filament. This, of course, is not the case, but a more detailed analysis in which this assumption is not made leads to practically the same result. The exact equation is

$$\frac{e}{m} = \frac{8V_b}{H_c^2}\frac{b^2}{(b^2 - a^2)^2} \qquad \text{(emu)}, \tag{5}$$

where e/m = ratio of charge to mass of electron in abcoulombs per gram,
V_b = plate potential in abvolts (fil pot = 0),
H_c = cutoff magnetic field in gauss,
b = radius of plate in centimeters, and
a = radius of filament in centimeters.

A summary of the more exact derivation of Eqs. (4) and (5) is given in the following section (fine print).

Consider an electron emitted by the filament with negligible initial velocity. It is exposed to a radial electric field which accelerates it toward the plate. In this process the electron moves through the impressed uniform magnetic

field H, which deflects it from its radial path *without*, however, *changing its speed*. Let r and θ be the polar coordinates of the electron at any point M between filament and plate as shown in Fig. 49-2. Its velocity v at this point does not lie along r because of the effect of the magnetic field. The radial and tangential components of v are $\dot{r}\left(\equiv \frac{dr}{dt}\right)$ and $r\dot{\theta}\left(\equiv r\frac{d\theta}{dt}\right)$. Hence the kinetic energy of the electron at point M is $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2)$.

If V is the potential of the electric field at point M and if the filament potential is taken as zero, then

$$\tfrac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) = eV \qquad \text{(energy equation)}, \tag{6}$$

since the magnetic field does no work on the electron, and since the initial kinetic energy of the electron is taken as zero.

The torque about the filament axis exerted by the magnetic field on the moving electron at point M is just $He\dot{r}r$. This torque must equal the time rate of change of angular momentum $\frac{d}{dt}(mr^2\dot{\theta})$. Hence

$$\frac{d}{dt}(mr^2\dot{\theta}) = He\dot{r}r \qquad \text{(torque equation)}. \tag{7}$$

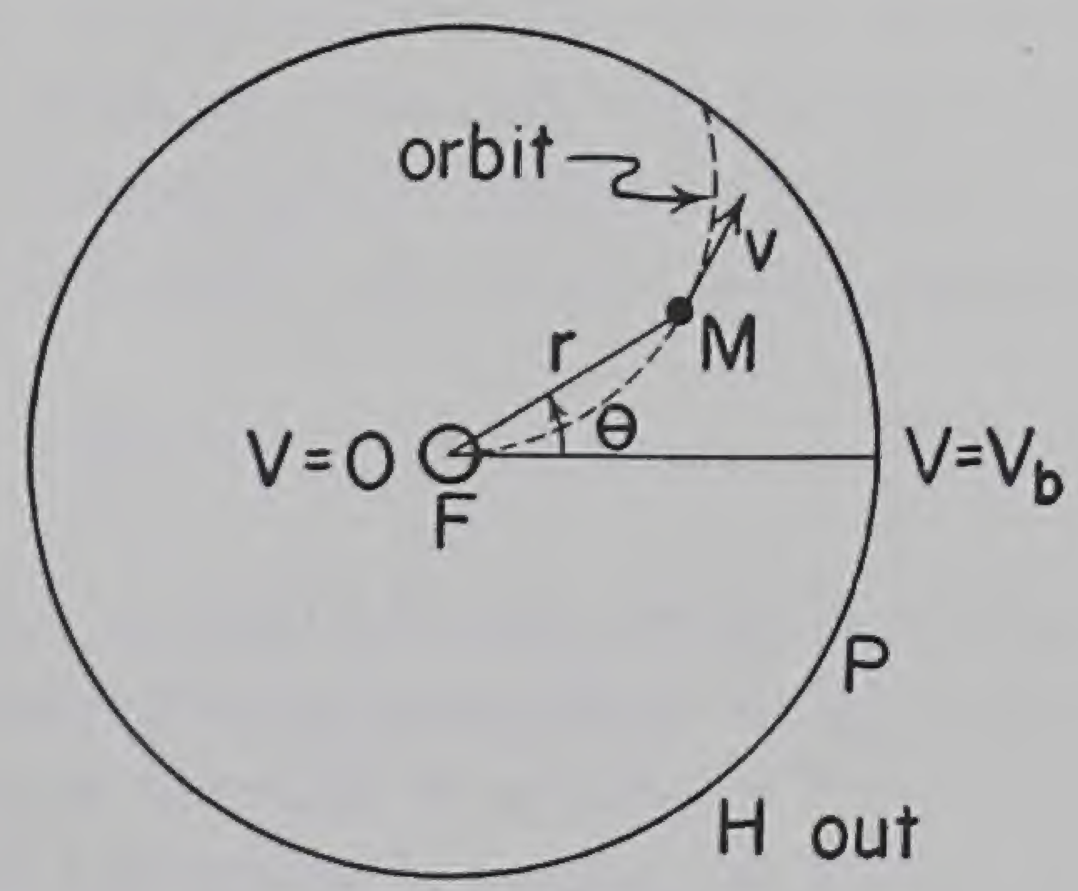


Fig. 49-2.

Integration of Eq. (7) gives

$$mr^2\dot{\theta} = \tfrac{1}{2}He(r^2 - a^2) \tag{8}$$

where a is the radius of the filament.

Equations (6) and (8) are the parametric differential equations of the electron orbit. Their solution shows that the orbit is a cardioid closely approximating a circular arc.

If $\dot{\theta}$ is eliminated between Eqs. (6) and (8), we get

$$\dot{r}^2 = 2\frac{e}{m}V - \frac{1}{4}\frac{(r^2 - a^2)^2}{r^2}\left(\frac{e}{m}\right)^2 H^2. \tag{9}$$

For $r = b$ Eq. (9) becomes

$$\dot{r}_b^2 = 2\frac{e}{m}V_b - \frac{1}{4}\frac{(b^2 - a^2)^2}{b^2}\left(\frac{e}{m}\right)^2 H^2. \tag{10}$$

Consider the value of $\dot{r}_b$ (radial velocity of electron at the plate) as H is increased from zero to a large positive value in Eq. (10). Since e/m, b, a, and V_b are fixed, as H increases from zero the right side of Eq. (10) starting at a fixed positive value decreases steadily toward zero. In this interval $\dot{r}_b^2$ is positive and hence $\dot{r}_b$ is real. This means that the electron orbit intersects the plate and hence a plate current exists. When H reaches a certain critical value H_c for which the right side becomes zero, then $\dot{r}_b = 0$. At this point the electron orbit is just tangent t the plate. For values of $H > H_c$ the right side becomes negative and $\dot{r}_b$ becomes imaginary. This means that the electron orbit does not reach the plate, hence the plate current is zero.

At the cutoff value of the plate current $H = H_c$ and $\dot{r}_b = 0$. Hence Eq. (10) reduces to

$$0 = 2\frac{e}{m}V_b - \frac{1}{4}\frac{(b^2 - a^2)^2}{b^2}\left(\frac{e}{m}\right)^2 H_c^2. \tag{11}$$

If this equation is solved for $\frac{e}{m}$, Eq. (5) is obtained. It is noteworthy that Eq. (11), and hence (5), is independent of space charge effects and is valid for both saturated and unsaturated plate currents.

In deducing Eq. (11) we have assumed a *constant* potential difference between filament and plate and a *uniform* magnetic field H throughout the tube. The first assumption is surely not valid because of the filament potential drop (about 4 volts). The second assumption, too, is generally not valid in experimental work. For these reasons, among others, we find that the cutoff of the plate current in the magnetron is not sharp but extends over a considerable interval. As a result it is difficult to assign a precise experimental value to H_c. Generally, the effect of a variation in H is more important than in V. Why? If Eq. (11) refers to conditions at the very center of the filament where H *is a maximum*, and if further, the right side of Eq. (10) is *positive* for all other sections of the filament, then H_c should be chosen at the very beginning of the break in the plate current.

The magnetron has several different uses. It may be used to control a current by means of a magnetic field or to measure the intensity of a uniform magnetic field. It may be used as a very-high-frequency

generator producing radio waves of only a few centimeters in length; the circling electrons in the tube act as harmonic oscillators. Finally, it may be used to determine the value of e/m for electrons as is done in this experiment.

In order to determine e/m by means of Eq. (5), it is necessary to know the plate potential, the diameters of filament and plate, and the critical magnetic field strength H_c. This last quantity is determined by measuring the plate current i_b of the diode (constant plate voltage e_b and filament temperature T) for a set of increasing or decreasing values of the magnetic field strength H. The value of H at the cutoff of i_b is H_c.

Some means must be used to determine the field H produced at the center of the solenoid by a given magnetizing current I. This field may be calculated approximately by means of the equation

$$H = \frac{4\pi NI}{10L}\left(1 - \frac{1}{2}\frac{D^2}{L^2}\right), \qquad D \ll L, \tag{12}$$

where H = strength of field at center of solenoid in gauss,
N = total number of turns,
L = length of solenoid in centimeters,
I = magnetizing current in amperes, and
D = average diameter of solenoid in centimeters.

It is assumed that there is no magnetic material in the neighborhood of the solenoid.

Perhaps a safer procedure is to measure the field strength at the center of the solenoid by use of a small test coil, magnetic-flux standard, and ballistic galvanometer. The method is essentially equivalent to that used in Experiment 41.

Errors: The chief error in this experiment is that in the value of H_c due to the lack of a sharp cutoff value for the plate current. This error is generally much larger than the combined errors of all of the other values involved in Eq. (5).

Under the conditions of this experiment, it seems to be best to take as H_c the value of H at the very beginning of the break in the plate current.

Method: The circuit for this experiment is shown in Fig. 49-3. The circuit for the FP400 diode (Fig. 49-3a) is essentially that used in Experiment 48. Figure 49-3b gives the circuit for the solenoid. The chief

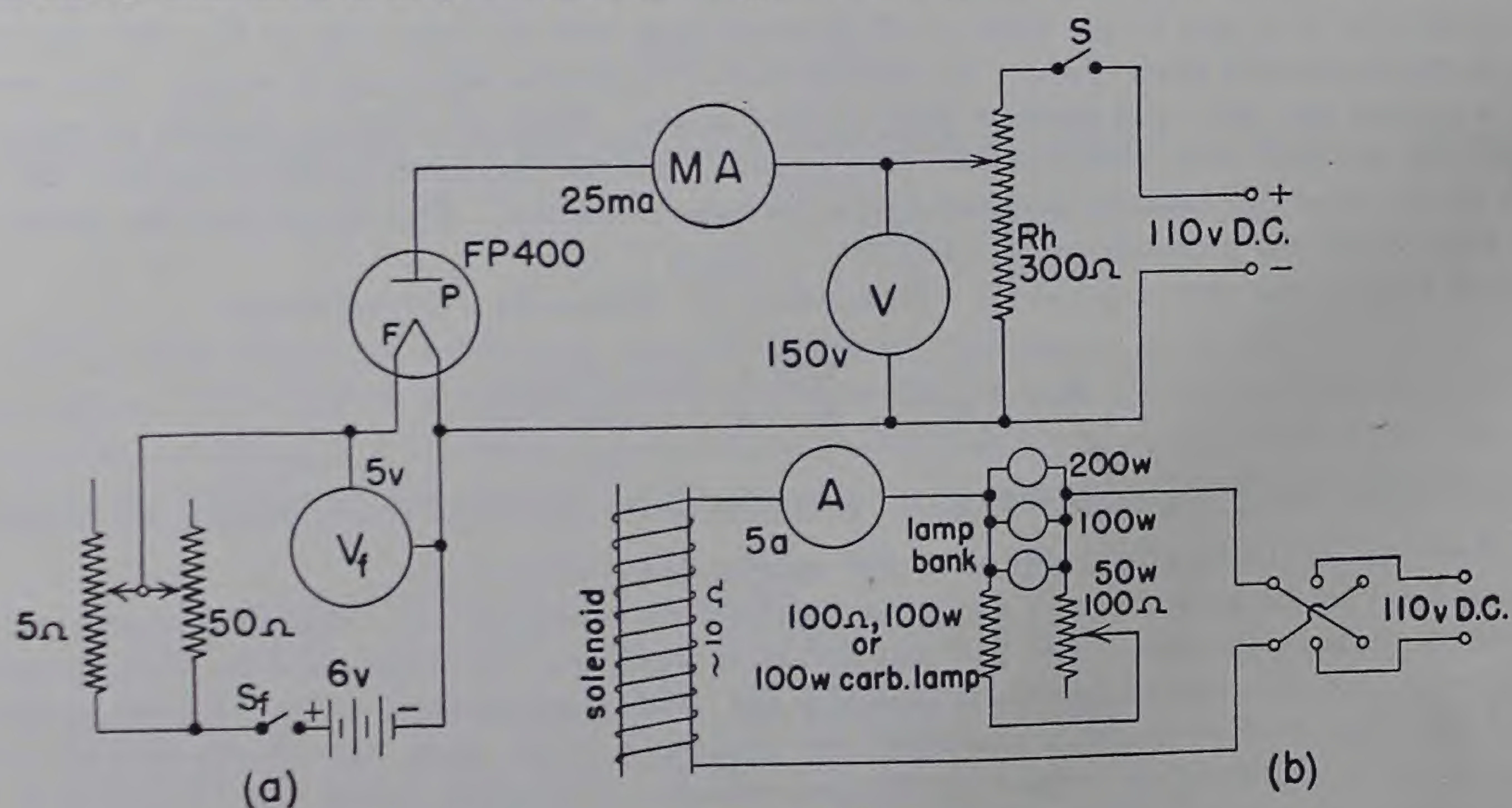


Fig. 49-3.

problem here is to arrange the circuit so that a fine adjustment of the magnetizing current in the solenoid is possible. The arrangement shown will work satisfactorily provided the solenoid has a resistance of about 10 ohms. Coarse adjustment of current is made by use of the lamp bank, fine adjustment by use of the 100-ohm variable rheostat.

Place the solenoid over the FP400 diode so that the center of the tube is at the center of the solenoid. Adjust the alignment as carefully as possible. Operate the tube at a constant filament voltage of 3.70 volts. Keep all iron away from the solenoid. Set the plate potential at 80 volts. Note the milliammeter reading for zero current in the solenoid. Increase the current I in the solenoid in approximately half-ampere steps by use of the lamp bank and note the value of this current when the plate current of the FP400 decreases sharply. Take a set of values of i_b (plate current) and I (solenoid current) in the neighborhood of this cutoff. Start with a value of I well below the cutoff value ($\frac{1}{2}$ amp below) and increase it in steps of 0.1 amp until the plate current just begins to decrease; then in steps of 0.05 amp until the plate current has dropped to about 20% of its initial value; then in steps of 0.1 amp until i_b is zero.

Repeat the above process using reversed magnetic fields.

Repeat the foregoing procedures for plate voltages of 60 volts and 40 volts keeping the filament voltage at 3.70 volts.

Replace the FP400 tube with a test coil and measure the field strength produced by the solenoid with a magnetizing current of 1.00 amp. Since $H = KI$ for the solenoid, this measurement determines K and hence H for all other values of I.

Computations: Plot i_b vs I for all three sets of data. Indicate on each graph the critical value of I. Estimate the error in each of the three critical values of I. Compute the corresponding values of H_c and ΔH_c.

Correct the observed plate voltages for filament potential drop. Calculate the value of e/m for electrons and the corresponding error in this value for each of the three sets of data. Average these values and compare with the accepted value of e/m.

Record: List apparatus and apparatus numbers. Tabulate e_b, i_b, and I for each of the three sets of data. Record filament voltage, solenoid constant K, and constants a and b. Give the computed values of e/m and their errors.

Experiment 60.

Vibrating Wire

Object: To generate stationary transverse waves in a wire. To determine the frequency of vibration of this wire in terms of the tension and mass per unit length of the wire and the wave length.

Apparatus: Music wire (0.016 gauge), clamps, pulley, weight holder and weights, analytical balance, 100-cm specimen of the wire, driving electromagnet, and meter stick. See Appendix II, Note M on the analytical balance.

Theory: The velocity v in centimeters per second with which a transverse wave pulse travels along a flexible wire is given by the equation

$$v = \sqrt{\frac{F}{m}}, \tag{1}$$

where F is the tension in the wire in dynes and m is its mass per unit length in grams per centimeter.

A train of running waves may be generated in the wire by driving some small portion of it back and forth at right angles to the direction of the wire. This may be accomplished in the case of an iron or steel wire by driving it with a small electromagnet placed close to the wire and energized with a 60-cps alternating current. The wire in the neighborhood of the iron core of the electromagnet is magnetized by induction and pulled toward the iron core 120 times per second. In this manner a train of waves of frequency 120 per second will be sent along the wire with a velocity given by Eq. (1). The wave length λ in centimeters of this wave train will be given by the well-known equation,

$$v = n\lambda, \tag{2}$$

where n is the frequency of the wave train.

We may eliminate v between Eqs. (1) and (2) and solve for n, thus getting

$$n = \frac{1}{\lambda}\sqrt{\frac{F}{m}}. \tag{3}$$

This equation enables us to determine n in terms of F, m, and λ.

While it is an easy matter to measure F and m, it is generally not feasible to measure λ in a running wave train. Under appropriate conditions, however, it is possible to set up so-called stationary or standing waves in the wire which enable us to make a precise determination of λ.

If the wire is fixed at both ends (as it is in this experiment), then the waves generated at any point along the wire will run along the wire to the fixed ends where they will be reflected back along the wire. The actual configuration of the wire at any instant is a combination of incident and reflected wave trains and may be very complex. However, under the proper conditions, the incident and reflected waves may combine in such a way as to produce stationary or standing waves. *This occurs when the distance between the fixed ends of the wire is just an integral multiple of a half wave length of the running waves.* The waves then "fit" into the length of the wire, so to speak, strongly reenforcing each other at certain points along the wire called loops or antinodes, and completely annulling each other at intermediate points called nodes. Under these conditions

the wire vibrates in segments as shown in Fig. 60-1. In this figure the wire is shown vibrating in three segments. The fixed ends of the wire must of necessity be nodes (positions of zero displacement). Nodes and loops alternate in position. It may be shown that the distance between two successive nodes (or two successive loops) is just equal to a half wave length. Hence, when these stationary waves are produced, it is an easy matter to pick out the nodal positions and thus measure the wave length of the generated wave train.

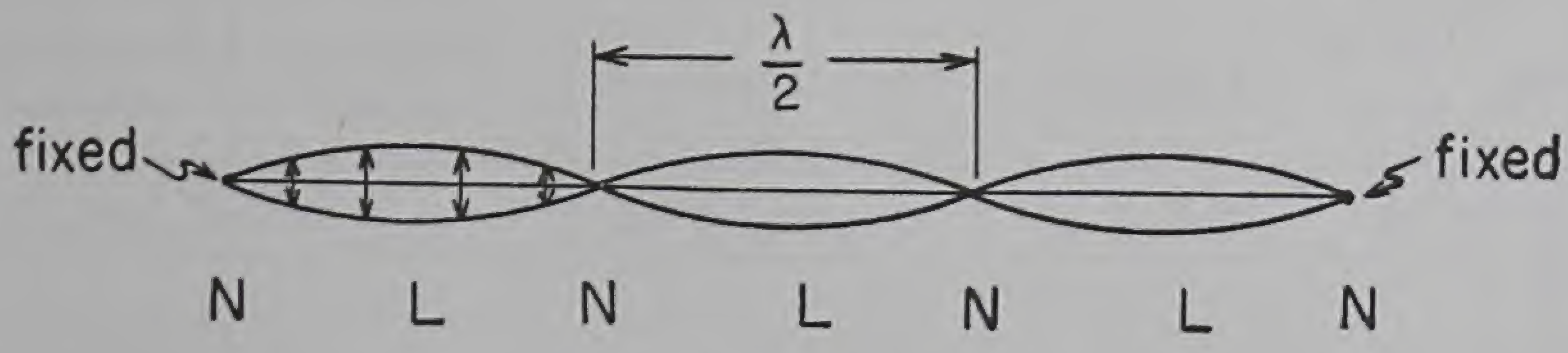


Fig. 60-1.

Method: Set up the apparatus as shown in Fig. 60-2. Connect the electromagnet to the 6-volt source of 60-cycle current. Place it near one end of the wire as shown, with its soft iron core a few millimeters away from the wire. Without adding any weights to the weight holder, pull down on the weight holder with your hand, gradually increasing the tension in the wire. If this is done carefully, and if the apparatus is working properly, the wire will suddenly vibrate strongly (usually in five segments) for a certain tension.

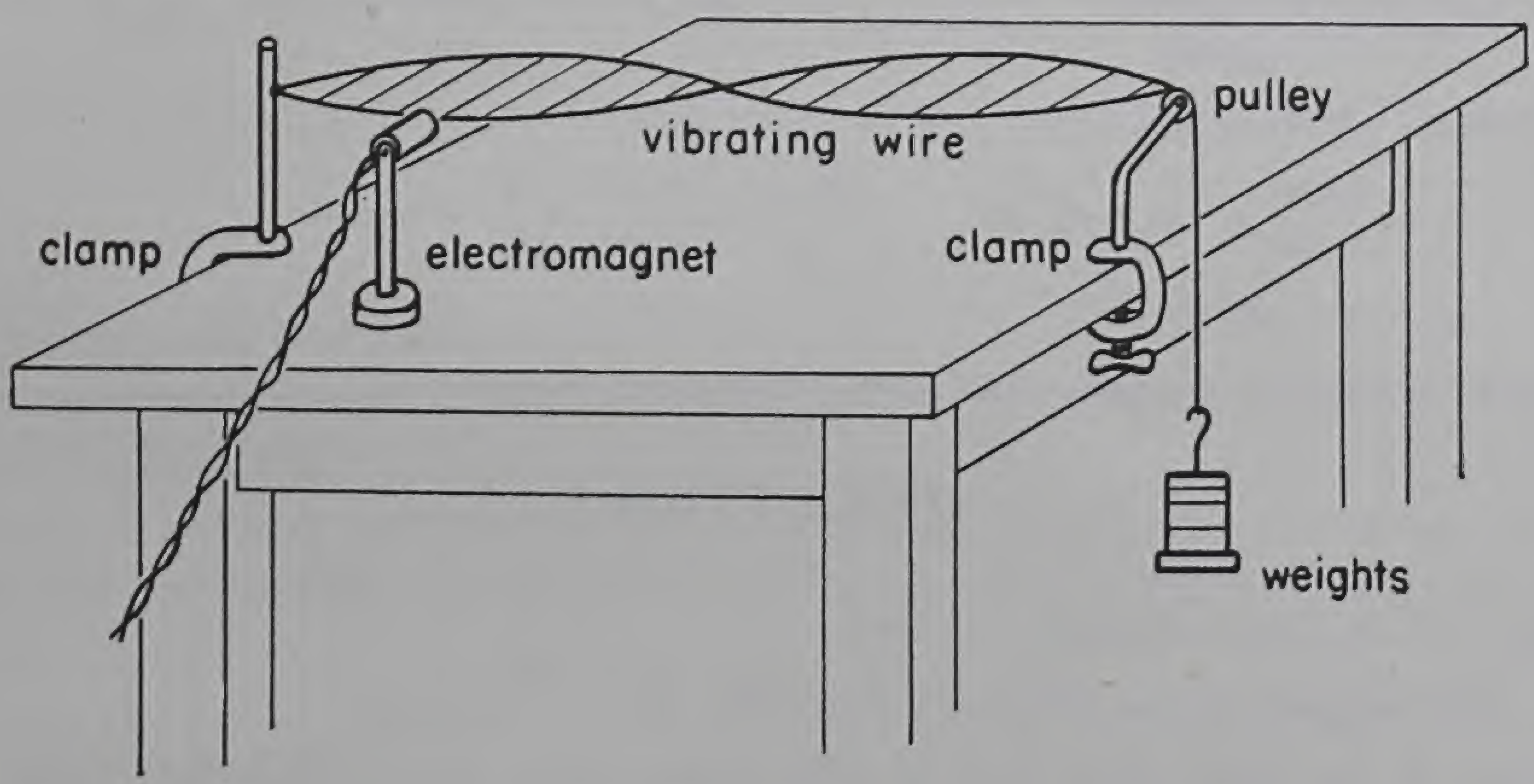


Fig. 60-2.

If the tension is increased further, the vibration will cease until the tension reaches a second critical value at which vibrations will again occur, with the wire now vibrating in four instead of five segments. Keep increasing the tension until the wire vibrates in three segments and then finally in two segments. It is not advisable to attempt to make the wire vibrate in a single segment because of the very large tension required.

After you have succeeded in making the wire vibrate in five, four, three, and two segments by applying force on the weight holder with your hand, repeat the same performance by adding weights to the weight holder. Record the weights (weight holder included) which give the best set of stationary waves for the case of five, four, three, and two vibrating segments. In each case estimate the amount by which the weight may be changed without appreciably reducing the stationary wave pattern. This will represent the indeterminate error in the tension in the wire.

Carefully measure the length of the wire between the clamp and the pulley to the nearest millimeter. Weigh the 100-cm sample of the wire on the analytical balance to the nearest milligram and determine the mass per unit length of this sample.

From these data compute for the case of each stationary wave pattern (1) the wave length, (2) the velocity of the wave train, (3) the frequency of the wave train. Also compute the indeterminate errors in these quantities using the estimated errors in the measured quantities. The frequency of the wave train in each case should be 120 vibrations per second. The error in the frequency determination should be less than 2% in this experiment.

Record: (Sample.)

App. No._______________

Mass of 100-cm sample = 1.015 ± 0.001 gm

Length of wire = 156.9 ± 0.1 cm

Standard frequency = 120.0 vib/sec

Obs	No. of loops	Load, gm	Wave length, cm	Velocity, cm/sec	Frequency, vib/sec
1.	5	590 ± 10	62.76 ± 0.04	7550 ± 70	120 ± 1
2.	4	....	....	....	....
3.	3	....	....	....	....
4.	2	....	....	....	....
				Ave	

$$\lambda = \frac{(2)(156.9)}{5} = 62.76 \text{ cm}; \qquad \frac{100\,\Delta\lambda}{\lambda} = \frac{10}{157} = 0.064\%$$

$$\Delta\lambda = \pm 0.04 \text{ cm}$$

$$v = \sqrt{\frac{(590)(980)}{0.01015}} = 7550 \text{ cm/sec}; \qquad \frac{100\,\Delta v}{v} = \frac{1}{2}\left(\frac{1000}{590} + \frac{0.1}{1.0}\right) = 0.9\%$$

$$\Delta v = \pm 70 \text{ cm/sec.}$$

$$n = \frac{7550}{62.76} = 120.3 \text{ vib/sec}; \qquad \frac{100\,\Delta n}{n} = \frac{7000}{7550} + \frac{4}{63} = 0.99\%$$

$$\Delta n = \pm 1 \text{ vib/sec}$$

Per cent difference between standard frequency and average measured frequency = _______________.

QUESTIONS

1. Show that Eq. (1) is dimensionally correct.

2. Develop the error equation corresponding to Eq. (3).

3. If the diameter of the steel wire used in this experiment were 1% larger than the diameter of the 100-cm sample whose mass was determined, what constant percentage error would be introduced into the value of n?

4. Show by diagram, or otherwise, that the distance between two successive nodes in a stationary wave is just a half wave length of the running wave.

Experiment 61.

Velocity of Sound. Resonance Tube

Object: To generate stationary sound waves in air. To determine the velocity of sound in air at room temperature from measurements of wave length for a given frequency. To compute the velocity of sound at 0°C.

Apparatus: Resonance-tube apparatus, tuning fork, rubber hammer, thermometer, meter stick.

Theory: The velocity of a compressional wave pulse in any medium is given by the equation

$$v = \sqrt{\frac{E}{\rho}}, \tag{1}$$

where v = velocity of the compressional wave,
E = volume modulus of elasticity of the medium, and
ρ = density of the medium.

Since this formula is seldom derived in general physical textbooks and since its development sheds considerable light on the mechanism of wave propagation, its derivation will be given below.

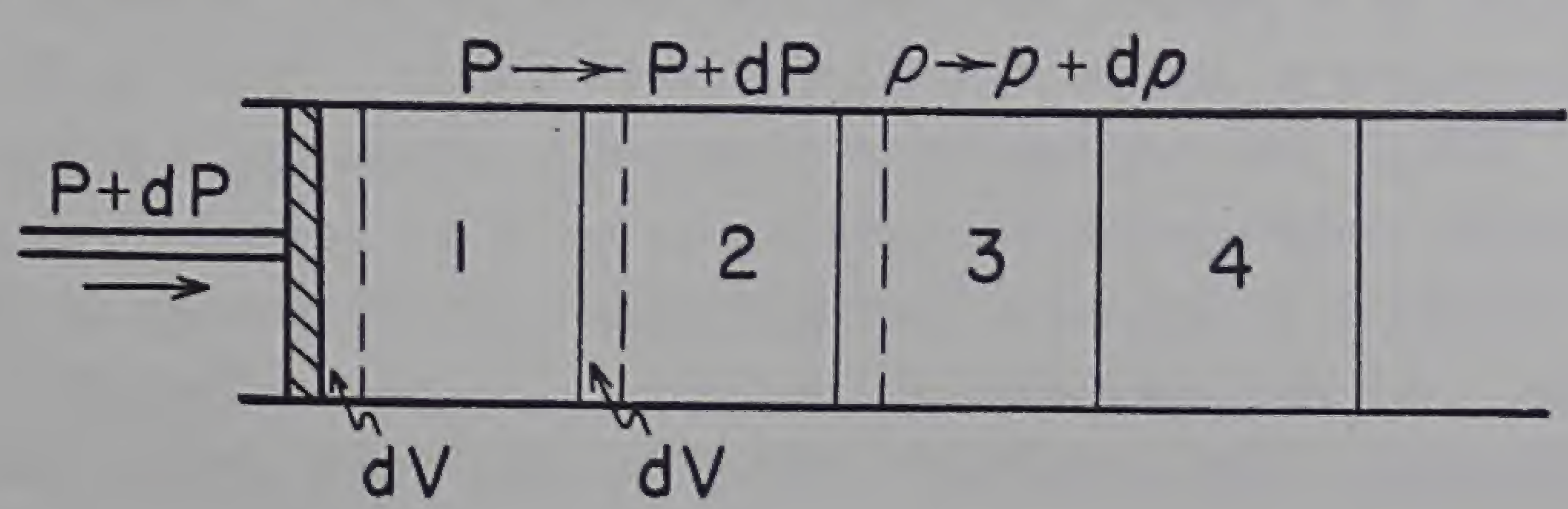


Fig. 61-1.

Let us imagine a portion of the medium (homogeneous and isotropic), through which we wish to send a compressional wave pulse, confined in a long rigid tube of unit cross-sectional area. At one end of this tube is a piston which may be used to compress the medium. See Fig. 61-1. Suppose that the normal density of the medium is ρ and its normal pressure P. Imagine the medium in the tube divided into sections each of unit volume and labeled consecutively 1, 2, 3, etc. Each of these sections will be 1 cm long since it is assumed that the cross-sectional area of the tube is 1 cm^2.

Let the piston now be moved to the right by the application of a pressure slightly greater than P, viz., $P + dP$. This will start a compressional wave front down the tube. We imagine that the moving piston first compresses the medium in section 1 by an infinitesimal amount dV until its pressure rises from P to $P + dP$. Thereafter section 1 moves with the piston and compresses section 2 by the amount dV, raising its pressure to $P + dP$. This process continues from section to section as the pressure wave front moves down the tube.

At the end of 1 second this pressure front will have affected v sections of the medium where v is the velocity of the pressure front. All sections to the right of this front will be unaffected. All sections to the left of this front will be compressed and moving to the right with the velocity of the piston. Since each of

these compressed sections has had its volume reduced by an amount dV, the total amount of compression at the end of 1 sec will be $v\,dV$. This will also be the distance that the piston has moved in 1 sec since the tube is of unit cross section.

We may now determine the value of v by applying the energy principle. The total work done by the piston in 1 sec must equal the increase in potential energy of the compressed sections of the medium, plus the kinetic energy of these sections. The work done by the piston in 1 sec is $(P + dP)v\,dV$. The corresponding increase in potential energy of the compressed sections is $(P + (dP/2))v\,dV$, *i.e.*, the average pressure times the change in volume. The kinetic energy of the compressed sections at the end of the second is $\frac{1}{2}\rho v(v\,dV)^2$. Hence

$$(P + dP)v\,dV = \left(P + \frac{dP}{2}\right) v\,dV + \frac{1}{2}\rho v(v\,dV)^2. \tag{2}$$

If we solve Eq. (2) for v, we get

$$v = \sqrt{\frac{\left(\frac{dP}{dV}\right)}{\rho}}. \tag{3}$$

But since dV is the decrease in volume of a unit volume of the medium, $\frac{dP}{dV}$ is just the volume modulus of elasticity of the medium, *i.e.*, E. Hence

$$v = \sqrt{\frac{E}{\rho}}. \tag{4}$$

This expression for v involves only the constants E and ρ of the *medium.* Therefore it is fairly evident that a compressional wave once started will travel through the medium at a rate which has nothing to do with the size or shape of the tube or with the subsequent motion of the piston after it has started the wave pulse. Furthermore Eq. (4) is applicable to any medium whether it be solid, liquid, or gaseous.

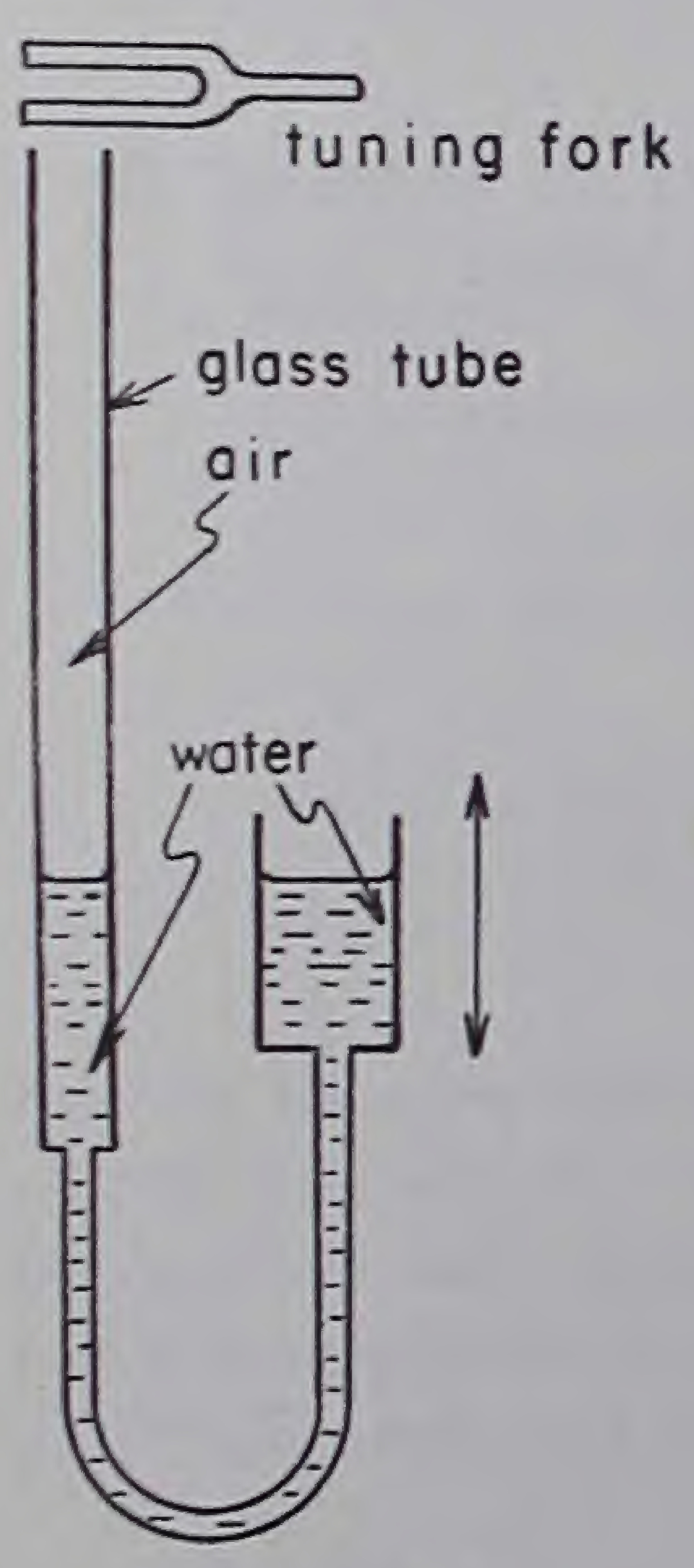


Fig. 61-2.

In this experiment the sound waves are generated by a vibrating tuning fork instead of a piston, the medium is air at room temperature, and the tube is a glass tube closed at one end by a column of water. See Fig. 61-2.

When the tuning fork is set into vibration, a train of waves consisting of alternate compressions and rarefactions in air is sent down the tube. This wave train is reflected at the water surface with a phase change of 180° and passes back up the tube. At the open end of the tube it is again reflected but with no phase change in this case. The resultant waves in the tube are a combination of incident and reflected wave trains and may be very complex just as in the case of transverse waves in a wire. But just as in the case of the wire, so in the case of an air column, it is possible to produce stationary waves under the proper conditions. The air column will then vibrate strongly in segments with a frequency equal to the frequency of the tuning fork. This occurs when the length of the air column is of such value that an *odd* multiple of quarter-waves "fits" the air column, since there must be a node at the lower end of the air column (at the surface of the water) and a loop or antinode near the open end of the tube. Under these conditions the air column resonates with the tuning fork and the intensity of sound from the system is considerably increased. This phenomenon of resonance enables us to determine when stationary waves are being produced in the air column.

The length of the air column may be varied by changing the level of the water in the tube. Thus the positions of the water surface at which resonance occurs may be determined.

It has been pointed out that an antinode must exist near the top of the tube. Analysis shows that its position is slightly above the top of the tube (about 0.6 the radius of the tube). If the water level in the tube is lowered from the top of the tube, resonance will occur when the position of the water level corresponds with the position of the first node. This position is sharply defined and may be accurately determined.

As the water level is further lowered, a second resonance point may be found which corresponds to the second node. In some cases additional nodes may be found depending upon the relation between the wave length and the tube length. The inter-nodal distance is just a half wave length. See Fig. 61-3. It is not advisable to try to determine the wave length using only the first antinode and the first node, since this distance ($\lambda/4$) cannot be accurately determined because of a lack of knowledge concerning the exact position of the first antinode. However, the distance between the successive resonance points (successive nodes) may be accurately determined and represents the value of $\lambda/2$.

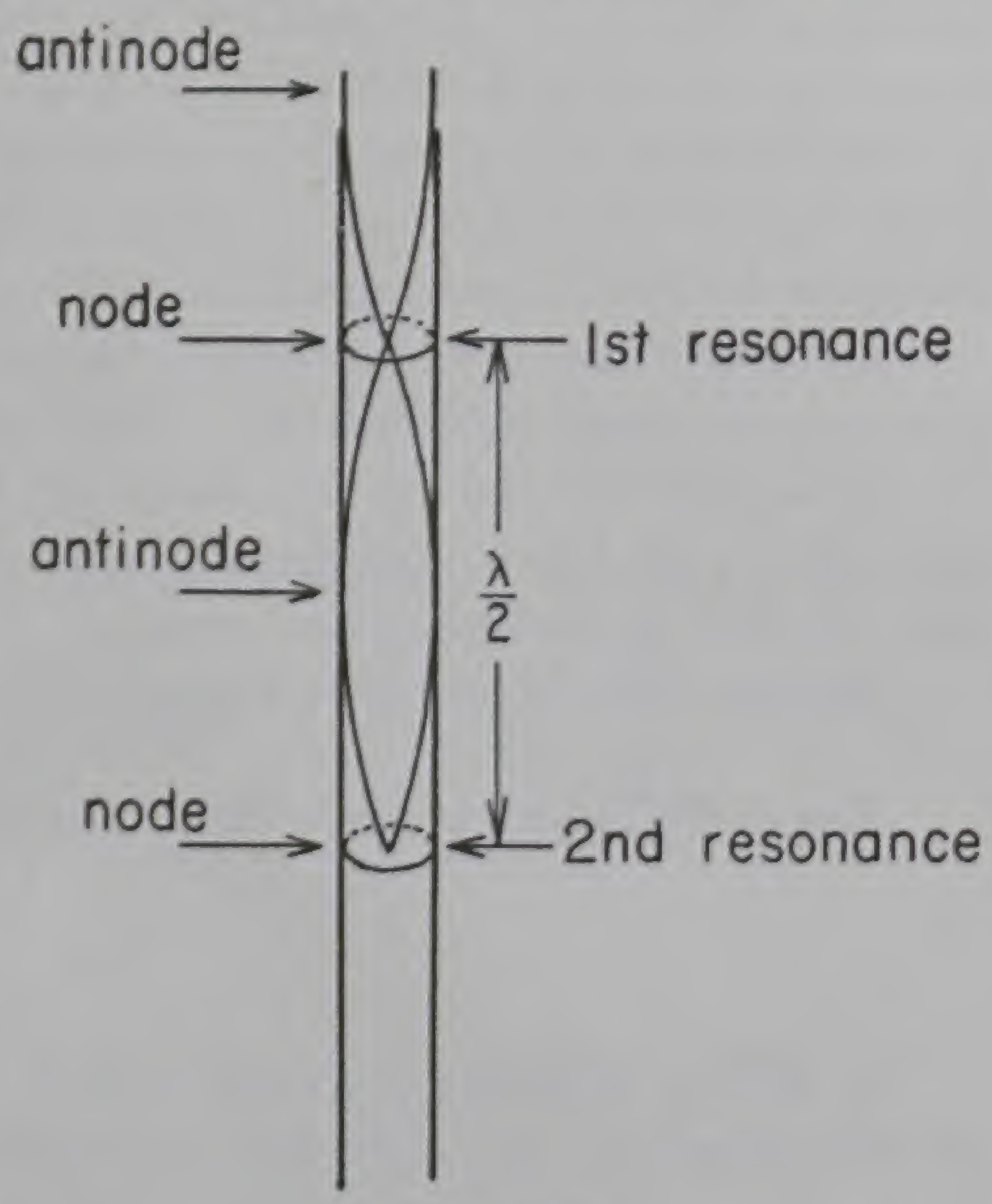


Fig. 61-3.

Once the wave length λ has been determined by a measurement of the inter-nodal distance, we may determine the velocity of sound in air at room temperature by means of the equation

$$v = n\lambda, \tag{5}$$

where n is the frequency of the tuning fork.

In order to compute the velocity of sound in air at 0°C it is necessary to make use of Eq. (4) in a modified form. If it is assumed that air obeys the general gas law and further that the compressions and rarefactions occurring in air when a sound wave passes through it are adiabatic in character rather than isothermal, then $E = \gamma P$ where γ (1.402 for air) is the ratio of the specific heat at constant pressure to that at constant volume and P is the pressure. The relation $E = \gamma P$ may be derived from the formula PV^γ = constant for an adiabatic change in the volume of a gas, by taking the logarithmic derivative of this latter expression. We get

$$\frac{dP}{P} + \gamma\frac{dV}{V} = 0 \qquad \text{or} \qquad -V\frac{dP}{dV} = \gamma P.$$

But by definition $E = -V\dfrac{dP}{dV}$, hence

$$E = \gamma P.$$

Equation (4) for air may then be written in the form

$$v = \sqrt{\frac{\gamma P}{\rho}}. \tag{6}$$

By the general gas law $P/\rho = (R/\mu)T$. Hence Eq. (6) becomes, after substituting for P/ρ,

$$v = \sqrt{\frac{\gamma R}{\mu}T}, \tag{7}$$

where R is the universal gas constant, μ is the molecular weight of air, and T is the absolute temperature.

It is clear from Eq. (7) that the velocity of sound in any gas is directly proportional to the square root of the *absolute* temperature. Therefore the velocity of sound in air at any temperature may easily be computed if it is known at some one temperature, for example, room temperature, by means of the formula

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}. \tag{8}$$

Method: Raise the water level in the glass tube until it is near the top of the tube. Start the tuning fork vibrating by striking it gently with a soft rubber hammer. (PRECAUTION: Never strike a tuning fork with a hard hammer. It may ruin the fork.) Slowly lower the water level while listening for resonance to occur. The amplification of the sound at resonance is quite pronounced even though the fork itself is emitting a scarcely audible sound. Once you have determined the approximate position of the first resonance point, make several trials by running the water surface up and down. When the point of maximum intensity is located, mark its position with one of the spring brass rings on the tube. Then lower the water surface

and, in a similar manner, locate and mark the next resonance point. Continue this process to the bottom of the tube. In each case make an estimate of the error involved in locating the resonance point.

With a meter stick held against the tube read and record the positions of the resonance points to the nearest millimeter. By subtraction determine the distance between successive resonance points, *i.e.*, $\lambda/2$. Estimate the error in this value of $\lambda/2$.

Determine the velocity of sound in air at room temperature by means of Eq. (5) using the rated frequency of the fork and the measured value of $\lambda/2$. Determine the error in this value of v, assuming that the error in the frequency of the fork is negligible.

By use of Eq. (8) determine the velocity of sound at 0°C from your experimentally determined velocity of sound at room temperature. Also determine the error in this velocity.

Compute the theoretical value of the velocity of sound in air at 0°C by use of Eq. (6) and compare this value with the experimental value at 0°C. The two values should agree within the limits of the experimental error in the measured value.

Repeat this experiment using a different set of equipment with a different tuning fork.

Record: Tabulate your data and results.

QUESTIONS

1. If it is assumed that the velocity v of a compressional wave in a medium depends only upon E and ρ for that medium and that this relationship has the form

$$v = E^x \rho^y,$$

show by dimensional argument that $x = \frac{1}{2}$ and $y = -\frac{1}{2}$.

2. Does the velocity of sound in air vary with the barometric pressure? Explain.

3. By use of Eq. (8) show that the velocity of sound in air in meters per second at a temperature of t°C is given approximately by the equation

$$v_t = 332 + 0.61t,$$

provided t is not large.

Experiment 62.

Velocity of Sound in Metal Rods

Object: To determine the velocity of sound in two different metal rods by use of a Kundt's tube. To compute the value of Young's modulus for these rods.

Apparatus: Metal rods, Kundt's tube, clamps, meter stick.

Theory: Stationary longitudinal waves may be generated in a metal rod by clamping it at its mid-point and stroking it lengthwise with a cloth moistened with alcohol or sprinkled with rosin. A series of disturbances (compressions and rarefactions) pass along the rod and are reflected from the free ends of the rod. Incident and reflected waves combine to form stationary waves. The rod will then vibrate strongly, emitting a shrill note. The fundamental mode of vibration of the rod will correspond to a stationary wave train in the rod which has an antinode at each of the free ends of the rod and a node at the clamped point, *i.e.*, the mid-point. See Fig. 62-1. In this case the length of the rod is just equal to a half wave length of the wave train, *i.e.* the distance between two successive antinodes.

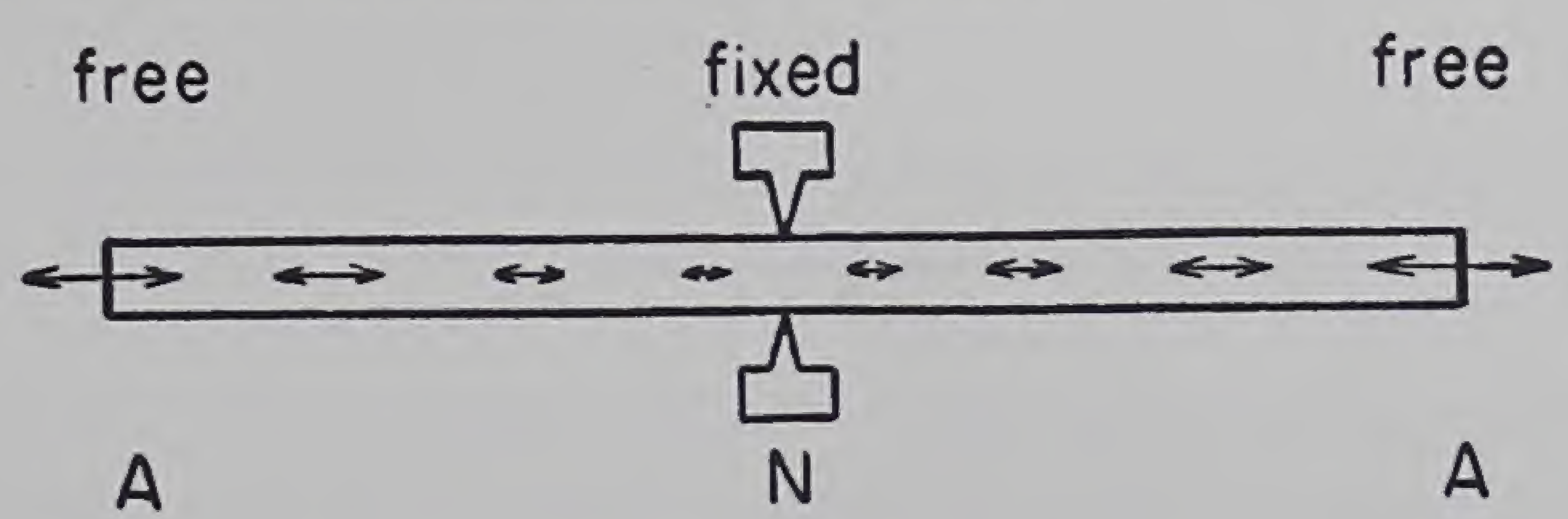


Fig. 62-1.

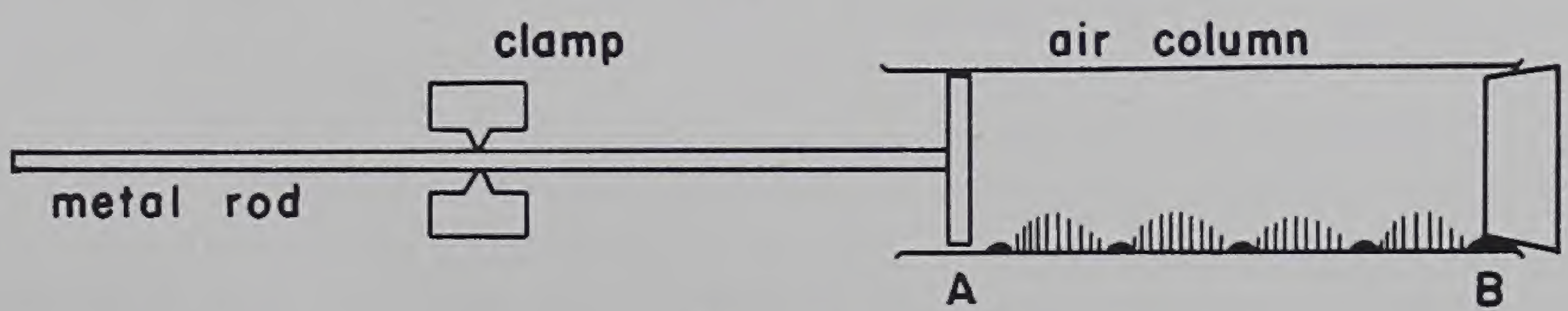


Fig. 62-2.

In order to make an experimental determination of the velocity of sound in this thin metal rod, a light metal-felt piston A is attached to one end and inserted into a glass tube as shown in Fig. 62-2. At the other end of the tube is a fixed plug B. Along the bottom of the tube is sprinkled some light powder such as cork dust or lycopodium powder.

The metal rod is set into vibration in the manner suggested above. The glass tube is adjusted in position until the air column AB is of such length as to resonate with the note emitted by the rod. Nodes and antinodes are formed in the air column which cause the powder at the bottom of the tube to collect in lateral ridges at the points of maximum disturbance, *i.e.*, at the antinodes. Thus it is an easy matter to determine the wave length of the sound in air. The velocity of sound in the metal rod, v_m, is given by the expression

$$v_m = n\lambda_m, \tag{1}$$

while the velocity of sound in air, v_a (at the same temperature), is given by

$$v_a = n\lambda_a, \tag{2}$$

where n = frequency of the vibrating rod and also that of the vibrating air column,
λ_m = wave length of the sound in the metal rod, and
λ_a = wave length of the sound in the air column.

If we divide Eq. (1) by Eq. (2) we get

$$\frac{v_m}{v_a} = \frac{\lambda_m}{\lambda_a}. \tag{3}$$

By means of Eq. (3) the velocity of sound in the metal rod may be determined in terms of the velocity of sound in air (regarded as known) and the ratio of the two wave lengths.

It is also possible to compute the velocity of sound in a thin metal rod by means of the equation

$$v_m = \sqrt{\frac{Y}{\rho}}, \tag{4}$$

where Y is Young's modulus for the rod, and ρ is the density of the rod. One might suppose from the analysis given in the theory of Experiment 61 that the above expression for v_m should involve the volume modulus of elasticity, E, rather than Young's modulus, Y. This supposition would not be correct since in that analysis it was assumed that the medium suffered no lateral change in dimension when a compressional pulse passed through it. This condition is realized when a disturbance originates in an elastic medium of great extent, or when the medium is confined to a tube with rigid walls. However, in this case, when a wave pulse travels along a thin rod, there is a small lateral expansion of that section of the rod undergoing compression. Because of this fact it may be shown that Young's modulus should be used rather than the volume modulus of elasticity. Equation (4) may, of course, be used to compute Y for a rod in terms of measured values of v_m and ρ.

Method: Set up the apparatus as shown in Fig. 62-2. Be sure that the metal rod is firmly clamped exactly in the center. Also the powder should be distributed evenly along the bottom of the tube between A and B.

Start with the piston A near the end of the tube. Stroke the metal rod with the cloth moistened in alcohol by grasping the rod near the clamp and then pulling away from the clamp. Slowly shorten the length of the air column during this process. When the air column between A and B is of the proper length, it will resonate with the vibrating rod and cause the powder on the bottom of the tube to collect in ridges across the tube, clearly defining the positions of the nodes and antinodes. The inter-nodal distance will be of the order of magnitude of 10 cm. Determine this inter-nodal distance accurately by measuring the distance between B (it must be at a node) and the node furthest away from it, and then dividing by the number of antinodes between these two nodes. Make an estimate of the error involved in this measurement of $\lambda_a/2$.

Repeat this process by shortening the air column still further until the next resonance point is reached.

Measure the length of the rod. This is $\lambda_m/2$. Estimate the error in this measurement. Determine the temperature of the room. Determine the velocity of sound in air at this temperature. The velocity of sound in air at 0°C may be taken as 33,170 cm/sec. The velocity of sound at any other temperature may be computed by use of the relation that the velocity of sound in air is directly proportional to the square root of the absolute temperature of the air. See Eq. (8) in Experiment 61. By use of these data and results determine the value of v_m for the metal rod. Also determine the error in this value.

Finally determine the value of Young's modulus, Y, for the rod by use of Eq. (4). The density of the material of which the rod is made (iron, brass, copper, or aluminum) may be found in Table C, Appendix III. Compare this value of Y with the accepted value. See Table M, Appendix III.

Repeat this experiment using a rod of different material.

Record: Record your data and results for this experiment in tabulated form.

QUESTIONS

1. Is there a node or an antinode of the vibrating air column at the piston *A* in this experiment? Explain.

2. Show how the apparatus in this experiment could be used to determine the velocity of sound in a gas such as CO_2, provided the velocity of sound in air is known.

3. The value of Young's modulus for a metal obtained in this experiment is likely to be somewhat larger than its value given in Table M, Appendix III. The latter value is an isothermal value while the former is an adiabatic value. How would this fact account for the difference in the value of *Y*?

Experiment 70.

Reflection and Refraction

Object: To study the reflection of light in a plane mirror; to study refraction of light in glass.

Apparatus: Plane mirror mounted in blocks, rectangular piece of plate glass, plate-glass prism, pins, rule, protractor, cork-topped stand.

Theory: The image formed by a plane mirror of an object in front of it may be located by tracing the light rays which seem to emanate from the image. Since light travels in straight lines, sighting at the image from two different directions, and projecting these two lines of sight until they intersect, will locate the image at the point of intersection. Other lines of sight must then pass through this same point of intersection if the image occupies a fixed position, regardless of the angle of view.

The same technique may be applied to the study of refraction of light in glass. In this case, however, what is seen is not a reflected image, but a refracted image. Sighting toward this image will enable the observer to determine the actual path the rays of light take in traveling from the object through the glass to the eye.

Method: *Part I. Reflection of Light by a Plane Mirror.*

A. Location of a point image by use of sight lines. Draw a line *AB* about 20 cm long across the center of a record sheet, place this sheet on the cork-topped stand, and stand the mirror with the edge of its reflecting surface on this line. Stick a pin vertically about 6 or 8 cm in front of the mirror to act as the object, *O*. (See Fig. 70-1.) Be careful that the pin is accurately vertical. The position of the image of this pin as seen in the mirror is to be located carefully.

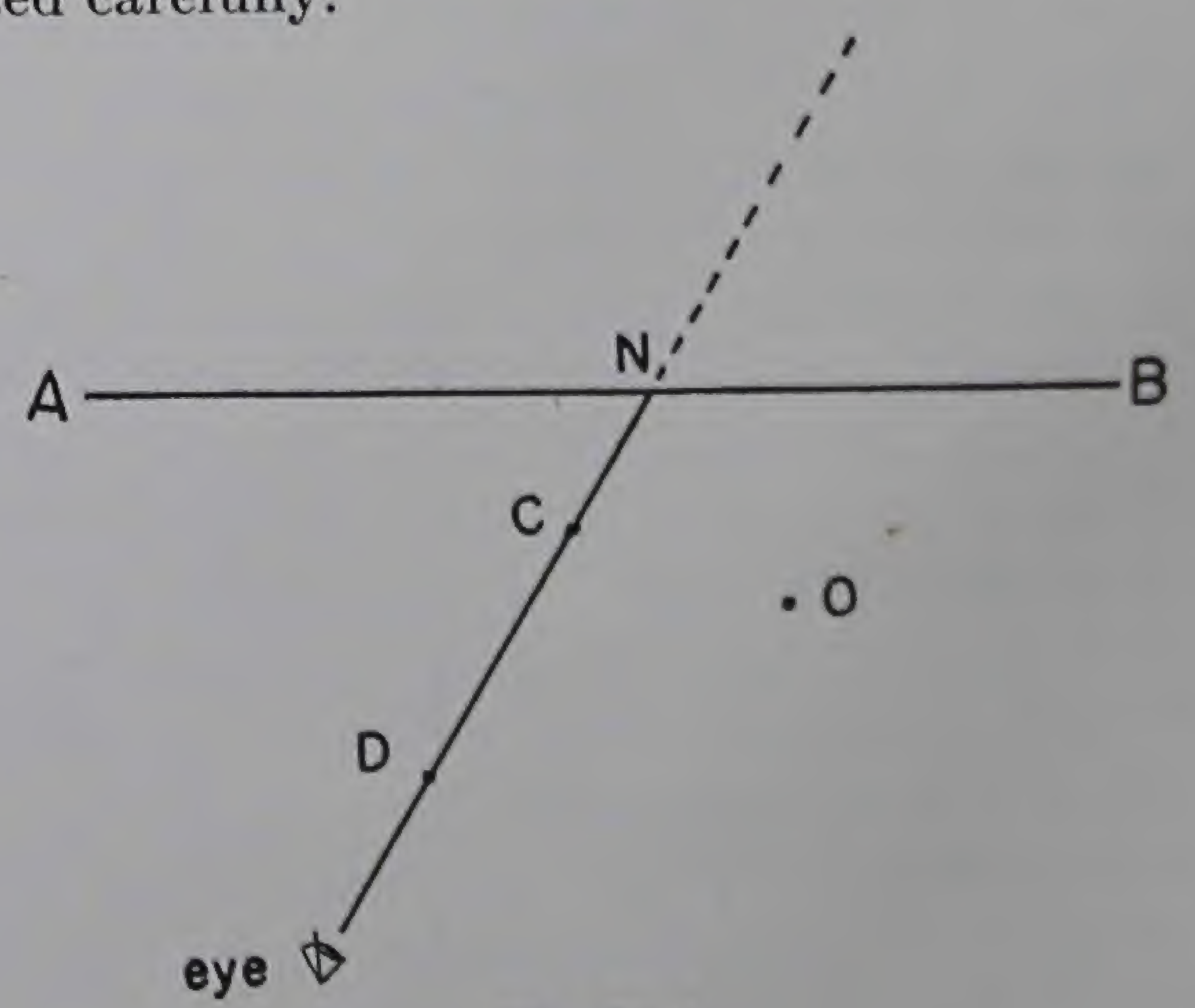


Fig. 70-1.

Stick a second pin near the mirror and a few centimeters to one side of the first pin (at a point such as *C*), and another pin (such as at *D*) 6 or 8 cm from *C* and exactly on the line which passes through *C* and the

image of the original pin. When this has been accomplished, the third pin will hide the second pin and the image of the first pin. The pins should be as nearly vertical as possible, and the eye in sighting should be placed on a level with the paper so that the line of sight is along the points of the pins. Label the points C and D. The image will lie somewhere on a line through C and D.

Without disturbing the position of the mirror, determine in the same way two other lines directed toward the image, at different angles with AB, one on each side of O. If the image has the same position when viewed from different directions, these lines (and all others similarly drawn) should intersect in the point which is the position of the image. Has the image a fixed position?

Let I denote the position of the image. Draw a line connecting I and O. What angle does this line make with AB? (Measure with the protractor.) How is it divided by AB? Describe definitely the position of the image with reference to the mirror and the position of the point object.

It is evident that when sighting along the line DC at the image, the light by which it was seen came to the eye along that line, having been reflected by the mirror at the point where DC meets it. Call this point N. Draw the incident ray ON and the perpendicular at N. Measure with the protractor and record in your figure the angles of incidence and reflection. Repeat this construction and measurement for the other two reflected rays.

Within what limits (expressed as a per cent) do your results agree with the law of reflection?

What is meant by the plane of the angle of incidence and of the angle of reflection? What is the plane of these angles in this experiment?

B. The position of a point image in a plane mirror will now be found by parallax. Draw a line AB on another record sheet as before, and this time stand the mirror so that its reflecting surface is in the vertical plane through AB but with its bottom edge about $\frac{1}{4}$ in. above the paper. Adjust it accurately. Stick a pin vertically 6 or 8 cm in front of the mirror. Hold a second pin behind the mirror, but do not stick it into the paper, and move it about until it is in such a position (using parallax) that the portion of it which is seen under the mirror (the eyes again being nearly on a level with the paper), lines up accurately with the portion of the image of the first pin which is seen at the same time *in* the mirror. Use both eyes, and move the head from side to side. When the correct position has been found, the section of the second pin and the image of the first pin will "fit" from all points of view. Let O_1 denote the position of the pin in front and I_1 the position of its image as indicated by the second pin.

Place the first pin at a different distance from the mirror, and again locate the image. Let O_2 and I_2 denote the position of this object and this image, respectively.

Draw lines connecting I and O in each case. What angle does each line make with the line AB? (Protractor.) How is it divided by AB? How do the results compare with those of Section A?

Part II. Refraction of Light in Glass.

A. Apparent displacement of objects seen through plate glass. Place a sheet of graph paper on the platform. Place the rectangular glass plate flat on the paper near the center in such a way that its long edge is parallel to the horizontal lines of the graph paper. With a *sharp* pencil draw a line around the glass plate. Now place a pin near the top of the page and to one side of the center. Place a second pin a few centimeters further down the page and to one side so that the line connecting them contacts the glass plate at a sharp angle.

Looking *through* the glass from the lower side of the page you can see both pins. As you move your head from side to side, you can find a line of sight along which the pins appear to be directly in line. This line of sight may be defined by placing a third pin between your eye and the glass plate where it appears to fall on the same line as the first two pins. This step must be repeated by placing a fourth pin near the bottom of the page so that it, too, appears to fall in this line of sight.

Use a straightedge and draw a line through the first two pin points to the edge of the glass plate. This line traces the path of the incident beam of light. Repeat this procedure by drawing a line from the lower edge of the glass plate through the third and fourth pin points. This line traces the path of the beam after it has passed through the glass.

The path of the light beam in the glass may now be traced by drawing a straight line between the point at which the beam enters the glass and the point at which it leaves the glass.

1. Find the relation between the incoming beam and the outgoing beam.
2. In what direction is the beam deviated in the glass?
3. Where is the angle of incidence? Where is the angle of refraction? Measure these in degrees with your protractor.

B. The refraction of light in a prism. Stick two pins near the center of a fourth record sheet about two-thirds as far apart as the length of one side of the plate-glass prism. Lay the prism flat on the paper between the two pins so that each side of angle A (the angle at which the identifying number is scribed) is touching one pin. With the eyes nearly at the level of the paper, view the setup from such an angle that the portion of the pin P on the far side of the prism which is seen *through* the prism is just hidden by the corresponding portion of the pin Q on the near side. See Fig. 70-2. Stick a third pin, O, in the paper to the

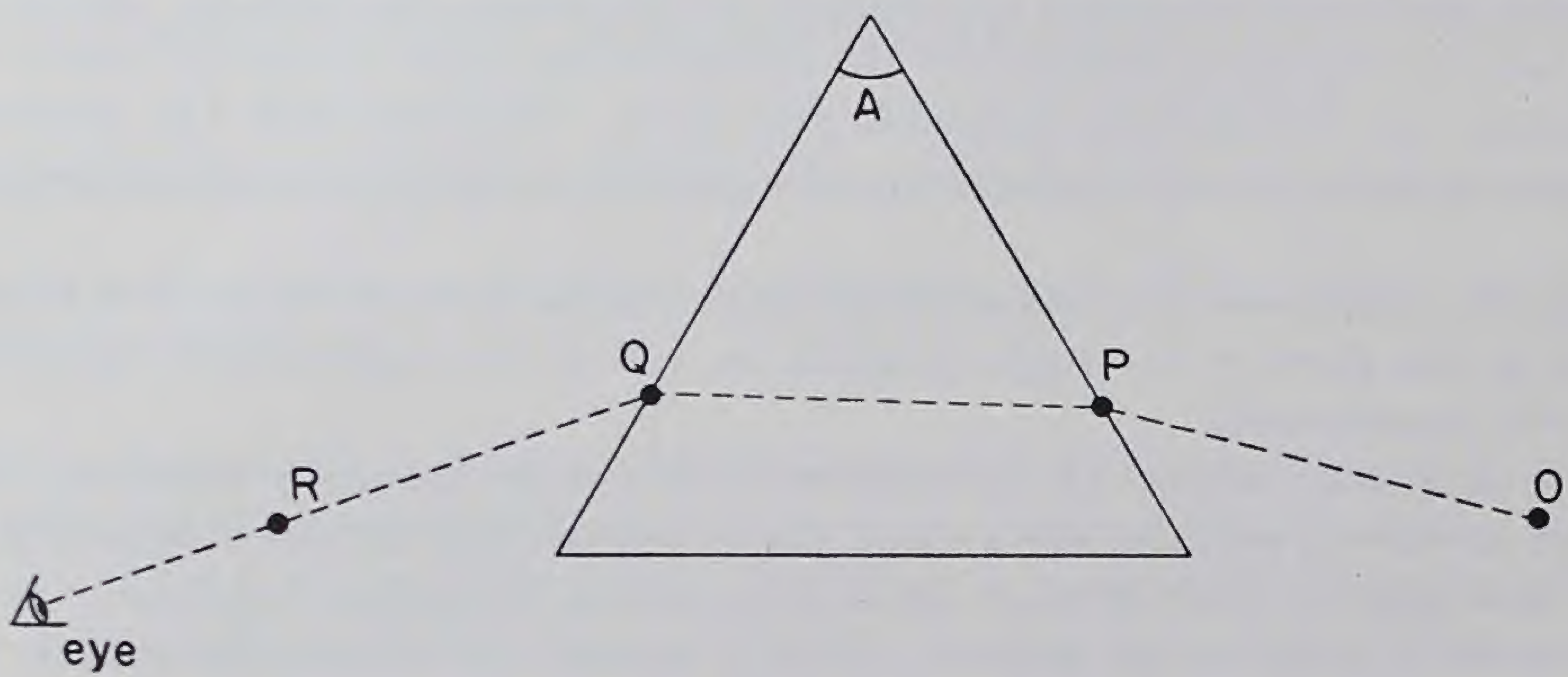


Fig. 70-2.

far side of the prism, several centimeters away from pin P, so that as seen through the prism it appears to be in the direct line of sight through pins Q and P. Stick a fourth pin, R, nearer the eye in the line of sight to pin Q. When properly arranged, and with the two far-side pins seen *through* the prism, all four pins must appear to be in a straight line so that only the pin nearest the observer can be seen.

Draw perpendiculars to the refracting surfaces at the points of entrance and emergence of the ray, and indicate the angles of incidence and refraction of the rays and the angle of deviation (the angle between lines OP and QR) caused by the prism. State the direction of deviation of the ray (toward or away from the perpendicular) in passing into and out of the prism. Measure and record the angle of the prism A and the angle of deviation.

QUESTIONS

1. In Part II-A, Question 3, of this experiment you measured the angle of incidence, i, and the angle of refraction, r, of a ray of light passing through a glass plate. Compute the index of refraction, n, of the glass by means of the equation

$$n = \frac{\sin i}{\sin r}.$$

2. In Part II-B of this experiment you measured the angle of deviation, D, of a ray of light passing through the glass prism. If this ray passes through the prism from P to Q in Fig. 70-2 in such a way that it is parallel to the base of the prism, then it may be shown that the angle of deviation is a minimum. Under these conditions, the index of refraction, n, of the glass prism is given by the equation

$$n = \frac{\sin \frac{1}{2}(D + A)}{\sin (\frac{1}{2}A)}.$$

Compute the index of refraction of the glass prism by use of this equation.

Experiment 71.

Index of Refraction by Apparent Elevation

Object: To determine the index of refraction of glass and various liquids by apparent elevation.

Apparatus: Microscope and stand, glass dish, liquids, piece of plate glass, vernier caliper.

Theory: When a small object which is covered by a layer of some transparent medium, such as glass or water, is viewed from directly above the surface, it appears to be nearer the surface than it actually is. The amount of this apparent elevation depends upon the thickness of the layer and the index of refraction of the layer of transparent material. The relation between these quantities is given by the equation

$$n = \frac{t}{t - e}, \qquad (1)$$

where n = index of refraction of the medium,
t = thickness of layer, and
e = apparent elevation.

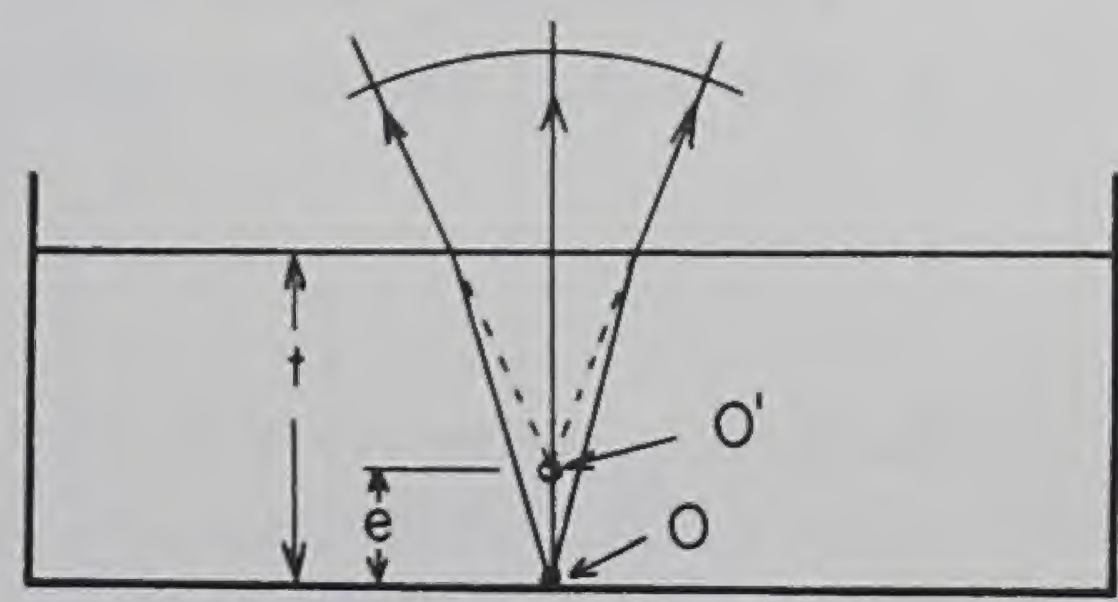


Fig. 71-1.

In Fig. 71-1 is shown a pencil of rays diverging from the source at O directly toward the upper surface of the medium of thickness t. At this upper surface these rays undergo refraction as they pass into the air and thus appear to come from point O'. It may be shown that the position of O' relative to O obeys Eq. (1) provided the light rays involved make practically normal incidence upon the upper surface. See any good general physics textbook for the development of this relation.

In order to determine n for the layer of material, it is only necessary to measure e and t. This may be done by using a traveling microscope (Note C, Appendix II) and a vernier caliper. The microscope is mounted directly above the layer of material with its axis pointing toward O. It may be focused on O, on O', or on the top of the layer by means of a rack and pinion which moves the entire microscope up or down in a fixed metal sleeve. The position of the microscope relative to the metal sleeve may be measured with a vernier caliper. Hence the distances t and e may be determined.

Method: *Part I. Glass.* Make a small ink spot upon a piece of stiff white paper. Place it on the microscope stand immediately under the microscope and focus the microscope on it with the rack and pinion. Be sure that the paper is flat against the microscope stand. With the vernier caliper measure the distance from the top of the metal sleeve to the top of the eyepiece of the microscope. In this process use the protruding shaft of the vernier caliper (see Fig. A-4). Call this reading r_1. Essentially it gives the position of O in Fig. 71-1. Take three readings of r_1 refocusing the microscope each time.

Then place the piece of plate glass over the spot and refocus the microscope until the image at O' is in sharp focus. Measure, three times, the new distance from the top of the metal sleeve to the top of the eyepiece with the vernier caliper. Call this reading r_2. It gives the position of O'.

Finally focus the microscope on the top surface of the piece of plate glass. In doing this it may be necessary to put a small ink spot on this top surface. Again measure, three times, the position of the micro-

scope relative to the sleeve. Call this reading r_3. It is evident that $t = r_3 - r_1$ and that $e = r_2 - r_1$, hence

$$n = \frac{r_3 - r_1}{r_3 - r_2}. \tag{2}$$

Calculate by means of Eq. (2) the index of refraction of the glass plate. From the estimated errors in r_1, r_2, r_3, determine the error in n.

Part II. Liquids. In the case of liquids use a flat-bottomed glass dish with a small scratch on the bottom of it (inside surface). This scratch will serve as the point O. Focus the microscope upon the scratch and record the vernier-caliper reading as in Part I. Be sure to make three trials. Then pour in the liquid, whose index of refraction is to be determined, to a depth of about 3 cm. Focus on the image of O at O' and again record the vernier-caliper readings. Finally sift a little powder (lycopodium) on to the surface of the liquid and focus the microscope on this powder. Record the vernier-caliper readings for this position of the microscope. From these data compute the index of refraction of the liquid. Determine the error in this value.

Repeat Part II for a second liquid.

Record: Tabulate your data and results.

QUESTIONS

1. Develop Eq. (1) by use of the wave front or curvature method (law of sagitta).
2. Show that the determinate-error equation corresponding to Eq. (2) is

$$\frac{\Delta n}{n} = \frac{-e}{t(t - e)} \Delta r_3 - \frac{1}{t} \Delta r_1 + \frac{1}{t - e} \Delta r_2.$$

3. Is the image at O' in Fig. 71-1 real or virtual? Explain.

Experiment 72.

Lenses

Object: To determine the focal lengths of converging and diverging lenses by different methods. To investigate the images formed by the converging lens.

Apparatus: Optical bench (Fig. 72-1), convex lens, concave lens, telescope, object which may be illuminated, plane mirror, vertical wires, lamp.

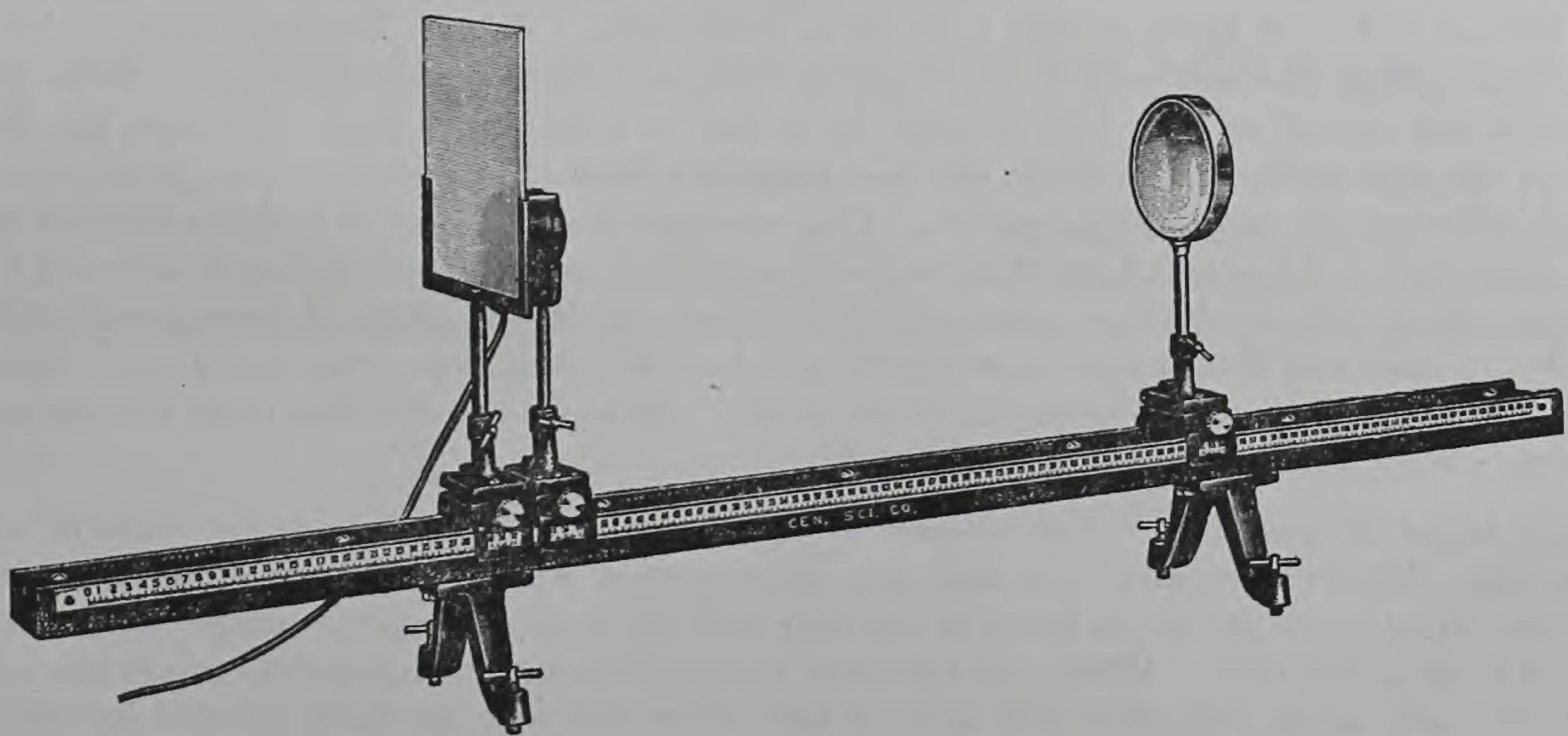


Fig. 72-1.

Theory: The locations of object and image in reference to a lens are given by the basic lens formula (for a thin lens)

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}, \tag{1}$$

where p = distance from the object to the center of the lens,
q = distance from the image to the center of the lens, and
f = focal length of the lens.

For positive values of q the image is *real*, and may be projected on a screen. For negative values of q the image is *virtual;* that is to say, the rays of light coming from the object through the lens *diverge* and the eye in viewing these rays sees them as though they came from an image located on the other side of the lens; such a virtual image cannot be projected onto a screen.

There are two general types of lenses: converging and diverging lenses. The former type is thicker in the middle than at the edges; at least one side of such a lens is *convex;* rays of light in passing through such a lens converge more (or diverge less) on the far side. The diverging lens is thinner in the middle than at

the edges, and at least one side is concave; light rays diverge more (or converge less) after passing through such a lens.

The *principal focus* of a lens is the point (in the case of a converging lens) on the axis of the lens through which all rays of light parallel to the principal axis pass when refracted by the lens, or (in the case of a diverging lens) *appear* to pass. The distance from the center of the lens to this point is called the *focal length* of the lens. A convergent lens has a positive focal length and is hence often called a positive lens. A divergent lens has a negative focal length, and is correspondingly called a negative lens. Thus it is clear that in Eq. (1) algebraic signs must be carefully observed.

From Eq. (1) it is clear that an object distance p which results in a positive image distance q implies that the object may be placed at a distance q and form an image at the distance p from the lens. (By convention, a real object distance is always positive; an image distance is positive if the image is on the opposite side of the lens from the object.) Two points such that an object at one produces an image at the other are called *conjugate foci.*

***Method:** Part I. Converging Lens.* Make three determinations of the focal length by each of the following methods.

1. *Focal length by parallel rays.* Rays of light from distant objects are essentially parallel. For the purposes of this experiment, a building or other object 50 yards or more away may be considered a distant object. Focus the image of such an object upon a screen. Since the incident rays are essentially parallel, the image formed will lie in the principal focal plane of the lens. *Why?* The best focus is obtainable if no extraneous light strikes the screen; it will be helpful if the room is darkened during this phase of the experiment with window shades up only high enough for a view of a distant object. Measure the distance from the center of the lens to the screen when the best focus has been obtained.

Focus a telescope on some distant object. The telescope is now adjusted to focus parallel rays entering it. Without changing the adjustment of the telescope, place it over the optical bench and aim it toward the lens. On the other side of the lens place an illuminated object. A piece of transparent ruler mounted across a hole in a card and illuminated from behind will do. Move the lens toward and away from this object until the object appears clearly in focus in the telescope. Measure the distance from the center of the lens to the object. *Why is this equal to the focal length?*

2. *Focal length by parallax. (Coincidence method.)* Set a plane mirror whose face is vertical close behind the lens. Before it set an object such as a short vertical wire which can be illuminated from the side. Move the wire until, with the eye in front of the wire and the lens, an inverted image can be seen in the air just above the tip of the wire. When this has been accomplished, the wire and its image are a focal length away from the lens, since only then will rays of light from the wire be made parallel by the lens and be reflected back on themselves by the mirror. Adjust the position of the wire until there is no parallax between it and its image. (See Appendix II, Note F.) Since the image is in the focal plane of the lens, the eye must be focused not on the lens but on the image before it.

3. *Focal length by conjugate foci.* Place the transparent-scale object near one end of the optical bench, and illuminate it from behind. Before it place the lens, and beyond this adjust the screen in a position to receive the image of the transparent scale. When a good focus has been achieved, the object and image are at conjugate foci of the lens (Fig. 72-2). To obtain a sharper focus, it will usually be helpful to cover all

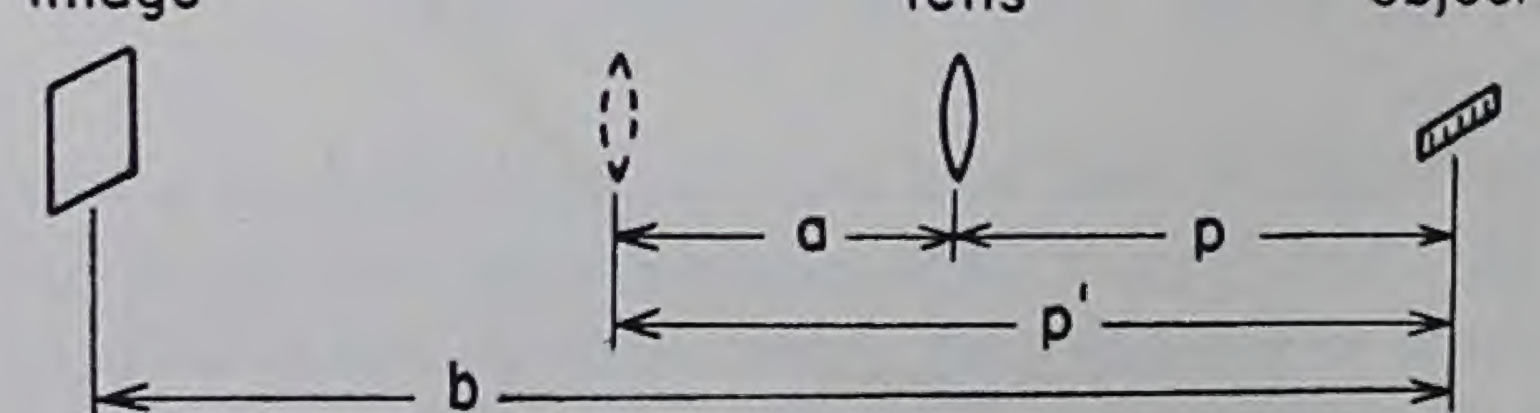


Fig. 72-2.

of the lens except for a small aperture at the center. This reduces spherical aberration, a defect of all lenses made of a single piece of glass and ground with spherical faces. Measure the distances from the object to

the center of the lens and from the center of the lens to the image (distances p and q). Find the focal length from these data.

Without moving either the object or the image after adjustment in the previous step, move the lens until another sharp image is cast on the screen. Again measure object and image distances (distances p' and q'). *How do these new distances compare with the previous measurements?* Find the focal length from these data.

Denoting by b the distance between the object and the image and by a the distance *between the two positions of the lens*, it may be shown that the focal length is given by

$$f = \frac{b^2 - a^2}{4b}. \tag{2}$$

Use Eq. (2) to find the focal length. Substituting an expression for a in terms of b and p in Eq. (2) and differentiating, it is found that the longest focal-length lens which will form an image a distance b from an object is one for which $f = \frac{1}{4}b$. In this case $q = p = 2f$. Equation (2) can be derived from Eq. (1) by substituting expressions for p and q in terms of a and b. *Perform this derivation.*

4. *Study the images formed by a converging lens* and make a table showing whether the image is real or virtual, erect or inverted, enlarged or reduced for the following distances of the object from the lens: more than twice the focal length, at twice the focal length, between the focal point and twice the focal length, at the principal focus, and between the principal focus and the lens.

Part II. Diverging Lens. Make three determinations of the focal length by each of the following methods.

1. *Focal length by virtual object.* Set up an illuminated object, *convex* lens, and screen as in Part I, Section 3, and obtain a sharp image on the screen. Now place the diverging lens between the first lens and the screen. Its diverging action will cause the image to lie farther from the convex lens than originally (Fig. 72-3). Move the screen out until the image is again in focus, first recording the original position of the

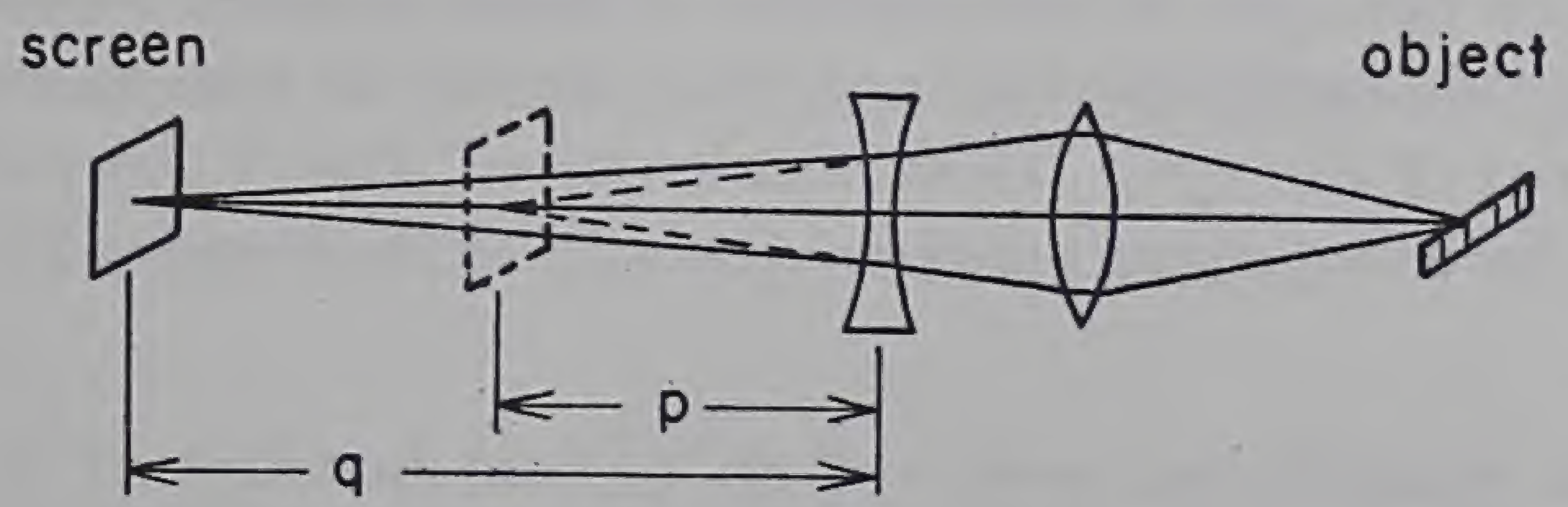


Fig. 72-3.

screen. Determine the distances between the center of the *concave* lens and the two positions of the screen. The position of the original image may be thought of as a virtual object (object distance negative) for the diverging lens. This results in a real image if the converging lens is "stronger" than the diverging lens. Thus the two positions of the screen are in fact conjugate foci of the diverging lens. Determine the focal length of the concave lens using Eq. (1).

2. *Focal length by parallax.* Place before the concave lens the shorter of the two vertical wires, and looking through the lens from the other side, locate its (virtual) image. Place the other vertical wire (tall enough so that 3 or 4 cm of its length are visible *above* the lens) in the general location of the previously located image. Looking at the image of the first wire *in* the lens and at the portion of the second wire visible *above* the lens, adjust the position of the taller wire by use of parallax (see Appendix II Note F) until its upper portion just coincides with the image. Measure the distances from the first wire to the center of the lens and from the center of the lens to the second wire. Calculate the focal length of the lens.

Record: Tabulate the six sets of values for the focal length of the converging lens and the two sets for that of the diverging lens, each set consisting of the three trial values and their mean. Calculate the indeterminate error in each case.

Answer all italicized questions.

Experiment 73.

Optical Instruments

Object: To construct and investigate the properties of the astronomical telescope and the simple compound microscope.

Apparatus: Optical bench, converging lens of long focal length and large aperture, two converging lenses of short focal lengths and small apertures, vertical wire or cross hairs, screen, two small translucent scales, large scale, lamp.

Theory: Two optical instruments constantly used in the scientific laboratory are the telescope and the microscope. Their primary function is to aid vision by presenting to the eye a "magnified" image of the object which is under examination. Thus we are able to see better with these instruments than with the unaided eye. In general the telescope is used to examine large distant objects, such as the moon, which cannot be brought to the position of most distinct vision (taken as 25 cm), but must be examined where they are. On the other hand, the microscope is generally used to examine small objects which may be moved about at will and hence may be brought to the position of most distinct vision. If these small objects are brought closer to the eye than this minimum distance, they cannot be seen clearly because of a lack of accommodation of the crystalline lens of the eye. It should be noted that if the power of accommodation of the eye were infinite (capable of focusing upon an object no matter how close to the eye), microscopes would serve no useful purpose.

Magnifying Power. The magnifying power of an optical instrument is essentially the ratio of the size of the image on the retina of the eye when the instrument is used to examine the object, to the size of the image on the retina of the eye when the unaided eye is used to examine the object. It is clear then that magnification occurs only when the size of the retinal image is increased by the use of an optical instrument. But the size of this retinal image is directly proportional to the angle subtended at the eye by the object or space image which is being examined, since the distance from the crystalline lens of the eye to the retina is fixed for a given individual. Hence a general definition for the *magnifying power* (*M.P.*) *of an optical instrument is the ratio of the angle subtended at the eye by the image produced out in space by the instrument, to the angle subtended at the eye by the object when viewed in its most favorable position by the unaided eye.* The student must be careful to distinguish clearly between linear magnification (the ratio of an actual dimension of an image to the corresponding dimension of the object) and magnifying power. If the moon, for example, is viewed through a telescope, the actual linear diameter of the image may be many times smaller than that of the moon, while the magnifying power, if vision is to be aided at all, would have to be greater than unity. The difference should be clear from the following discussion of the simple magnifying lens.

Suppose a small object OO' is placed just inside the principal focus F of a thin converging lens LL' as shown in Fig. 73-1a. A *virtual* image, erect and enlarged, will be formed at II'. The relation between object distance p, image distance q, and focal length f will be

$$\frac{1}{p} - \frac{1}{q} = \frac{1}{f} \qquad \text{(negative sign used because image is virtual).}$$

If the eye is placed immediately behind the lens it will "see" the image II' provided $q \geq 25$ cm. For $q < 25$ cm the eye would not be able to focus on the image because of a lack of accommodation. The angle β in the figure is essentially equal to the angle subtended at the eye by the image II' since the eye is placed immediately behind the lens.

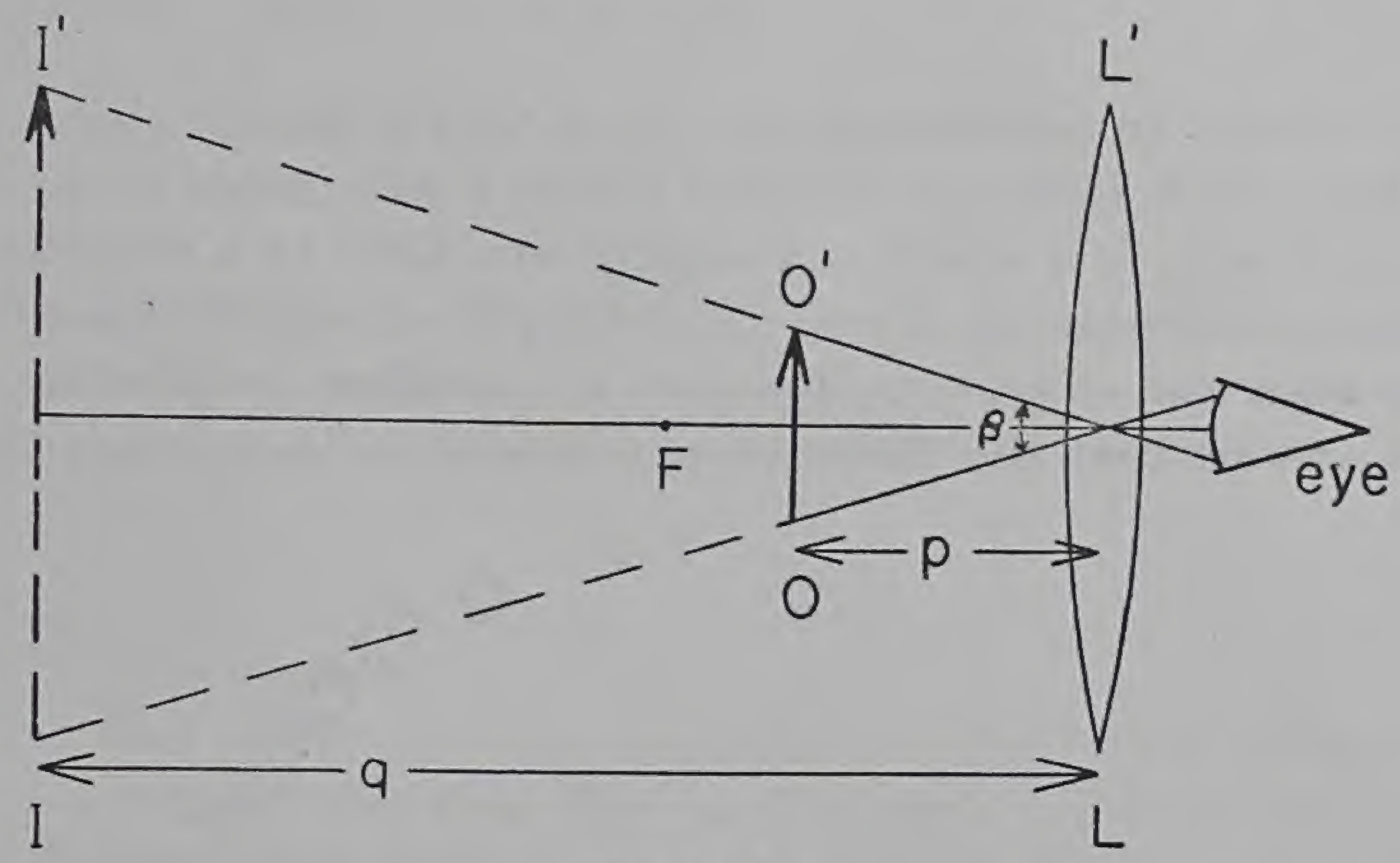


Fig. 73-1a.

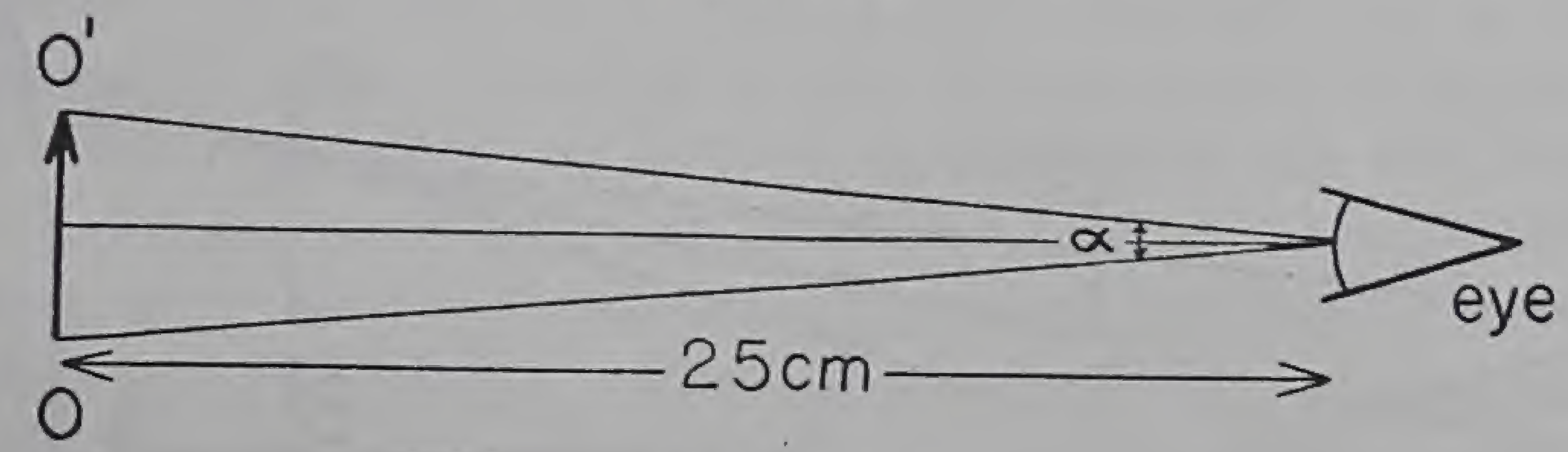


Fig. 73-1b.

If now the same object OO' is to be examined without the visual aid of the lens LL', it must be moved out to a distance of 25 cm from the eye for distinct vision. It then subtends an angle α at the eye as shown in Fig. 73-1b.

The magnifying power of the lens is then given by the relation

$$\text{M.P.} = \frac{\beta}{\alpha}. \tag{1}$$

Since the angles α and β are small, the following relations are valid:

$$\beta = \frac{II'}{q} = \frac{OO'}{p}, \tag{2}$$

$$\alpha = \frac{OO'}{25}. \tag{3}$$

If we substitute these values of β and α into Eq. (1) and simplify, we get

$$\text{M.P.} = \frac{25}{p} = 25\,\frac{f+q}{fq}. \tag{4}$$

Since q must lie on the interval $\infty \geq q \geq 25$ cm, it is clear that the M.P. may vary from its maximum value

$$\text{M.P. (max)} = \frac{25}{f} + 1 \qquad \text{(image at 25 cm)}, \tag{5}$$

to its minimum value,

$$\text{M.P. (min)} = \frac{25}{f} \qquad \text{(image at infinity)}. \tag{6}$$

If f is small compared to 25 cm, as it generally is, it will be seen that the two values of M.P. differ very little.

As a rule, when an object is viewed through a magnifying glass, one has not the faintest idea where the image lies. As with objects, it is possible to view images over a range of distances (25 cm to infinity). In order to avoid eyestrain it is advisable to focus on the object in such a way that the image first appears at infinity. In this way parallel rays rather than divergent rays enter the eye. This reduces the M.P. slightly but avoids eyestrain.

The Telescope. The telescope, an instrument used to observe a distant object, consists essentially of (1) the *objective*, a converging lens which forms (in its focal plane) a real image of the distant object, and (2) an *eyepiece* which, in the simplest case, is a single converging lens used as a magnifying glass to examine this real image. The virtual image produced by the eyepiece may lie at any distance from it between 25 cm and infinity. Since the eye muscles are subject to the least strain when parallel rays fall on the eye (virtual image at infinity), we shall assume that the telescope is adjusted so that this is the case.

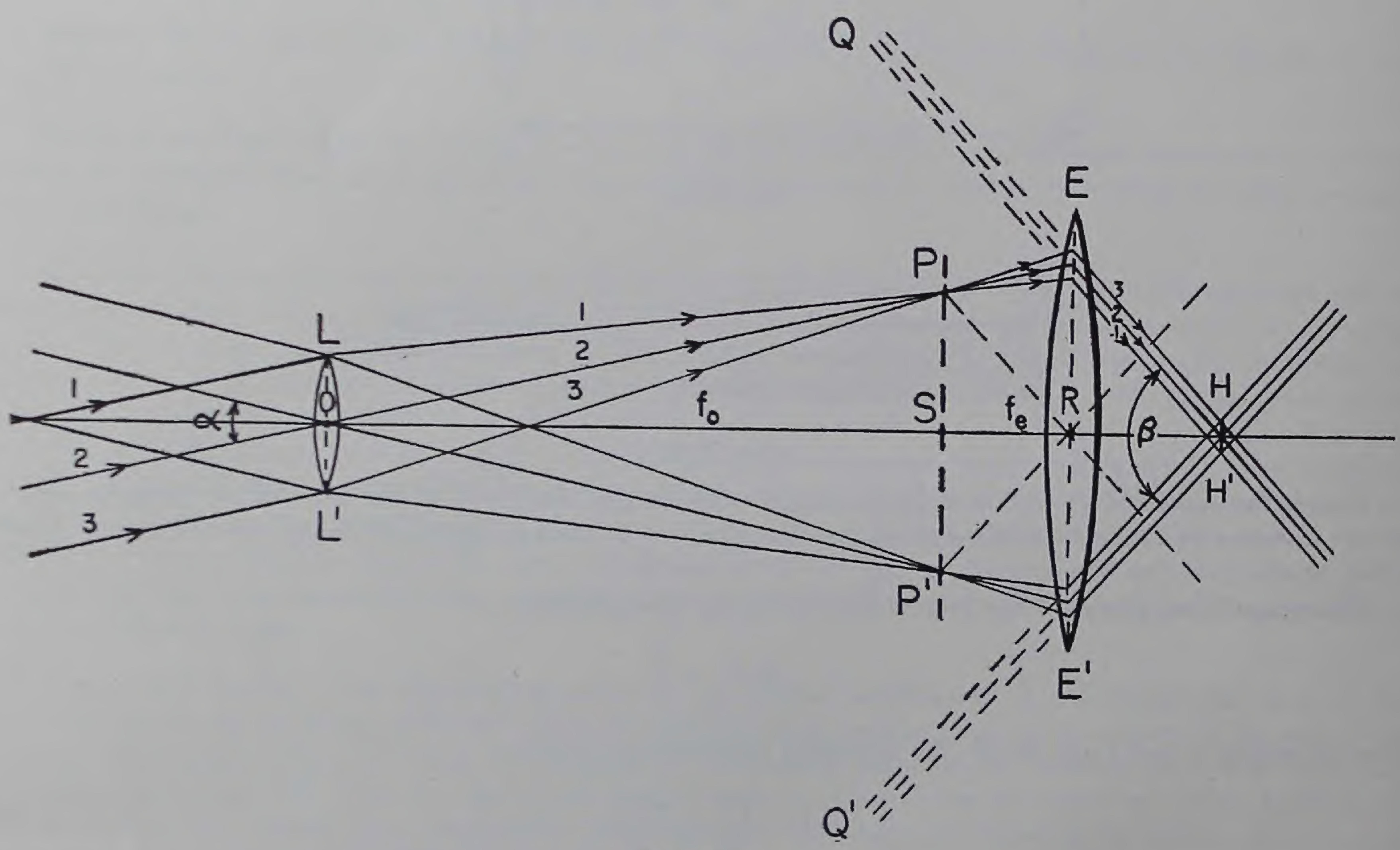


Fig. 73-2.

Figure 73-2 shows the optical paths of two sets of rays through a telescope having an objective lens LL and an eyelens EE'. The size of the eyelens is greatly exaggerated in this diagram because the angles which the rays make with the principal axis OR are greatly exaggerated. Rays 1, 2, 3 are a set of rays, practically parallel, coming from a point on the lower edge of the distant object (not shown). These rays pass through the objective lens LOL' and are brought to a focus at P in the focal plane of the objective lens. Corresponding parallel rays from the uppermost edge of the distant object are brought to a focus at the point P'. Thus a *real inverted image* of the distant object is formed in the focal plane of the objective lens. $OS = f_o$, the focal length of the objective lens.

This real image PP' is examined by means of the eyelens or eyepiece EE' which is so adjusted in position that it forms a virtual image QQ' of the real image PP' at a very great distance from EE'. Thus the distance $SR = f_e$, the focal length of the eyelens. Also the rays 1, 2, 3 emerging from the eyelens are, for all practical purposes, parallel rays. When the eye of the observer is placed behind the eyelens EE', these parallel rays enter the eye and the eye sees the final virtual image QQ' essentially at infinity. It is evident that this final

virtual image will be inverted with respect to the distant object. This is a characteristic of the astronomical telescope as distinguished from the terrestrial telescope.

The magnifying power of the telescope is simply the angular size of the final virtual image divided by the angular size of the distant object. The distant object obviously cannot be brought to the position of most distinct vision, *i.e.*, 25 cm. Hence one may write

$$\text{M.P.} = \frac{\beta}{\alpha}, \tag{7}$$

where β and α are now the angles shown in Fig. 73-2. If it is remembered that these angles are very small in the case of the telescope (greatly exaggerated in the figure) it is an easy matter to show that $\beta/\alpha = f_o/f_e$; and hence

$$\text{M.P.} = \frac{f_o}{f_e}. \tag{8}$$

This is left as an exercise for the student.

An examination of Fig. 73-2 shows that the emergent parallel rays coming from the two opposite edges of the distant object all pass through the area HH', to the right of the eyelens. This area is called the *exit pupil* or *eye ring* of the telescope. It may be shown that through this area pass not only the rays mentioned above but all rays coming from the object which can be seen through the instrument. The exit pupil thus defines the place where the eye should be placed to see the largest range of the object. Further consideration of Fig. 73-2 shows that the exit pupil HH' is, in fact, nothing but the *real image* of the objective lens LL' formed by the eyelens EE'. Its position and size therefore may be computed by treating LL' as an object in front of the eyelens ($p = f_o + f_e$) and by finding the position and size of the image of this object. By use of the lens formula the student may show that the distance of the exit pupil from the eyelens is $f_e[1 + (f_e/f_o)]$ and that its diameter HH' is given by the relation

$$HH' = LL'\left(\frac{f_e}{f_o}\right). \tag{9}$$

Since the magnifying power of the telescope is, by Eq. (8), equal to f_o/f_e, it follows from Eq. (9) that

$$\text{M.P.} = \frac{f_o}{f_e} = \frac{LL'}{HH'}. \tag{10}$$

Thus the M.P. of the telescope is also equal to the diameter of the objective lens divided by the diameter of the exit pupil. By properly illuminating the objective lens its real image (exit pupil) may be projected on a screen and measured.

A study of the effects of diffraction in telescopes shows that it is desirable to have objectives with large diameters in order to increase resolving power (ability to separate objects very close together). The consequent increase in the size of the exit pupil may be offset by increasing the focal length f_o of the objective, and hence increasing the M.P. of the telescope.

The Microscope. The magnifying power of a magnifier consisting of a single lens is of the order of 10 or 15. When higher magnifying powers are desired, use is made of the microscope. In its simplest form it consists of a short-focus converging objective lens L_o which forms a real image (enlarged and inverted) of the small object placed just outside its principal focus, F. This real image is examined by means of an eyelens (ocular) L_e which forms an enlarged virtual image of the real image. See Fig. 73-3. The small object is PRQ. A set of rays emanating from point Q passes through the objective L_o and is brought to a focus at Q_1. Another set emanating from point R on the principal axis is brought to a focus at point R_1. Thus an enlarged and inverted real image $P_1R_1Q_1$ is formed by L_o. This real image is placed in the principal focal plane of the eyelens L_e, so that the eye placed in back of L_e sees an enlarged virtual image $P_2R_2Q_2$ (not shown) at infinity.

The magnifying power of the microscope is the ratio of the angular size of the final image to the angular size of the object when placed 25 cm from the eye. The student may show that this reduces to the expression

$$\text{M.P.} = m\frac{25}{f_e}, \tag{11}$$

where m is the linear magnification of the image P_1Q_1, *i.e.*, P_1Q_1/PQ.

There exists an exit pupil or eye ring HH' for the microscope just as for the telescope; and, as before, this exit pupil is the real image of L_o formed by L_e.

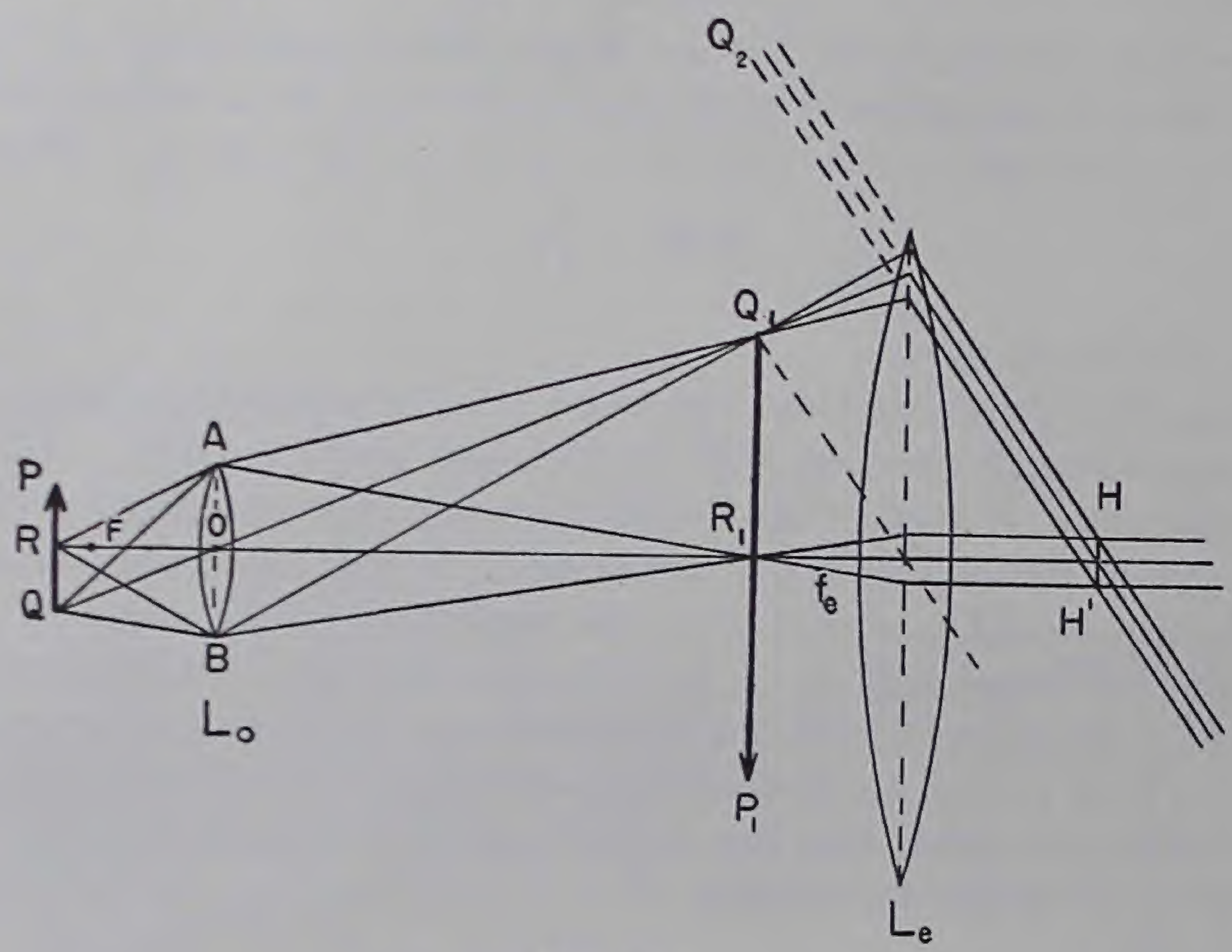


Fig. 73-3.

An important quantity occurring in connection with microscopes, is the *numerical aperture* (N.A.) of the microscope. It is given by the relation

$$\text{N.A.} = n \sin \widehat{ARO}, \tag{12}$$

where n is the index of refraction of the object medium (generally air but not always) and $\widehat{ARO}$ is the slope angle of the outermost ray from an axial point on the object. (See Fig. 73-3.) The magnifying power of the microscope may be expressed in terms of the N.A. of the microscope and the diameter of the exit pupil by the relation

$$\text{M.P.} = \frac{50\ \text{N.A.}}{HH'}. \tag{13}$$

Method: In the following work several precautions must be taken. The lenses and other items of equipment used must be carefully aligned so as to lie all on the same axis and have their principal planes perpendicular to this axis. Poor alignment generally aggravates image distortion. In adjusting for parallax, the distortion, a result of lens imperfections and poor alignment, may cause a false movement if the eye is moved too far. Adjustments of this sort must be made with small head motions. The distortion may be partially eliminated by use of a diaphragm, which will cut down spherical aberration in the lens to which it is applied; it will at the same time, however, cut down the brightness of the images seen.

Each observer should make an independent determination of each quantity involved in the following instructions. The mean of the readings should be computed in each case.

Part I. Astronomical Telescope. Using a distant building or tree as an object, focus an image on a screen with the long-focal-length lens (the objective lens). Remove the screen and place the vertical wire in the same location. Make any final small adjustment necessary to eliminate parallax between the image and the wire. (The eye is placed behind the wire looking toward the lens past the wire. The image, how-

ever, will be *at* the wire, and the eye must be focused for this distance.) Behind the wire place a short-focal-length lens (the eyelens) at a distance slightly greater than its focal length. Place the eye immediately behind this eyelens and slowly move the eyelens *toward* the objective lens until a distinct virtual image of the distant object and vertical wire first appears. Focusing in this manner ensures that parallel rays enter the eye and prevents eyestrain. Since the wire and the first image coincide, the second lens thus provides an enlarged image of the object with the image of the wire superimposed to act as a cross hair.

Record the distances from the objective lens to the wire and from the wire to the eyepiece. As previously noted in the theory, the magnifying power of the telescope is the ratio of these two distances. Calculate the magnifying power.

Aim the telescope at the large scale in the laboratory. Focus the telescope as indicated above by moving the eyelens in, from a position too far out. With both eyes open, one seeing the scale directly and one seeing it through the telescope, aim the telescope until the images on the two retinas of the eyes merge into one. The scale as seen directly will seem small and be superimposed on the larger image as seen through the telescope. (NOTE: It may be difficult to accomplish the desired superimposition of images at first. The brain, confronted with seemingly inconsistent data, tends to rationalize, and pretends that one image or the other fails to exist. If this should happen, close the eye whose image is being accepted so that the other image again appears, and then try once more for the dual image.) Any parallax between the two scale images should be eliminated, before finding the magnifying power, by slightly moving the eyepiece in a direction such as to bring the final image to the same position as the large scale. Estimate the number of the smaller scale divisions which are superimposed on each larger division. This number is the magnifying power of the telescope. Compare this with the value previously calculated.

Illuminate the objective lens from the side, and project its real image formed by the eyepiece on a screen behind the eyepiece. This image is the exit pupil or eye ring of the telescope. The ratio of the actual diameter of the objective lens to the diameter of its image on the screen is equal to the magnifying power of the telescope. Compute this value and compare it with the two previous values.

Repeat this part of the experiment using another eyelens of somewhat different focal length than the original eyelens.

Part II. Compound Microscope. Place the small scale near one end of the optical bench, and illuminate it from behind. Before it place the shortest focus lens (the objective lens) at a distance somewhat greater than its focal length. Project its real image upon the screen. Remove the screen and place the vertical wire in the same location. Make any final small adjustment necessary to eliminate parallax between the image and the wire. Behind the wire place the medium-focal-length lens (the eyelens or ocular) at a distance slightly greater than its focal length. Focus by moving the eyelens in, from a position too far out. Why? Since the wire and the first image coincided, the original object should now be seen greatly magnified, with the vertical wire seemingly coincident with it to serve as a cross hair. Record the locations of all items: object scale, objective lens, vertical wire, and eyepiece. Also record the diameter of the objective lens and the diameter of the exit pupil or eye ring. Compute the magnifying power of the microscope by use of Eq. (11). Compute the N.A. of the microscope. It is $OA/\sqrt{(OA)^2 + (OR)^2}$ (see Fig. 73-3). Why? Compute the M.P. of the microscope by use of Eq. (13). Compare these two values of the M.P.

The magnifying power of a microscope may be determined directly in the following manner: Hold a scale of the same type as the object scale about 25 cm from the eye when the other eye is viewing the object scale through the microscope. Adjust the focus of the eyelens until there is no parallax between the two scale images, one seen directly and the other seen through the microscope. Superimpose the two scale images as in the case of the telescope. Estimate how many divisions of the scale viewed directly correspond with one division as viewed through the microscope. This number is the M.P. of the microscope. Would you expect this M.P. to be exactly the same as the calculated M.P.? Explain.

Record:

	Apparatus Numbers	*Focal Lengths*
Long-focus lens	____________	__________
Medium-focus lens	____________	__________
Short-focus lens	____________	__________

Item	*Obs No. 1*	*Obs No. 2*	*Ave*
Part I. Astronomical Telescope			
Position of objective lens			
Position of real image			
Position of eyepiece			
Distance objective lens to wire			
Distance wire to eyepiece			
M.P. (calc)			
M.P. (by direct obs)			
Objective-lens diameter			
Exit-pupil diameter			
M.P. (calc)			

Item	*Obs No. 1*	*Obs No. 2*	*Ave*
Part II. Compound Microscope			
Position of object scale			
Position of objective lens			
Position of real image			
Position of eyepiece			
Distance OR			
Distance OR_1			
M.P. [calc by Eq. (11)]			
M.P. (by direct obs)			
Diameter objective lens			
Diameter exit pupil			
N.A.			
M.P. [calc by Eq. (13)]			

QUESTIONS

1. By use of Fig. 73-2 and Eq. (7) develop Eq. (8).

2. Develop Eq. (9) by finding the position and the size of the image, HH', of the objective lens, LL', formed by the eyepiece lens, EE'.

3. Develop Eq. (11).

4. By use of Eqs. (12) and (13) show that the M.P. of a microscope may be increased by using an oil-immersion objective lens, *i.e.*, by filling the space between the object (PRQ) and the objective lens (AB) in Fig. 73-3 with oil.

Experiment 74.

Index of Refraction with Spectrometer

Object: To determine the index of refraction of a glass prism for the mercury green line by spectrometer measurements of the prism angle and angle of minimum deviation.

Apparatus: Spectrometer, mercury arc, glass prism, level.

Theory: It is possible to make a very accurate determination of the index of refraction, n, of glass for a given wave length of light by use of refraction through a glass prism.

Suppose that a ray of monochromatic light (light of a single wave length) is incident upon one face of a glass prism at point M as shown in Fig. 74-1. This ray will be refracted as it passes from air into the glass

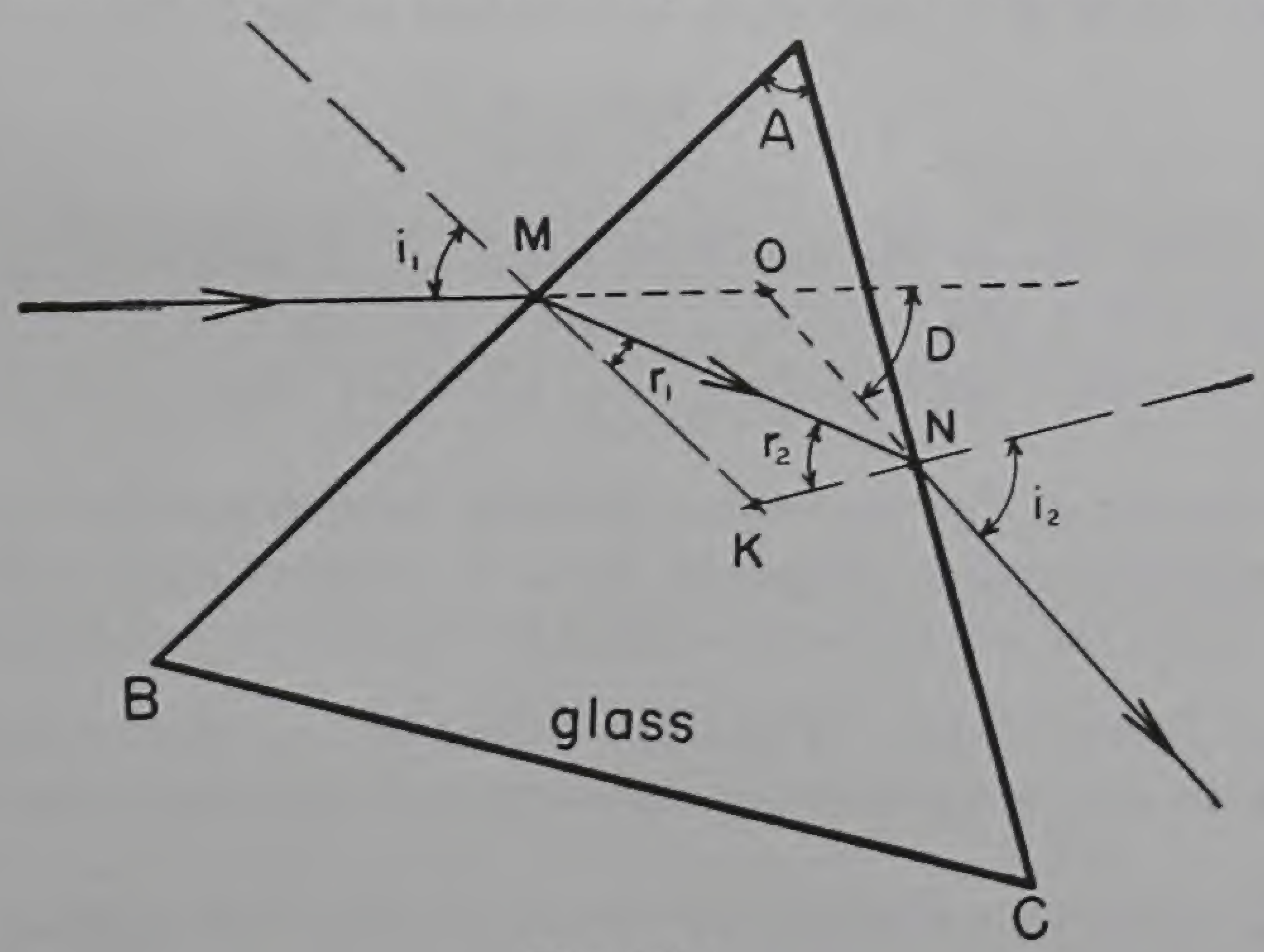


Fig. 74-1.

and will proceed through the glass along some line MN. At point N it will again be refracted as it emerges from the glass into the air on the other side of the prism. The angle D in the figure represents the total deviation of the ray resulting from its passage through the prism, *i.e.*, D is the angle between the incident ray and the emergent ray. It may be shown from the geometry of the figure that

$$D = (i_1 - r_1) + (i_2 - r_2),$$

where i_1 = angle of incidence,
r_1 = corresponding angle of refraction,
i_2 = angle of emergence, and
r_2 = corresponding angle of refraction.

It may also be shown by geometry that $r_1 + r_2 = A$ where A is the angle of the prism as shown. Hence

$$D = i_1 + i_2 - A. \tag{1}$$

The complete proof of these formulas is left as an exercise for the student.

For a given prism, the magnitude of the angle D depends only upon the incident angle i_1. This is evident since for a given prism of fixed n and A the angles r_1, r_2, and i_2 may be calculated once i_1 is given. D may, therefore, be regarded as a function of i_1.

By actual trial it is found that there is one value of i_1, and only one, which makes D a minimum. When this is the case the angles i_1 and i_2 must be equal, for, if they were not equal, there would exist two different angles giving minimum deviation, *i.e.*, i_1 and i_2. This follows from the fact that it is always possible to reverse the direction of the ray through the prism without changing the value of D.

In this experiment the condition of minimum deviation is secured by trial. Under this condition

$$\left.\begin{aligned} i_1 &= i_2 = i; \\ r_1 &= r_2 = r = \frac{A}{2} \end{aligned}\right\}. \tag{2}$$

It follows that Eq. (1) may be written

$$D_m = 2i - A, \tag{3}$$

where D_m = angle of minimum deviation,
i = angle of incidence or emergence, and
A = prism angle.

The index of refraction n of the glass prism is given by the defining relation

$$n = \frac{\sin i}{\sin r}. \tag{4}$$

By use of Eqs. (2) and (3) the value of n may then be written in the form

$$n = \frac{\sin \frac{1}{2}(D_m + A)}{\sin (A/2)}. \tag{5}$$

Relation (5) provides us with one of the standard methods of determining the index of refraction of a substance, such as glass, when it is in the form of a prism. By use of a spectrometer the angles A and D_m may be measured with great accuracy, and hence a very accurate value of n may be obtained.

***Method:** Part I. Adjustment of Spectrometer.* Details concerning the construction and the approximate adjustment of the spectrometer are given in Note L, Appendix II. The student should read this section in the Appendix and make the adjustments called for before proceeding further in this experiment.

Part II. Measurement of Prism Angle. Place the prism on the spectrometer table with its refracting edge near the center of the table and with its base approximately perpendicular to the axis of the collimator as shown in Fig. 74-2.

Illuminate the slit of the collimator C with light from a lamp bulb or from a mercury arc. Make the slit fairly wide to begin with so that there will be no difficulty in seeing it. Parallel rays of light coming from the collimator will then fall upon both faces of the prism and be reflected. Before trying to find these reflected rays with the telescope, try finding them with your eyes. You should be able to see a clear image of the collimator slit by reflection from either face of the prism. If you cannot find these images with your eyes, there is little chance that you will be able to do so with the telescope. Some adjustment of the position of the prism on the table may be necessary during this process.

Clamp the prism table so that it cannot turn with the telescope. Seek the two images of the collimator slit by using the telescope. These should be found with the telescope before any readings are attempted. Then set the cross hairs of the telescope on the center of the image of the collimator slit in one of the faces of the prism, *e.g.*, in position T. In order to get an accurate setting it is advisable to make the collimator slit very narrow. Read both verniers to the nearest minute of arc and record these readings. Distinguish between the two vernier readings by labeling them. Move the cross hairs off the collimator image by using the slow-motion screw of the telescope; then move them back on to the image. Again read and record the position of the telescope.

Next, unclamp the telescope and move it to the position T', setting the cross hairs on the other image

of the collimator slit. Repeat the process described in the foregoing paragraph. In this part of the experiment it is essential not to move the prism, prism table, or collimator. Why?

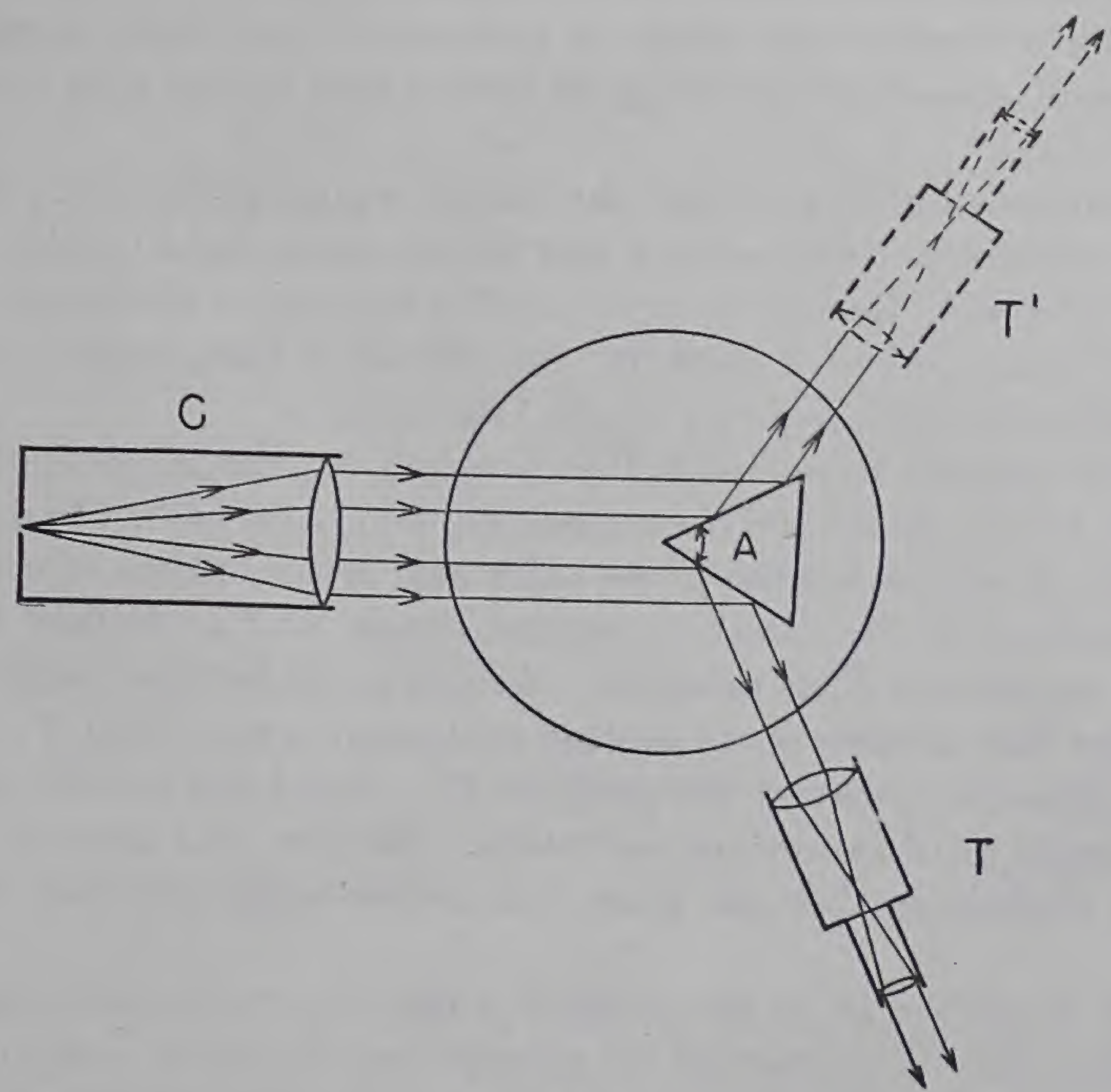


Fig. 74-2.

By subtraction determine the angle through which the telescope has been turned. This is twice the angle of the prism. Why? Make an estimate of the error involved in measuring the angle of the prism. NOTE: All prisms used in the laboratory are very nearly equiangular; hence your prism angle should be very nearly 60°. Use as angle A the angle in which the identifying number is scribed.

Part III. Measurement of Angle of Minimum Deviation. Set the prism on the spectrometer table with its refracting edge near the center of the table and with its base making an angle of about 30° with the collimator axis as shown in Fig. 74-3.

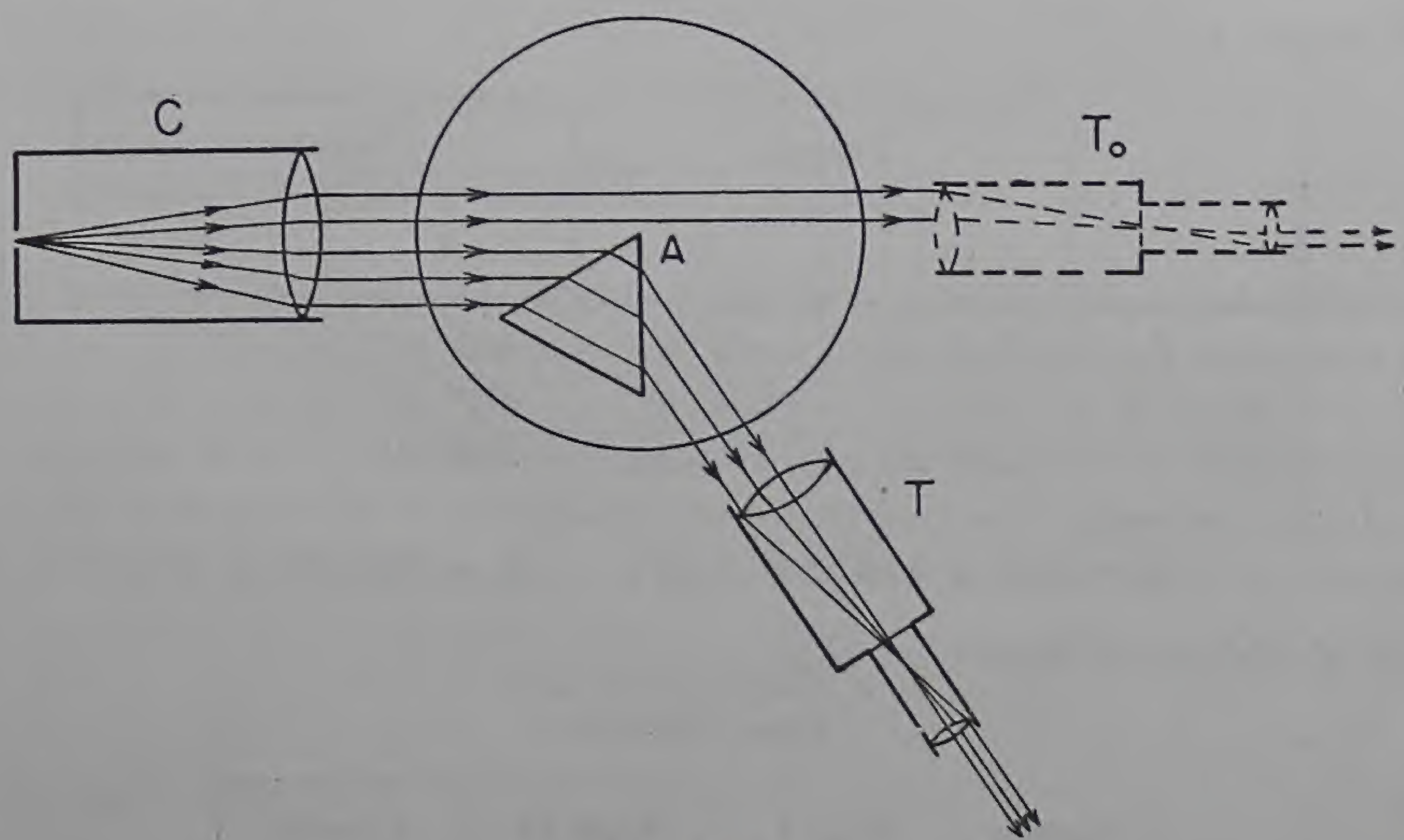


Fig. 74-3.

Illuminate the slit of the collimator with light from the mercury arc. Some of the parallel rays from the collimator will be incident upon the first face of the prism and be deviated in their passage through the

prism toward the telescope in position T. Others will miss the face and pass directly toward the telescope in position T_o, provided the first prism face does not extend too far across the spectrometer table.

With the collimator slit fairly wide try to find the image of the slit with your eye in position T_o, and then in position T. There will be only one image at position T_o but there should be a whole series of slit images at position T because of dispersion by the glass prism; that is, one gets the line spectrum of the mercury arc in this position.

Set the telescope in the position T_o and find the direct image of the slit. When this image has been found, make the slit of the collimator very narrow and set the cross hairs at the center of the slit using the slow-motion screw of the telescope. Read and record both verniers for this position of the telescope. Then move the cross hairs away from, and then back to, the center of this image. Again read and record the verniers. These readings give the zero position of the telescope.

Now swing the telescope around to position T and search for the green line of the mercury arc. This line is quite intense and can hardly be missed unless you happen to be color blind. Set the cross hairs of the telescope on this line. Then slowly rotate the prism table and prism in that direction which causes the green line to move toward the position of the direct or central image at T_o. Follow this line with the telescope until the line has moved as far toward T_o as possible. At this point further rotation of the prism table in the *same* direction will cause the line to reverse its motion and move away from T_o. Set the cross hairs of the telescope on the green line when it is nearest the position T_o. Read and record both verniers for this position of the telescope, *i.e.*, the position of minimum deviation. Repeat this process two more times. In each trial rotate the prism table slightly so that the green line moves away and then back to its position of minimum deviation.

The angle of minimum deviation D_m is the smallest angle obtainable between the two angular positions T_o and T of the telescope and may be computed by subtraction of vernier readings (*the same vernier*) for the two positions. Compute this angle and estimate the error in its determination.

By use of Eq. (5) compute the index of refraction for the green mercury line (λ = 5460.74 A). Use a five-place log table for this calculation. Compute the error in the index of refraction using the method given in the sample record.

Record: (Sample.)

App. Nos. Prism No. 8
Spectrometer No. 6
Mercury arc No. 5
Wave length (green line) = 5460.74 A

Part II. Prism Angle A.

	Vernier		Vernier	
	A	*B*	*A*	*B*
Left T	246° 13′	66° 3′	246° 15′	66° 2′
Right T'	126° 17′	306° 9′	126° 18′	306° 12′
Diff	119° 56′	119° 54′	119° 57′	119° 50′

Ave Diff = 119° 54′ ± 2′, A = 59° 57′ ± 1′

Part III. Angle of Minimum Deviation D_m.

Zero Position

Vernier	Trial I	Trial II	Average
A	193° 48′	193° 45′	193° 46′
B	13° 39′	13° 41′	13° 40′

DEVIATED POSITION

Vernier	Trial I	Trial II	Trial III	Average
A	245° 34′	245° 38′	245° 32′	245° 37′
B	65° 25′	65° 21′	65° 28′	65° 27′

DEVIATION

Vernier	Dev pos	Zero pos	D_m
A	245° 37′	193° 46′	51° 51′
B	65° 27′	13° 40′	51° 47′
		Ave 51° 49′ ± 2′	

$$D_m = 51° \, 49' \pm 2'$$

$$n = \frac{\sin \frac{1}{2}(D_m + A)}{\sin \frac{1}{2}A} = \frac{\sin 55° \, 53'}{\sin 29° \, 58'} \qquad \begin{array}{r} \bar{1}.91798 \\ \bar{1}.69853 \\ \hline 0.21945 \end{array}$$

$$n = 1.6575$$

In order to compute the error in n, we may proceed in the ordinary manner by developing the error equation. In this case the error equation is complicated by the presence of trigonometric functions. Those students who are familiar with the calculus may find it worth-while developing the error equation to fit this case. For the sake of variety we shall proceed in a different manner.

The errors in angles D_m and A are given as $\pm 1'$ and $\pm 2'$, respectively. In order to get the error in n we have only to recalculate the value of n using for D_m its average value $\pm 2'$, and for A its average value $\pm 1'$. This new value n' $(= n + \Delta n)$ minus the original value will then represent the error in n. The signs to be chosen for ΔD_m and ΔA should be such as to make n as large as possible. A little thought indicates that D_m should be increased by $2'$ and A decreased by $1'$. Hence

$$n + \Delta n = \frac{\sin 55° \, 53' \, 30''}{\sin 29° \, 57' \, 30''} \qquad \begin{array}{r} \bar{1}.91802 \\ \bar{1}.69842 \\ \hline 0.21960 \end{array}$$

$$= 1.6581$$

$$\therefore \Delta n = \pm 0.0006$$

In calculating the error in the result by this method it is essential that the calculations be made very carefully without arithmetical mistakes because the error is the difference between two relatively large numbers which are very nearly equal.

Finally it should be pointed out that the error obtained in this experiment is probably too small because no account is taken of the fact that the spectrometer was only approximately in adjustment. Lack of complete adjustment will probably increase the error, but it is difficult to say by how much.

QUESTIONS

1. Develop Eq. (1) using Fig. 74-1.

2. By use of the calculus show that D is a minimum in Eq. (1) when, and only when, $i_1 = i_2$.

3. In Part II (measurement of prism angle) it frequently happens that the observer is able to see with his eyes a clear image of the collimator slit by reflection from either face of the prism but is unable to "see"

the image by use of the telescope. By use of Fig. 74-2 explain this anomaly. (HINT: Suppose prism lies nearer collimator than shown.)

4. In Part III (measurement of angle of minimum deviation) the refracting edge of the prism is supposed to be placed near the center of the prism table. Why is this necessary?

5. By use of the calculus develop the error equation corresponding to Eq. (5).

6. The angle between T and T' in Fig. 74-2 is twice the angle of the prism. Prove this by use of a diagram and the laws of reflection.

Experiment 75.

Wave-length of Light. Diffraction Grating

Object: To measure the wave lengths of some of the prominent spectral lines in the mercury-arc spectrum with the diffraction grating.

Apparatus: Spectrometer, transmission diffraction grating, level, mercury arc.

Theory: A very useful device for the production of spectra and the measurement of wave length is the diffraction grating. It consists essentially of a series of narrow parallel slits very close together. When a beam of light is transmitted by such a grating, each of the apertures becomes the source of secondary waves according to Huygens' theory. These secondary waves recombine and by interference produce new wave fronts which form the spectrum of the incident beam.

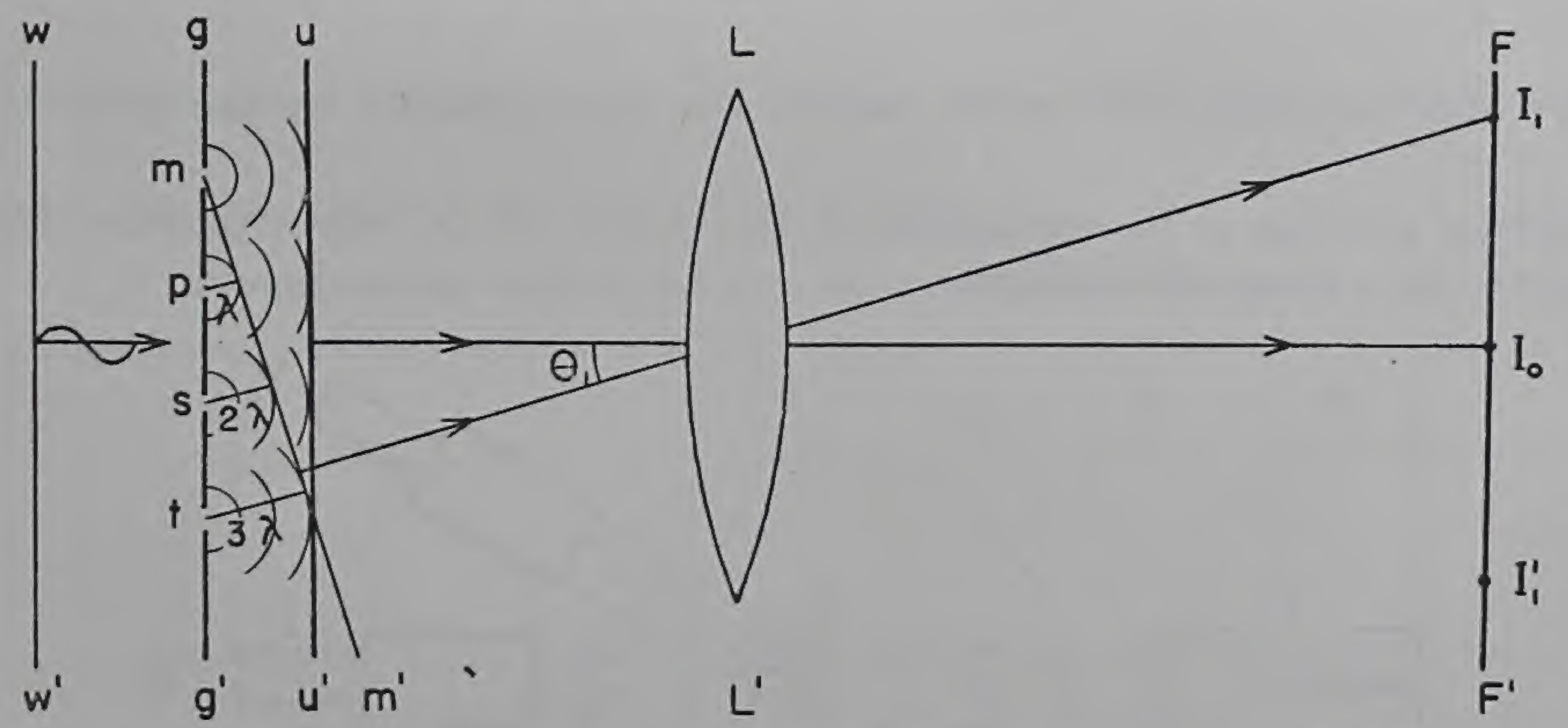


Fig. 75-1.

Figure 75-1 shows the action of the diffraction grating. Thus gg' represents a cross section of the grating, and the openings m, p, s, t, etc., represent the slits which may be thought of as extending at right angles to the plane of the paper. Let ww' represent a plane wave of monochromatic light falling upon the grating at normal incidence. When this wave front reaches the grating, the openings m, p, s, t become new sources of cylindrical waves whose circular cross sections are shown in the figure. If we draw the envelope to all these secondary waves after they have traveled a few wave lengths away from the grating, we obtain the plane wave surface uu' which we should have had even if the grating had not been present. The converging lens LL' will bring this plane wave to a focus at I_o (the central image) in the principal focal plane of the lens. Thus an image of the distant line source (illuminated slit) giving rise to the plane wave ww' will appear at I_o.

But uu' is not the only envelope that can be drawn in the figure. A surface mm' can also be drawn which is tangent to the secondary waves. It is so drawn that its distance from p is one wave length, from s two wave lengths, etc. Hence mm' may be regarded as another wave front which, after passing through the lens, will be brought to a focus at point I_1 in the focal plane of the lens. Thus an image of the source

(first-order image) will also appear at I_1. A similar argument shows that there should be another first-order image I_1' as far below I_o as I_1 is above it.

Additional wave fronts may be constructed, giving higher order images. For example, a second-order image may be formed by a wave front drawn from m and passing the opening at p at a distance of 2λ. In general the intensity of the images falls off rapidly as the order increases.

An examination of Fig. 75-1 shows that the wave front which forms the first-order image at I_1 makes such an angle θ_1, with the grating that the following equation holds:

$$\lambda = a \sin \theta_1, \tag{1}$$

where λ = wave length of the light,
$a = mp$ = grating space, and
θ_1 = angle between gg' and mm'.

Equation (1) may be generalized for any higher order image by writing

$$n\lambda = a \sin \theta_n, \tag{2}$$

where n is an *integer* representing the order of the image and θ_n is the angle between the plane of the grating and the wave front producing the image. Since rays are perpendicular to wave fronts, we may say that θ_1, for example, is also the angle between the set of parallel rays entering the lens and forming the central image at I_o and a corresponding set forming the first-order image at I_1.

By use of a spectrometer it is possible to measure the diffraction angles θ_n with considerable accuracy, and hence determine λ.

If the source of light is not monochromatic but emits light of various wave lengths, then the diffraction grating will sort out these wave lengths forming spectra of different orders. In particular, the light from a mercury arc when passed through a diffraction grating forms a distinctive line spectrum which may be analyzed.

Method: Before starting this experiment, adjust the spectrometer by the method outlined in Note L, Appendix II.

Place the diffraction grating in its mounting at the center of the spectrometer table with its plane as nearly perpendicular to the axis of the collimator as can be judged with the eye.

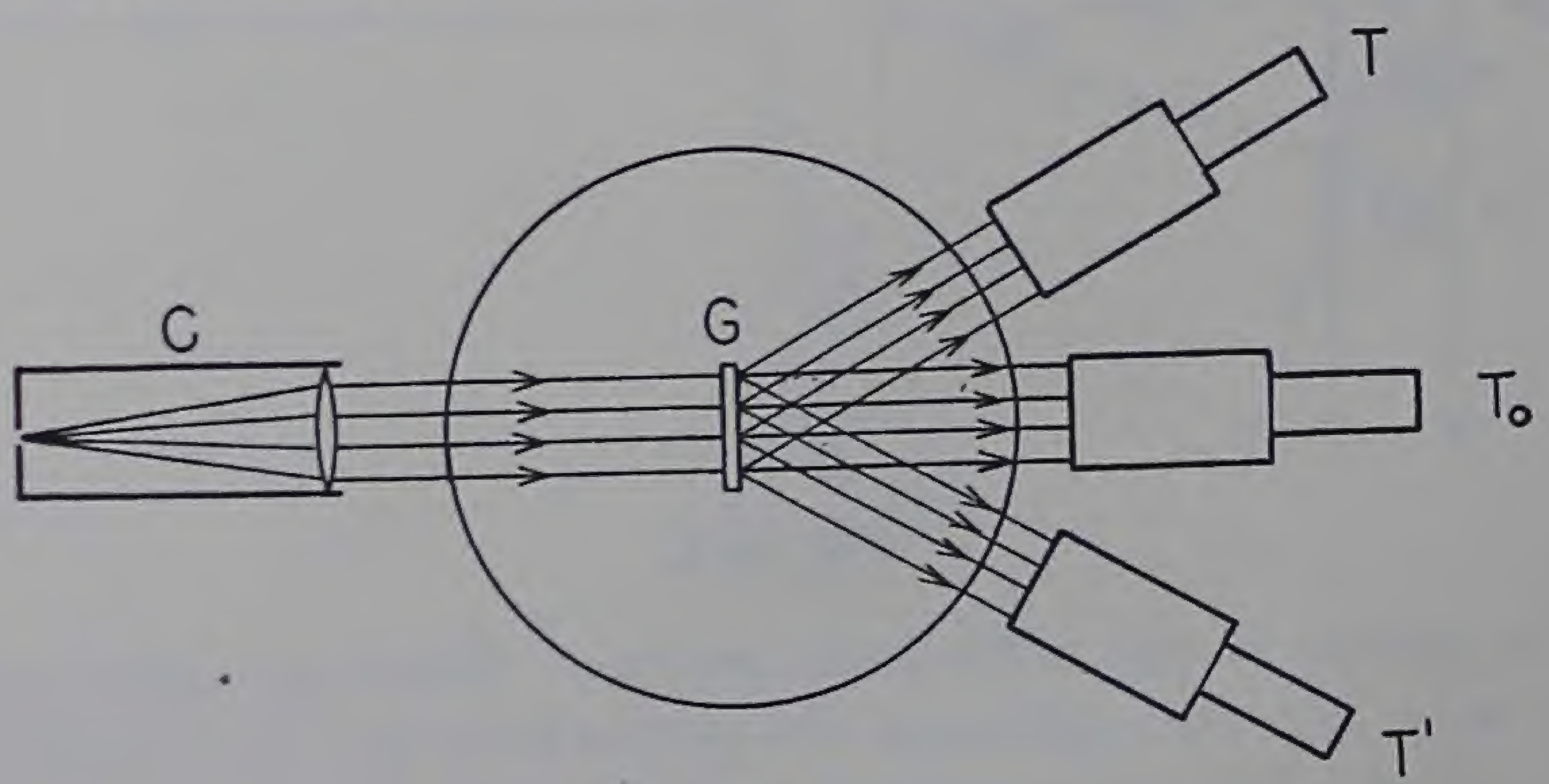


Fig. 75-2.

Illuminate the slit of the collimator with light from a mercury arc. Parallel rays of light will emerge from the collimator lens and fall perpendicularly upon the grating G, as shown in Fig. 75-2. These rays will be diffracted by the grating, forming bright line spectra of the mercury arc on both sides of the central image at T_o. Look for these spectra with the eyes before trying to find them with the telescope. It should be possible to see two or three orders of spectra. There are several rather prominent lines in the mercury spectrum: a bright yellow line (doublet), an intense green line, and several blue and violet lines, one of which is fairly intense. See Table H, Appendix III.

Swing the telescope into the position T, so that it is directed toward the *first*-order spectrum. Make the slit of the collimator quite narrow. Then set the cross hairs of the telescope in turn upon the brightest

violet line, the bright green line, each of the yellow lines (these are very close together and may not be resolved unless the collimator slit is very narrow). In each case record both vernier readings to the nearest minute of arc. Then rotate the telescope into the corresponding position at T' on the other side of T_o. Take a similar set of readings. Be sure to set the cross hairs on the same lines to right and left of T_o.

Repeat the above process for these same lines in the second-order spectrum.

In order to answer Question 5 it is necessary to take readings on the T_o position of the telescope. It is not necessary to use these readings in the main body of this experiment since the angle between the T and T' positions may be taken as twice the angle of diffraction in the case of each line.

Compute the wave length in angstroms of each of the mercury lines measured, using the number of lines per inch stamped on the grating mounting. In each case determine the amount of error in the wave length, using estimated errors in the diffraction angles. It is advisable to use a five-place log table to make the calculations. Errors in the wave lengths may be calculated as they were in Experiment 74.

Record:

Spectrometer No.________________

Grating No.________________

No. of lines per inch________________

Grating space in angstrom units $= \dfrac{2.540 \times 10^8}{\text{lines per inch}} =$ ________________

YELLOW LINE NO. 1–FIRST ORDER

Vernier	Pos T	Pos T'	θ_1	$\Delta\theta_1$
A				
B				
			Ave	

Make similar tables for the other lines and other orders.

	FIRST ORDER				SECOND ORDER				
Mercury lines	θ_1	$\Delta\theta_1$	λ	$\Delta\lambda$	θ_2	$\Delta\theta_2$	λ	$\Delta\lambda$	Ave λ
Yellow, No. 1									
Yellow, No. 2									
Green									
Violet									

Per cent difference between measured λ and standard value:________________

____

____

____

QUESTIONS

1. Show that the error equation corresponding to Eq. (2) is

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta a}{a} + \cot\theta_n\,\Delta\theta_n,$$

where $\Delta\theta_n$ is expressed in radians. It is assumed that there is no error in n, the order of the spectrum.

2. By use of the error equation given in Question 1 show that the wave length of a given spectral line can be determined most accurately by making observations on the image of highest order that appears in the spectrum.

3. Discuss the differences between the grating spectrum and the prism spectrum. Which color is deviated the most in each case?

4. What will be the angular displacement in minutes of arc between the lines of the sodium doublet in the second-order spectrum produced by a transmission grating having 14,500 lines per inch?

5. If the normal to the grating makes a small angle i with the axis of the collimator, show that a constant error is introduced which makes all calculated values of λ too large. The correction factor in this case is

$$\cos\left(\frac{i}{\cos\theta}\right) = \cos\left(\frac{\frac{1}{2}\delta\theta}{1 - \cos\theta}\right),$$

where θ is the average diffraction angle and $\delta\theta$ is the difference between the T_oT and T_oT' angles in Fig. 75-2. Compute this correction factor for the green line in this experiment. Is it significant?

Experiment 76.

Spectroscopic Analysis

Obiect: To calibrate a prism spectroscope and to determine the wave lengths of certain spectral lines.

Apparatus: Spectrometer, prism, mercury arc, bunsen burner, various salts.

Theory: When a beam of white light is sent through a prism of a spectroscope as shown in Fig. 74-3, it is dispersed into a spectrum, the violet rays (short wave lengths) being deviated most and the red rays (long wave lengths) being deviated least. This is due to the fact that the index of refraction of glass is a function of the wave length, decreasing with increasing wave length. This means that the deviation of a ray of light produced by a glass prism is a function of the wave length of the light. Therefore, the prism spectroscope may be used to determine wave lengths if it is first calibrated by sending rays of known wave lengths through it and observing their deviations. A graph showing the relation between these wave lengths and their corresponding deviations can then be used to determine any unknown wave length in terms of its deviation. The spectrum of the known wave lengths is called the comparison spectrum.

In this experiment the principal lines in the mercury-arc spectrum will be used as the comparison spectrum.

Method: Adjust the spectrometer by the method given in Appendix II, Note L. Use the mercury arc as a source and adjust the prism for minimum deviation of the green line in the mercury-arc spectrum (5461 A). Then clamp the collimator and prism table in position allowing only the telescope to rotate. These adjustments must not be altered during the remainder of the experiment.

Calibration of Spectrometer. Set the cross hairs of the telescope successively on the prominent lines of the mercury-arc spectrum, reading the position of the telescope in each case. Include in the telescope settings its zero position for the undeviated beam from the collimator. In making a setting, the collimator slit should be as narrow as possible and the cross hairs set on the center of the line. The mercury lines used should include the violet line (4358 A), the green line (5461 A), the yellow doublet (5770 A, 5791 A), and the orange lines (6152 A, 6232 A). Plot a graph with ordinates as wave lengths in angstroms and with abscissas as angular deviations. This constitutes the calibration curve for the spectroscope.

Determination of Wave Lengths. Remove the mercury-arc source and replace it with a bunsen burner and screen. Do not place the burner too close to the collimator slit. Place on the screen the salt to be analyzed and observe the spectrum emitted by the heated salt. Set the cross hairs of the telescope on all the lines emitted and compute the deviations for these lines. By use of the calibration curve determine the wave lengths of these lines. Determine the elements in the salt which give rise to these lines. Refer to Table H of Appendix III.

Obtain the absorption spectrum of a salt solution and estimate the wave lengths of the absorption bands by use of the calibration curve.

It will not be necessary to make an error analysis in this experiment.

Record: Tabulate your data and results. Answer Question 1.

QUESTIONS

1. With your calibration data from the spectrum of the mercury arc, plot a second graph with $1/\lambda^2$ as ordinates and deviations as abscissas. Discuss the advantages, if any, of this type of calibration curve.

Experiment 77

Photometer

Object: To measure the candlepower of an incandescent lamp; to determine its efficiency under various conditions; to measure the absorption coefficient of an absorbing screen.

Apparatus: Grease-spot photometer equipped with voltmeter (0—150) and voltage controls for standard and test lamps, *or* photovoltaic-cell photometer similarly equipped plus photovoltaic cell (Weston photronic cell) and reflecting galvanometer; standard lamp; unknown lamp; ammeter (0—1); absorbing screen.

Theory: A point source of light gives off equal amounts of light flux in all directions. The intensity (amount of flux per unit area) is evidently a function of the distance from the source, and in fact is *proportional to the inverse square of the distance.* This behavior is described by the following formula:

$$I = K\frac{CP}{r^2}, \tag{1}$$

where I = intensity of illumination,
K = proportionality constant,
CP = strength of the source in candlepower, and
r = the distance from the source to the surface illuminated.

With proper choice of units the proportionality constant may be made unity. A unit of intensity much used is the foot-candle, and is the intensity of illumination at a surface placed 1 ft from a standard candle—that is, a source with a strength of 1 candlepower. In this case Eq. (1) reduces to

$$I = \frac{CP}{r^2}. \tag{1'}$$

The strength of a lamp in a given direction is defined as the candlepower of a point source which would replace it, *i.e.*, which would give the same illumination in the same direction.

The photometer is an instrument designed to compare the strengths of two different sources. The method is generally to adjust their respective distances from a screen until the intensity of illumination from one source is equal to that from the other. One such type is the grease-spot photometer, in which the two sources are placed on opposite sides of a paper screen which has several grease spots on it. Light is transmitted through the paper more readily where it is spotted than where it is not, and thus if the screen is lighted from one side and viewed from the other, the spots appear brighter than their surroundings. On the other hand, if viewed from the same side as the illuminating source, the spots appear darker, since the light is transmitted rather than reflected. When, however, the screen is illuminated equally from both sides, the grease spots tend to disappear, since light transmitted from one side is counteracted by light transmitted from the other. In practice, arrangements are made to look at *both* sides at once, and since the two light sources usually differ slightly in color, adjustment is made to provide *equal contrast* between the grease spots and their surroundings on both sides of the screen. A simple method of viewing both sides simultaneously is illustrated in Fig. 77-1.

Another type of photometer utilizes a photovoltaic cell, whose electrical output is dependent on the intensity of the light falling upon its face. It is possible to calibrate such a cell with its associated meter to read the actual illumination intensity, and this is done in some types of "light meters" used in photography. However, in order to get results which are independent of the cell's output versus illumination-intensity characteristic, the cell should be used merely to detect a certain level of illumination intensity. In such use,

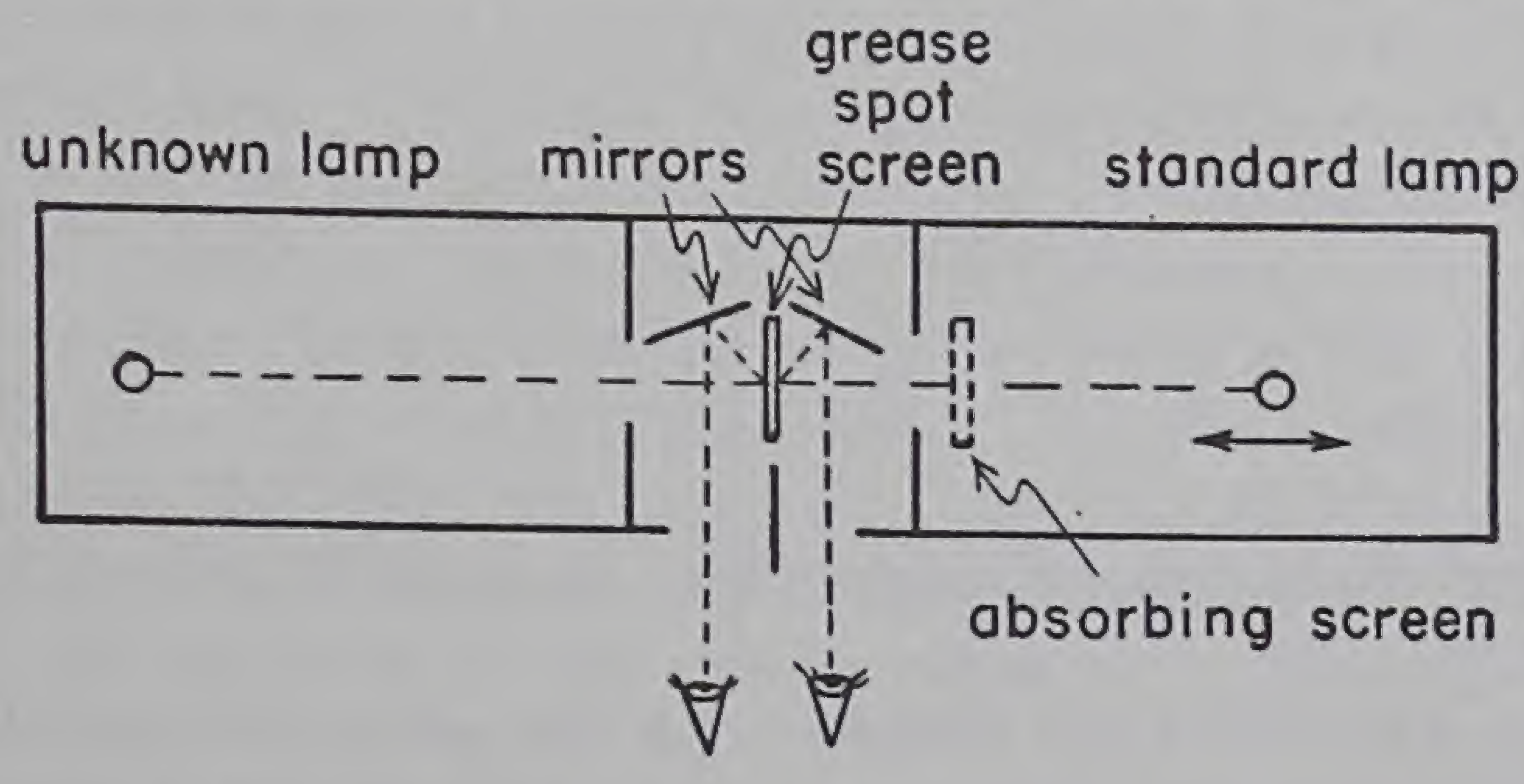


Fig. 77-1.

the relative distances of two sources from the cell are varied until the cell's output is the same in either case. The photometer may be constructed as in Fig. 77-1 with the photovoltaic cell mounted in the center in place of the grease-spot screen in such a way as to swivel to face either direction. Alternatively, the photometer may be one-half as long, with the cell facing only one way and the two sources to be compared using the same socket successively. The latter type is preferable because of its simpler construction, and Method B assumes its use.

If, in the following treatment, the subscript s refers to the standard lamp, against which we are to compare the unknown lamp designated by subscript x, it is clear that at balance for the photometer,

$$I_x = I_s. \tag{2}$$

That is to say, using Eq. (1′),

$$\frac{CP_x}{(r_x)^2} = \frac{CP_s}{(r_s)^2}, \tag{3}$$

or

$$CP_x = CP_s \frac{(r_x)^2}{(r_s)^2}. \tag{3′}$$

It is also clear that the ratio of the distances, squared, is unitless, and that therefore the distances may be measured in any convenient units.

An absorbing material has the property of reducing the intensity of light passing through it by a fixed *fraction* for a given thickness of the material, regardless of the original intensity. Thus, a given absorbing screen may pass 80% of all incident light. The intensity of the light as it passes through the material diminishes in accordance with the law

$$I = I_o e^{-as}, \tag{4}$$

where I = the intensity of light a distance s into the medium,
I_o = the intensity of the incident light,
e = the base of natural logarithms, and
a = the absorption coefficient.

It is readily seen that for a given thickness, t, the ratio of the transmitted light to the incident light is a constant:

$$\frac{I}{I_o} = e^{-at} = A. \tag{5}$$

If such a screen is placed between the standard lamp and the detector of a photometer, the intensity is now reduced and is given by

$$I_s' = A \frac{CP}{(r_s')^2}. \tag{6}$$

In order to find A and hence a with the grease-spot photometer, a balance is obtained between the standard lamp and the unknown lamp without the screen, in which Eqs. (2) and (3) hold. Then without changing I_x, another balance is obtained with the screen in place, at which time the following is true:

$$I_x = I_s'.$$

But then also,

$$I_s = I_s',$$

or

$$\frac{CP}{(r_s)^2} = A\,\frac{CP}{(r_s')^2},$$

or

$$A = \frac{(r_s')^2}{(r_s)^2}. \tag{7}$$

Then, knowing the thickness of the screen and using Eq. (5), it is possible to compute the absorption coefficient, a.

The procedure of finding A is somewhat simpler with the photovoltaic-cell type of photometer. Using either lamp, for convenience the standard lamp, toward the far end of the photometer, a reading is obtained on the galvanometer connected to the photovoltaic cell. The screen is inserted between the lamp and the cell and the lamp moved up to produce the same reading. Clearly then, $I_s = I_s'$, and the rest of the computations are as previously indicated.

One method of designating the efficiency of an electric lamp is to rate it in terms of candlepower strength per watt of electrical power consumed. Much of the radiation from an incandescent lamp is in the infrared, and as such is lost for lighting purposes. However, the hotter a body is, the greater is the fraction of its radiation in the visible region of the spectrum; and hence the efficiency of an incandescent lamp increases with its temperature. The total power dissipated, of course, increases with the temperature, and thus, in this experiment, the relationship of efficiency with power input is to be measured. The latter will be measured by finding the current and the voltage of the test lamp.

Method: *A. With Grease-spot Photometer.*

Examine the photometer, noting what adjustments and scales are available. The theory on which this experiment is based depends on the lamps being essentially point sources of light. Deviation from this state becomes appreciable (1%) if the illuminated source (filament) subtends an angle of more than about 10° at the screen. Thus ordinary 110-volt incandescent lamps should not be used closer to the screen than about 20 cm.

The electrical system consists of the following elements: main power switch, a voltmeter, a selector switch connecting the voltmeter across the terminals of either the standard or the unknown lamp, a rheostat in series with each lamp, and an ammeter in series with the unknown lamp.

Part I. Efficiency. Note the calibration of the standard lamp and the voltage at which it is calibrated; set this value by means of the voltmeter and the rheostat for the standard lamp. Record these values.

Set the voltage of the unknown lamp to 120 volts, and record this and the corresponding current. Adjust the position of the standard lamp to obtain balance at the screen. Attempt to get the *contrast* between the grease spots and the surroundings equal. Record the distances of the lamps from the screen. Move the standard lamp away, and adjust this distance for balance again, the other observer making the setting. Record this value.

Reduce the voltage at the unknown lamp by about 5 volts and check the standard-lamp voltage. Again record current, voltage, and standard-lamp distances. Repeat in steps of about 5 volts to the lowest voltage at which balance can still be obtained.

Part II. Absorption Coefficient. Place the absorbing screen in the space provided between the standard lamp and the screen. Bring the standard lamp to within about 25 cm of the oil-spot screen, and adjust the voltages of one or both lamps until a balance may be obtained with a small additional motion of the standard lamp. Record the voltages and this position, r_s'. Remove the absorbing screen, and adjust the *position*

of the standard lamp until a balance may again be obtained, but without changing the voltage of either lamp. Record this distance as r_s.

Again insert the absorbing screen, and change the voltage of one of the lamps so that the balance point is about 3 or 4 cm removed from its original point. Record the new values of voltages and r_s'. Remove the screen and find a new value of r_s without changing the voltages.

Set the voltage of the standard lamp to its *standard* value, and measure the strength of the unknown lamp at one of the voltages used in Part I, but with the absorbing screen in place. Record the voltages and the standard-lamp distance. With the vernier caliper measure the thickness, t, of the absorbing screen.

B. With Photovoltaic-Cell Photometer. Examine the photometer. The photovoltaic cell connects directly to a deflecting galvanometer whose resistance is of the order of 100 ohms, and whose sensitivity is of the order of 10^{-8} amp/mm/m. The technique of measuring light intensities is to obtain a *reference deflection* on the galvanometer from the unknown lamp, and then match it with an equal deflection from the standard lamp by moving the standard lamp to the proper distance from the cell. Before commencing the experiment, see to it that no light falls on the photovoltaic cell; then adjust the galvanometer to zero deflection.

Note what adjustments and scales are available. The theory on which this experiment is based depends on the lamps being essentially point sources of light. Deviation from this state becomes appreciable (1%) if the illuminated source (filament) subtends an angle of more than about 10° at the screen. Thus ordinary 110-volt incandescent lamps should not be used closer to the screen than about 20 cm.

The electrical system consists of the following elements: main power switch, a voltmeter, a rheostat in series with the lamp socket, and an ammeter in series with the socket.

Part I. Inverse Square Law. It will be of interest to perform an approximate check on this law under the circumstances of this experiment. Set the voltage at the lamp to its lowest value, and move the lamp to within about 20 cm of the photovoltaic cell. Adjust the voltage of the lamp until a large deflection is obtained on the galvanometer, about full scale. Record the voltage and distance of the lamp, and the deflection on the galvanometer. Now move the lamp 10 cm farther away, keeping the voltage constant. Read and record the galvanometer deflection. Repeat this process at distance increments of 10 cm until the lamp is as far away from the cell as the apparatus permits. Plot the galvanometer deflection against the reciprocal of the square of the distance. The plot should be a straight line for accurate inverse-square-law behavior. There are at least three factors present which would tend to prevent such ideal behavior. One of these is the fact that the output of the photovoltaic cell depends on the resistance in the external circuit, and is linear only for zero load resistance, being fairly linear for 100 ohms, and quite unlinear for 1000 ohms. The student should list two other factors causing nonlinearity.

Part II. Efficiency. Insert the unknown lamp in the socket and move it to the far end of the photometer from the photovoltaic cell. Adjust the voltage to the maximum value available (about 120 volts) and then move the lamp to a position where it gives a large deflection on the galvanometer. This should be more than half scale with the lamp in the farthest quarter of its travel. Record the galvanometer deflection, and the voltage and current in the unknown lamp, as well as its distance from the cell. Move the lamp away, and adjust for the reference deflection again, the other observer making the setting. Record this distance.

Reduce the voltage at the unknown lamp by about 5 volts, and record this and the corresponding current. Move the lamp toward the photovoltaic cell until the galvanometer deflection once more reaches the same value as it had earlier. Record the new distance from the cell as obtained by each observer. Repeat in steps of about 5 volts until the lamp is about 20 cm from the cell.

Remove the unknown lamp from the socket and insert the standard lamp. Note the candlepower of the standard lamp and the voltage at which it is calibrated; set this value by means of the voltmeter and rheostat. Record these values. Move the standard lamp until the deflection obtained on the galvanometer is the same as in the previous part of the experiment. Record this distance. Move the lamp, and adjust for the reference deflection again, the other observer making the setting. Record this value.

Part III. Absorption Coefficient. Place the absorbing screen in its place between the standard lamp and the photovoltaic cell. Bring the standard lamp to within about 25 cm of the cell, and adjust its voltage

to some convenient value which gives a large deflection of the galvanometer. Record the voltage and position of the standard lamp and the deflection of the galvanometer. Move the lamp away and then readjust its position to obtain the same deflection on the galvanometer, the other observer making the adjustment. Record this value of r_s' also. Move the lamp to the far end of its travel, and remove the absorbing screen. Adjust the position of the standard lamp to get the same galvanometer deflection. Repeat with the other observer making the adjustment. Record these values of r_s. Be sure the voltage at the lamp has not changed during these measurements.

Make a second trial, having changed the voltage of the lamp so that the distance r_s' is 3 or 4 cm different from what it was in the first trial for the same galvanometer deflection. With the vernier caliper, measure the thickness, t, of the absorbing screen.

A method which is more direct, but which assumes linearity of response of the photovoltaic cell–galvanometer system, is the following: set the standard lamp at such a distance from the photovoltaic cell as to give a large deflection, d_o, on the galvanometer. Record this deflection. Insert the absorbing screen between lamp and cell, and record the new galvanometer deflection, d. Since linearity of response is assumed, $d/d_o = I/I_o$, and Eq. (5) may be applied directly to find A and hence a. Compare these results with those obtained from the method of the previous paragraphs.

Methods A and B: *Computations. Efficiency.* For each setting, find the mean value of the distances, compute the strength of the unknown lamp, its power consumption, and its efficiency in candlepower per watt. Write the determinate-error equation corresponding to Eq. (3′) and find the indeterminate error, given that the calibration of the standard lamp is accurate to within ±3%. Obtain the indeterminate error in the efficiency. Plot the candlepower of the test lamp versus the power consumption. Plot the efficiency of the test lamp versus the voltage applied.

Absorption. Find the values for the fraction of light absorbed from the data of each trial in this part, and compute the corresponding values of the absorption coefficient. Find the mean value of each, and compute the indeterminate error in the values for A and for a, noting that the determinate-error equation corresponding to Eq. (5) is

$$\Delta a = -\frac{\Delta A}{tA}, \tag{5a}$$

assuming the error in t to be negligible. Under what circumstances would the use of this absorbing-screen technique be necessary?

Record: Tabulate data and results.

QUESTIONS

1. Light of intensity I passing through an absorbing sheet of material of infinitesimal thickness ds has its intensity reduced by an infinitesimal amount dI. It is found that

$$dI = -aI\,ds,$$

where a is the absorption coefficient of the material. By integrating this equation show that Eq. (4) is obtained.

2. Interpret the curves which you have drawn. What conclusion can be drawn as to the most efficient condition under which to operate a lamp?

3. What limits the voltage at which it is profitable to run a lamp?

4. Given a photovoltaic cell and a lamp socket at a fixed distance from it. Explain how the candlepower of an unknown lamp could be determined for various lamp voltages with this equipment and a standard lamp. (HINT: The standard lamp is used to calibrate this "light meter.")

Experiment 78.

Photoelectric Tube

Object: To study some of the characteristics of photoelectric cells.

Apparatus: Light-tight box containing the following: light source (lamp bulb with concentrated filament), photocell socket, provision for moving lamp bulb with respect to photocell, scale for measuring distance between light and cell, and filter holder; reflecting galvanometer; dial resistance box; potentiometer (200 ohms or over) with switch; potentiometer (100 ohms) with switch; 4-volt storage battery; source of 115 volts d.c.; vacuum phototube 929; gas phototube 930; voltmeter (0—3 volts, d.c.); voltmeter (0—150 volts, d.c.); set of Wratten filters selected from list in Table Q, Appendix III.

Theory: Certain metals emit electrons when light falls on their surfaces. This was first observed by Hertz in 1887 in his experiments on radio waves. He noted that a spark occurred across a spark gap in his receiver when ultraviolet light from his transmitter fell on the metal balls forming the gap. A year later Hallwachs was able to show that the spark occurred because the ultraviolet light caused the metals to emit negative electricity, aiding the spark to start. If such a sensitive metal surface is placed in an evacuated tube with a collector plate nearby, then completion of the circuit between emitter and collector outside the tube would permit the establishment of a current. In the conventional sense, this current is from collector to emitter inside the tube, but the electron current, of course, is from the emitter to the collector. If the collector is made positive with respect to the emitter, the current becomes larger with larger potential until a point is reached beyond which no further increase of potential causes an increased current. The collector is now receiving all the electrons emitted by the light-sensitive surface, the effect of space-charge having been overcome. (See Experiment 43 on the Vacuum Tube for information on this topic.)

If the tube is filled with an inert gas at a low pressure instead of being evacuated, the current will level off as before as the potential increases but will begin to rise again at higher voltages between emitter and collector. In this case, the electrons being drawn over by the collector gain enough energy to ionize gas molecules, which are then drawn to the negative plate of the tube. This adds to the net current in the tube, although the photo-emission current is not increased. If this ion current becomes heavy enough, the light-sensitive surface may be destroyed by the bombardment of positive ions.

The amount of electron current emitted by the light-sensitive surface varies directly with the intensity of illumination falling on it. This relation is exact over a very wide range of light intensities, from approximately 0.0001 to 10,000 foot-candles. Apparent deviations from this linear relationship may be caused by proximity of the sensitive surface to the walls of the tube and by space charges.

The electron current also depends on the wave length of the light falling on the emitter, and the dependency is not in general a simple one. Different metals have different spectral distribution of preferred frequencies, and the emission rises to peaks at these light frequencies. For each metal, however, there is a maximum wave length of light beyond which no emission occurs regardless of intensity. This *photoelectric threshold* depends essentially on the work function of the material of the emitter. According to Einstein, the energy, ϵ, of a photon of light is directly proportional to the frequency, ν, of the light. This relation is given by the fundamental equation

$$\epsilon = h\nu \quad \text{(ergs)}, \tag{1}$$

where h is Planck's constant. An electron, to escape from a metal surface, must obtain a certain minimum amount of energy, the *work function* of the metal. In order for light, therefore, to cause the ejection of any electrons from the surface, the frequency must be such that a photon of light has more energy to give up to an electron than the value of the work function of the metal. Any excess energy appears as kinetic energy of the ejected electron.

$$h\nu = w_o + \tfrac{1}{2}mv^2, \tag{2}$$

where w_o is the work function of the metal. Only light above a certain minimum frequency can do this, hence the existence of a photoelectric threshold. The threshold frequency is that which is just sufficient to eject an electron with no kinetic energy. Thus the threshold frequency, ν_o, is given by the relation

$$h\nu_o = w_o. \tag{3}$$

The peaks of emission current occurring at different light frequencies come about because of selective absorption; *i.e.*, the sensitive surface is not black, and hence does not absorb all wave lengths of light with equal ease. Hence, some phototubes have peak outputs for light in the infrared region of the spectrum, some in the ultraviolet, and some in the visible region.

Method: Wire the circuit as indicated in Fig. 78-1, but do not connect the 115-volt d.c. or the batteries until the instructor checks your circuit. Note that by means of the 100-ohm potentiometer fine control is available to apply from -2 to $+2$ volts to the photocell. With the 200-ohm potentiometer, any additional voltage from zero to $+115$ volts is available with coarser control. The dial box R is set to about the critical damping resistance of the reflecting galvanometer, approximately 300 ohms for a meter with about 100 ohms resistance and 10^{-8} amp/mm/m sensitivity. A reversing switch may be inserted in the voltmeter circuit for convenience in reading negative voltages. When the circuit is connected, adjust the voltage to zero, and with the photocell in total darkness, adjust the galvanometer deflection to zero, or note its reading and make a corresponding zero correction on all following readings.

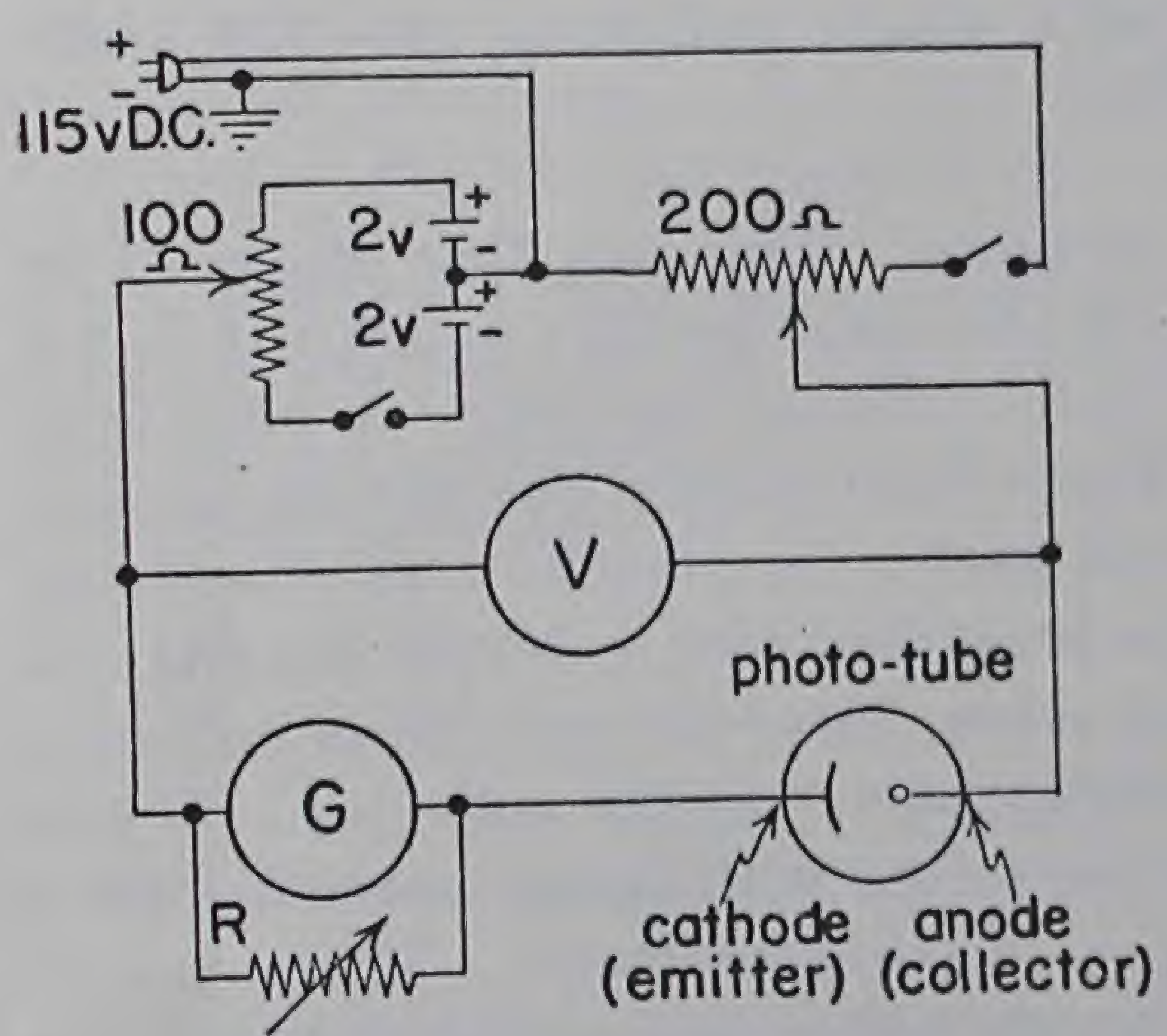


Fig. 78-1.

Part I. Output versus Intensity. Place the *vacuum-type photocell* in the socket, and adjust the distance between the photocell and the light source to about 20 cm. Adjust the voltage of the light source to obtain a large deflection on the galvanometer (nearly full scale) when the photocell voltage is 100 volts. Record the voltage and distance of the light source and the galvanometer deflection. Increase the distance between photocell and light source by an amount equal to about one-eighth the total motion available, and with the light-source voltage the same as before, record the new distance from the photocell and the resulting deflection of the galvanometer. Repeat at similar distance increments until the photocell and light source are as far apart as the equipment permits.

Decrease the voltage of the light source, adjust the photocell voltage to 40 volts, and repeat the previous procedure with the gas photocell instead of the vacuum photocell.

For each tube, plot galvanometer deflection versus the reciprocal of the square of the distance between cell and light source. The plots should be straight lines if the output values of the photocells are directly proportional to light intensity.

Part II. Photocell Current-Voltage Characteristics. With the gas photocell in place and with the light source in some medium position and at the brightness used in Part I, set the photocell voltage at -2 volts and read and record the galvanometer deflection (probably zero). In 0.4-volt steps, change the photocell voltage up to plus 2 volts, recording the galvanometer deflection for each voltage. Take further readings at the following voltages: 3, 5, 7, 10, 15, 20, 30, 40, 50. Take readings at 60 and 70 volts if the galvanometer remains on scale, increasing the voltage cautiously while watching the galvanometer. Reduce the photocell

voltage to −2 volts, and replace the gas phototube with the vacuum phototube. Take readings as before, each 0.4 volt between −2 and +2 volts. Take further readings at the following voltages: 3, 5, 7, 10, 15, 20, 30, 50, 100.

Plot galvanometer deflections versus phototube voltage for each cell. Discuss each section of each curve in terms of the theoretical aspects involved.

Part III. Spectral Distribution of Response. Move the light source as far from the photocell as the apparatus permits, and adjust it to its rated voltage. With the *vacuum phototube* in place, and with 100 volts applied to it, note the galvanometer deflection. Adjust the distance of the light source until the galvanometer registers nearly full-scale deflection. Insert one of the Wratten filters between the light source and the phototube and note the galvanometer deflection. Repeat with each filter in turn, recording the deflection for each filter.

In order to get a satisfactory measure of phototube response to light of different wave lengths, it is necessary to correct the galvanometer deflections at the various wave lengths for certain frequency-dependent factors which affect the photo output. These factors are (1) the spectral intensity distribution of the light emitted by the source, (2) the width of the band of wave lengths transmitted by the filter, and (3) the transmission coefficients of the filter at these wave lengths. To make such a correction it is only necessary to divide each galvanometer deflection by a factor which standardizes the deflection to a value it would have if the following were true: (1) intensity distribution of light uniform over the spectrum, (2) transmission band widths of all filters the same, and (3) transmission coefficients of all filters the same. These standardized deflections are then proportional to the spectral sensitivities of the tube at the corresponding wave lengths.

Refer to Table Q, Appendix III, and record the wave lengths and standardizing factors for the filters used. Standardize your galvanometer deflections and plot the standardized deflections against the wave lengths. Discuss the implications of the resulting curve. From the curve, determine the approximate cutoff wave length, and from it determine the work function of the emitter surface in electron volts. (See Table L, Appendix III, for values of constants and energy conversion factors.)

Record: Record data and results in tabular form.

546 546

Appendix I.

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Appendix II.

Notes on Equipment

A. The Vernier

1. The vernier is a convenient attachment for determining accurately a fraction of the finest division on the main scale of a measuring instrument. A portion of a typical main scale of an instrument without a vernier is shown in Fig. A-1. Usually the main scale is fixed in some manner and a sliding index indicates the position on the scale corresponding to the measurement in question.

In Fig. A-1, the finest division on the main scale is one-tenth of a centimeter, and by the position of the index it is known that the measurement in question is between 2.3 and 2.4 cm. We can *estimate* the fraction

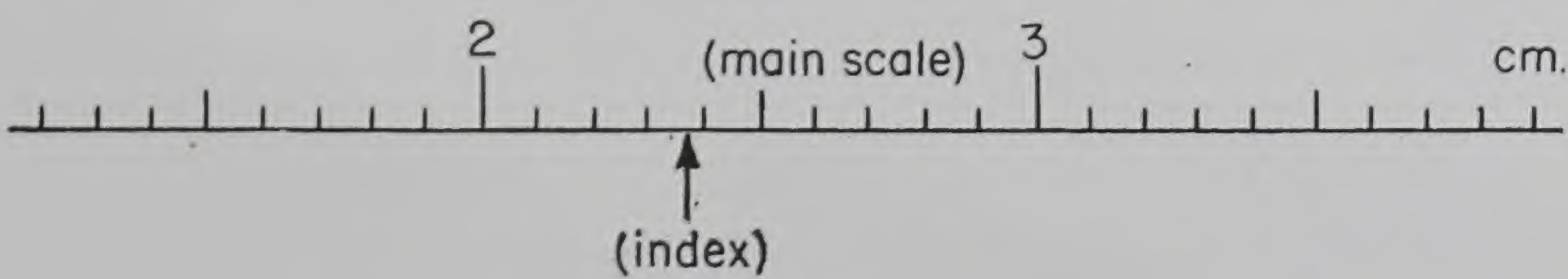


Fig. A-1.

of the division to be seven-tenths so that the measurement is 2.37 cm, with the first decimal place known accurately and the second one estimated.

The vernier scale is an auxiliary to the main scale, and its divisions are different from those of the main scale but related to them in a simple manner. The zero mark of the vernier scale takes the place of the simple index in the illustration above. However, by the use of the vernier scale, the fraction of the smallest division on the main scale can be read accurately.

In Fig. A-2 is shown the same main scale as in Fig. A-1 with the sliding index now replaced by a vernier scale. Note that the same measurement (2.37 cm) is indicated.

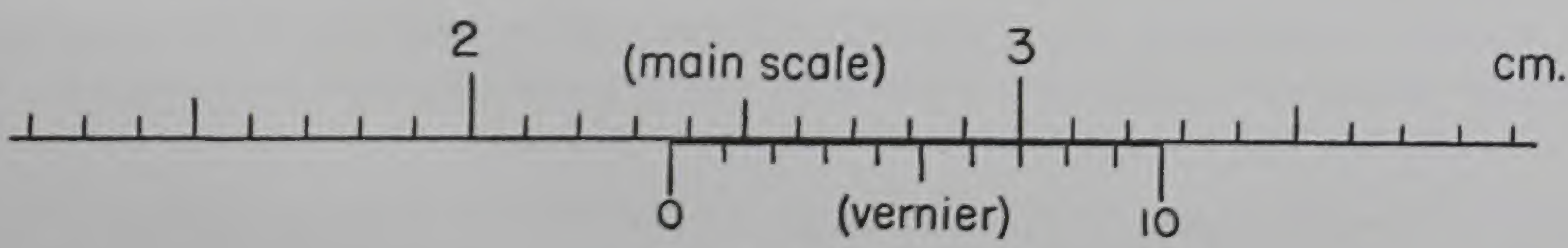


Fig. A-2.

In the case of a centimeter scale such as the one shown, the most convenient fraction of the smallest division of the main scale is a tenth. In order to make such a measurement, the vernier scale is so graduated that ten of its divisions correspond exactly to nine of the main scale divisions. That is to say, each vernier division is only nine-tenths as long as a main scale division. In Fig. A-3 we see the vernier with its number-one mark exactly corresponding to a mark on the main scale (in this case the 2.8-cm mark). Since the vernier division is only nine-tenths as long as a main division, the vernier's *index* must be one-tenth of a main division to the right of the 2.7-cm mark on the main scale. Remembering that the vernier index shows the actual measurement, this reading must, therefore, be 2.71 cm.

Now if the sliding vernier were moved another tenth of a main division to the right, the vernier's number *two* mark would be exactly opposite a mark on the main scale. (*Which* mark on the main scale, of course, does not matter, since it is the position of the vernier *index* which gives the reading.) This means now that the vernier index is exactly two-tenths of a main scale division to the right of the 2.7-cm mark on the main scale. The actual reading, therefore, is now 2.72 cm.

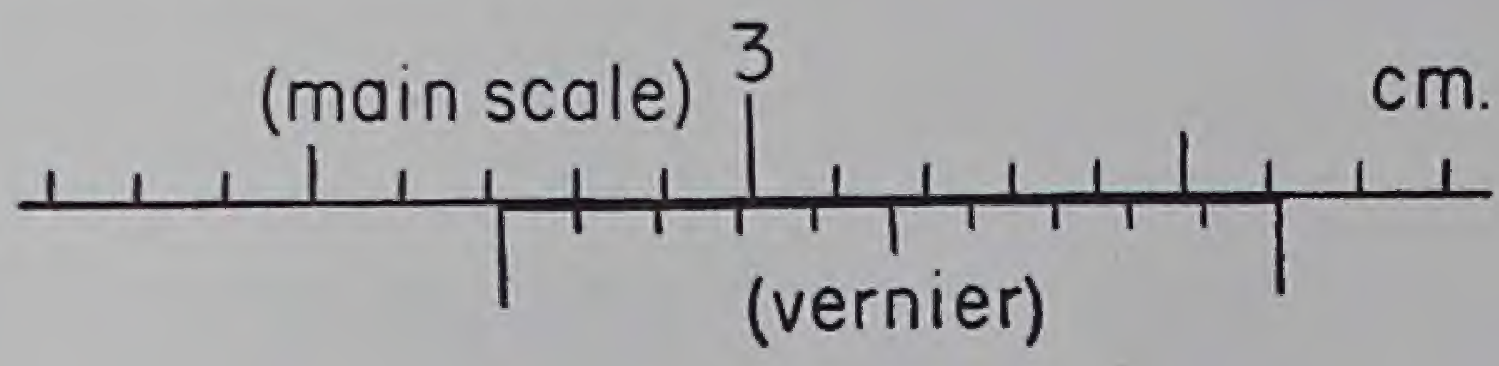


Fig. A-3.

Now the difference in length between a main division and a vernier division is called the *least count* of the vernier. In the case just studied the least count was 0.01 cm. *It is also clear that the closest measurement that can be made accurately* (coincidence of a vernier line and a main line) *is just the least count.* We have seen that when the number-one mark of the vernier is exactly opposite a mark on the main scale the vernier *index* is a distance equal to the least count to the right of the next lower main scale mark; when the number-two mark of the vernier coincides with a main scale mark the index has moved two times the least count; and so on. Thus, by noting which vernier line coincides with a main scale mark we know immediately what fraction of a main division the vernier index has moved.

Now, looking back at Fig. A-2, which shows the same reading as Fig. A-1, we can read accurately that the measurement is 2.37 cm where now the last figure is read directly, not estimated.

2. *Vernier Caliper.* In Fig. A-4 is shown an ordinary vernier caliper, measuring the diameter of a cylinder which is placed between the jaws of the caliper. The left-hand jaw is fixed to the main scale (a)

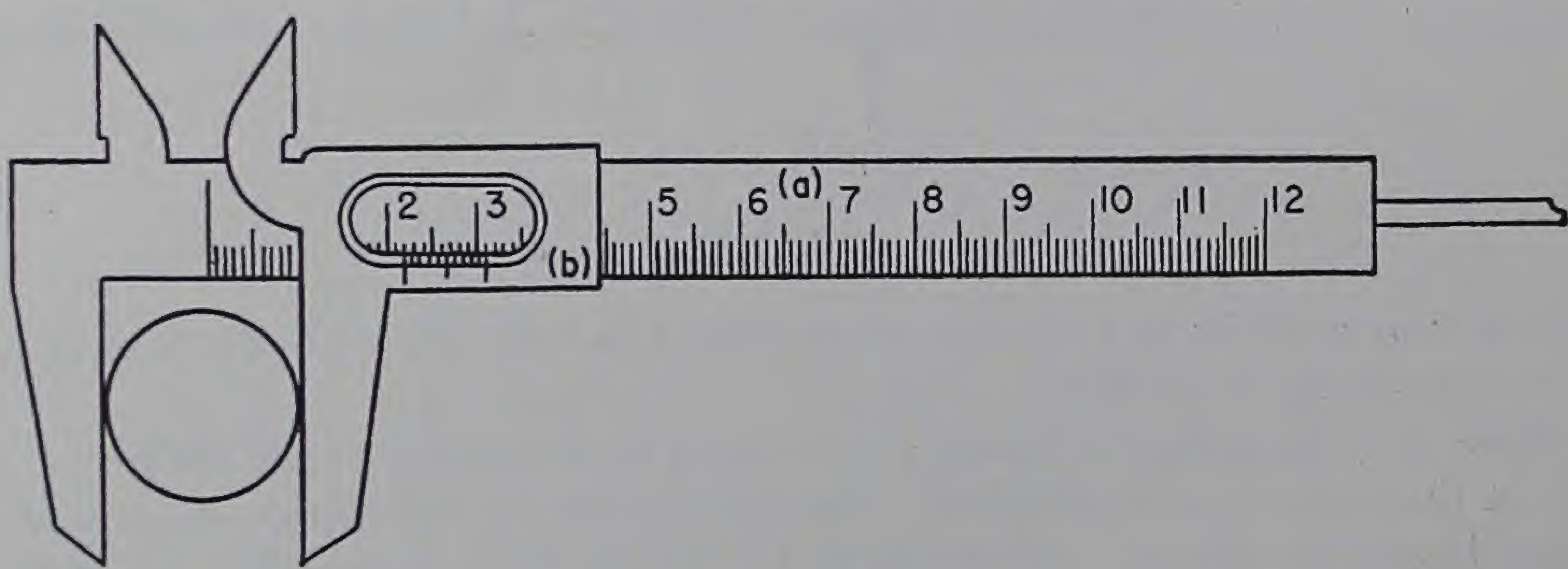


Fig. A-4.

and is perpendicular to it. The right-hand jaw is parallel to the left-hand jaw, but can slide along the main scale and carries with it the vernier scale (b).

When the two jaws touch each other, the index of the vernier scale should be exactly coincident with the zero mark on the main scale. The two upper jaws are for measuring *inside* dimensions of a cavity, while the protruding shaft at the right is for measuring the depth of holes.

3. *Zero Error.* The actual dimension of an object will be the difference in readings when it is measured and when the calipers are closed. Ideally the caliper will read exactly zero when closed; if it does not there is a *zero error* which must be taken into account. This zero error will, of course, be the same for all measurements and must be subtracted from all readings. If, with the jaws closed, the reading is positive, the zero error is said to be positive.

4. *Angular Measurement.* The verniers used on instruments which measure angles are constructed in the same manner as the one described above. A common main scale on a spectrometer is one which is marked every third of a degree. The vernier scale may run from minus two divisions up to 20 divisions with each half division marked. With such an arrangement, the vernier scale from zero to 20 covers exactly 13° on the main scale. That is to say, 40 vernier divisions equal 39 main divisions, so the least count is 1/40th.

Since each main scale division is 20′ of arc, each vernier mark will then indicate 1/40th of this or $\frac{1}{2}'$. Thus the vernier will read directly in minutes and half minutes if it is numbered from zero to 20. The negative 2′ are for convenience. Note that the vernier reading must be added to the reading of its index. That is, if the index shows 65° 20′ plus, and the vernier shows $12\frac{1}{2}'$, the measurement is 65° $32\frac{1}{2}'$.

B. The Micrometer

1. In its simplest form, the micrometer is simply a screw, very accurately made so as to be uniform along its entire length, moving in a fixed nut. The pitch of the screw is known, and the head of the screw moves forward by the amount of this pitch for one complete revolution. A scale is attached to the screw like a wheel to an axle, and by means of it the fraction of a revolution can be measured. (See Fig. A-5.) For

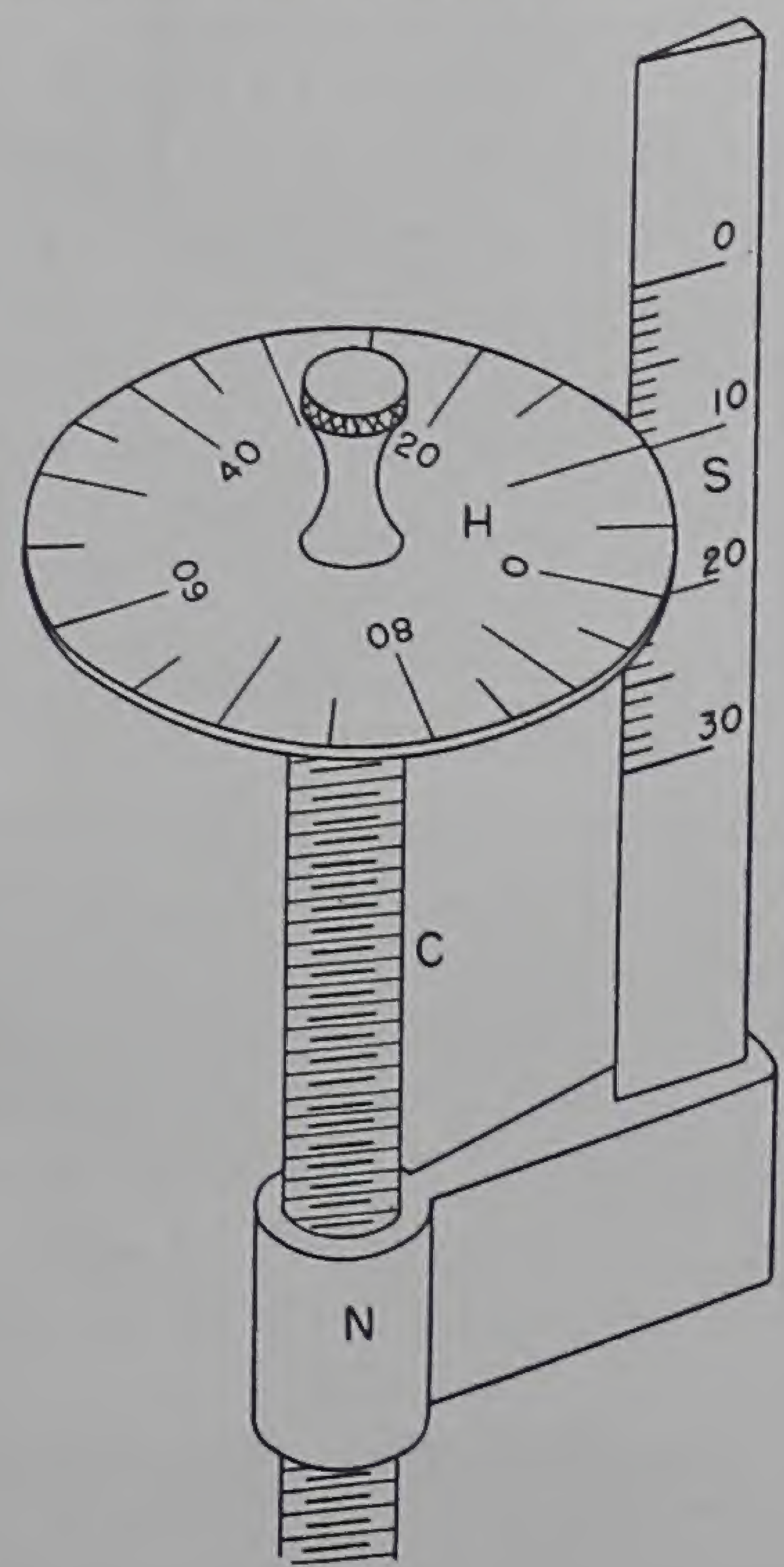


Fig. A-5.

instance, if the scale is divided into 100 parts, then turning the screw and the scale through one such part will advance the screw 0.01 of its pitch. A fixed scale is usually mounted on the nut in which the micrometer screw turns, to count the number of whole revolutions. The moving scale is called the "micrometer head."

2. *Micrometer Caliper* (Fig. A-6). The micrometer caliper is a modification of the simple micrometer principle. The fixed nut and scale are part of the frame F. The knurled knobs K and K' and the moving

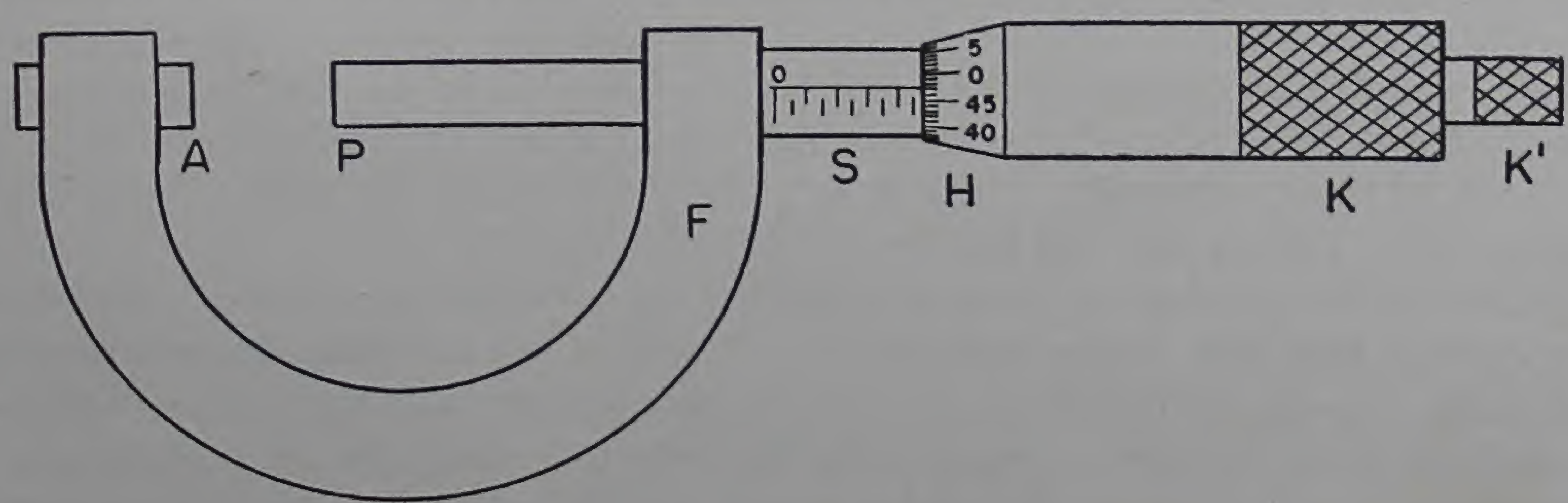


Fig. A-6.

scale H are mounted on the accurate screw. The pitch of the screw in the better calipers is $\frac{1}{2}$ mm. That is, one complete turn of K will advance the point P of the screw half a millimeter toward the anvil A, which is a part of the frame, and may or may not be adjustable.

On a micrometer with this pitch, the movable scale H is divided into 50 parts, so that each part represents 1/50th of $\frac{1}{2}$ mm, or 0.01 mm. In other words, in *two* complete turns, the moving scale turns past 100 divisions, and advances the screw 1 mm. To indicate half a millimeter (one turn) the fixed scale S has alternate marks set below the full millimeter marks. See the sketch Fig. A-7. In this sketch the movable

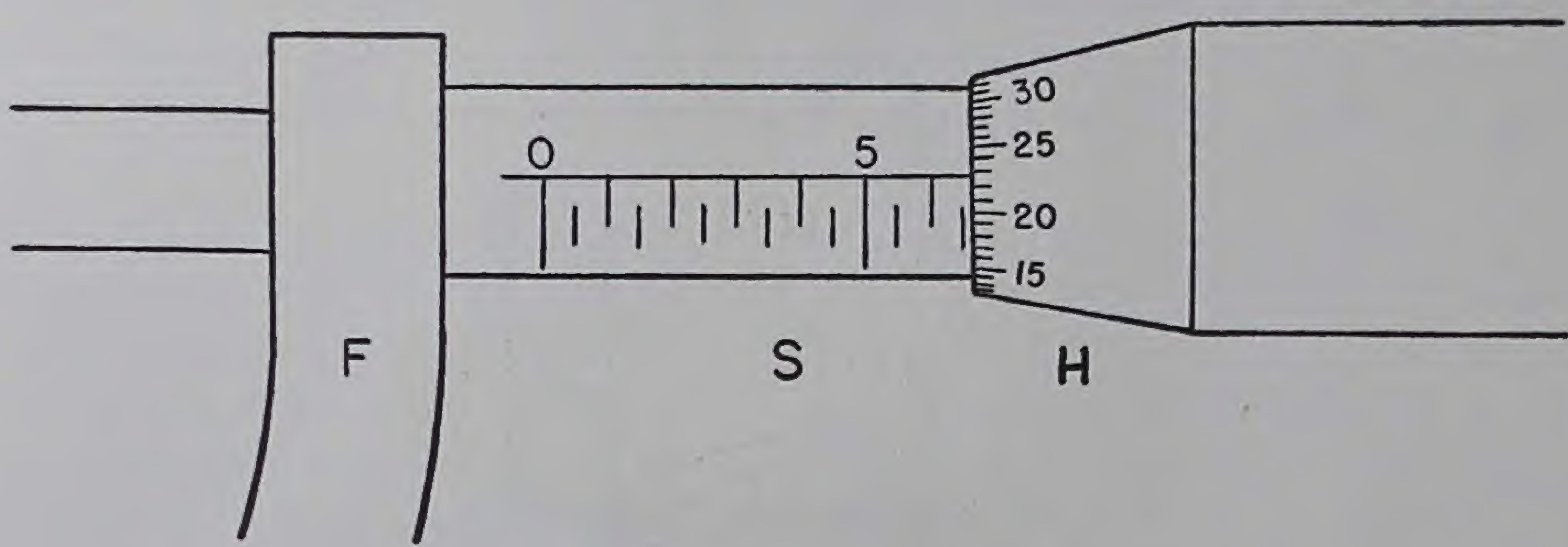


Fig. A-7.

scale has turned beyond a half-millimeter mark. The reading (remembering to estimate one-tenth of the smallest scale division) is 6.727 mm.

The zero error must be subtracted from the reading made with the calipers. This error is determined by closing the caliper—bringing the point P of the screw in contact with the anvil A. If the reading of the scales is positive, the zero error is positive.

In making any measurements, the knob K' is to be used so as not to force the instrument. This knob will continue to turn after the screw stops, with a little friction or with the clicking of a ratchet. Closing the jaws on the measured object *gently* until this point is reached will ensure the same force being applied at each measurement, and prevent damage to the screw and jaws. The object measured should be held loosely so that it will be able to align itself perpendicularly to the jaws.

C. The Traveling Microscope

The traveling, or micrometer, microscope is used to measure short distances with accuracy. It consists of a low-power microscope mounted on a moving chassis which slides along a carefully constructed track. See Fig. A-8. The sliding motion is produced by a micrometer-screw arrangement. (See the preceding section, Appendix II-B.)

In the usual case the pitch of the thread is $\frac{1}{2}$ mm, and the movable scale (the "micrometer head") has 100 divisions. Thus to move the microscope a full millimeter, the head must be turned twice, or past 200 divisions. This imposes special care on the user, since each division has a value of 1/200th of a millimeter; it means that the reading on the micrometer head must be divided by 2 to get the motion in hundredths of a millimeter. The index mark M will indicate on a fixed scale the travel in full millimeters, and its position must be noted carefully to find whether the micrometer head is in the first or second turn of the millimeter in question. The reading of the index in Fig. A-9 is 14.5 plus. The micrometer head indicates 63.3% of a revolution, or 31.65 hundredths of a millimeter. The index shows the head to be in the second revolution, so our reading, upon adding the two figures, is 14.8165 mm.

The main precaution to be taken in using this instrument concerns *lost motion*. The screw is purposely cut so that there is some free play in the instrument. It will be noticed that the micrometer head may be turned back and forth through several divisions without causing any motion of the chassis. It is obvious, therefore, that readings must always be taken *with the screw turning in the same direction*. Whether this is to be clockwise or counter-clockwise is decided by noticing which direction gives the closest check on zero. That is to say, when the micrometer head is on zero, the index M should be exactly on a line or halfway

between two lines of the fixed scale. In taking a reading, if the screw is accidentally turned too far, back it off about a turn, and then approach the setting from the same side as before.

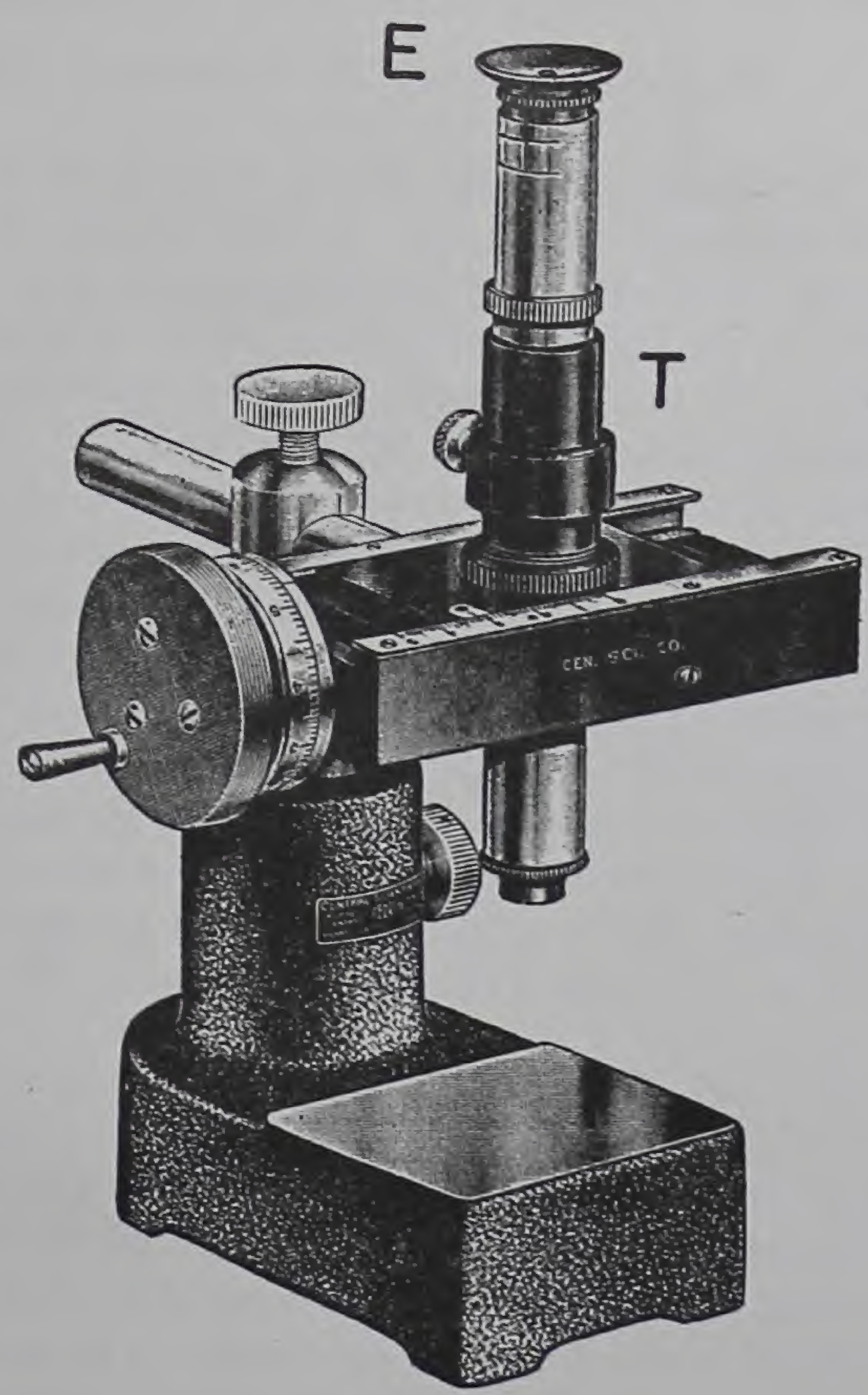


Fig. A-8.

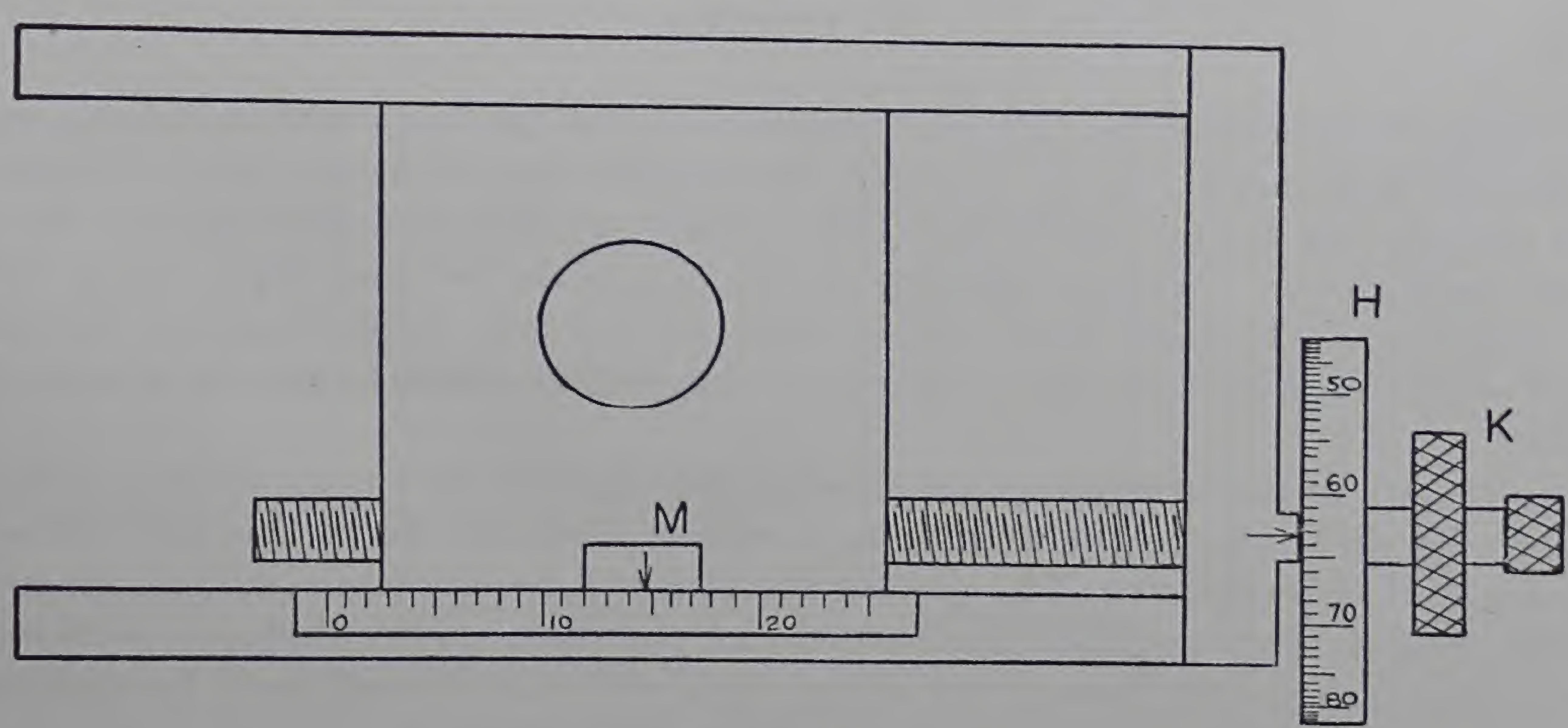


Fig. A-9.

Do not handle the micrometer head *H*, but always turn the screw by means of the knurled knob *K*.

In using the microscope, the eyepiece *E* (Fig. A-8) must first be adjusted so that a sharp image of the cross hairs will be formed. Then the main tube *T* must be moved up and down (moving the whole microscope) until the point under observation is in good focus. This can be checked by seeing whether there is

any parallax, that is, whether the cross hairs seem to move with respect to the image when the eye is moved from side to side. If such parallax is found, it must be eliminated by further adjustment of the eyepiece, and by refocusing the microscope. See Note F.

D. The Mercury Barometer

The mercury in the mercury barometer partially fills an evacuated glass tube and a reservoir which is open to the air. The atmospheric pressure is balanced by the column of mercury, and the height of this column is, of course, measured from the surface of the mercury in the reservoir. This surface must be brought to a fixed known height in order for the calibration of the column to be accurate. To do this, a small ivory pointer is provided above the mercury surface in the reservoir, and an adjusting screw, located below the reservoir, will raise or lower the mercury in the reservoir.

To use the barometer, first turn this adjusting screw until the surface of the mercury is just at the level of the tip of the ivory pointer. This has been accomplished when the point and its image in the mercury appear just to touch each other. Then stand with the eye just at the level of the top of the mercury column and adjust the movable tube surrounding the column by means of the knob at the right-hand side of the barometer, until the bottom of the tube is just at the level of the meniscus of the mercury; that is, until light from behind the barometer is just shut off by the tube's edges coming to the level of the mercury. A vernier attached to this moving tube will then allow the height to be read to the twentieth of a millimeter.

For accurate work, the effect of temperature on the density of mercury and on the length of the scale must be taken into account. To reduce the readings as obtained by the above directions to standard conditions (the barometer is calibrated at 0°C) use Table F in Appendix III.

E. The Laboratory Timer

The laboratory timer in use at the Physics Department of the University of Minnesota is controlled by a synchronous electric motor which drives a set of contacts. In the laboratory is a telegraph sounder which clicks each 30 sec. A warning, which starts 2 sec before each click, is given by a light which flashes at half-second intervals, the fifth flash coinciding with the click. The accuracy of the system, it should be noted, depends only on the frequency control of the electric power. The error is probably less than 0.05%.

F. Parallax

This phenomenon is defined as the apparent displacement of one body with respect to another when the position of the *observer* is changed. As an example, two pencils may be held at arm's length in line with one eye but with one pencil a few inches behind the other. Hold one with the point up, and the other above and behind it with the point down. Without moving the pencils, move the head from side to side. The pencils appear to shift with respect to each other, but as they are brought closer together the shift becomes less pronounced, until when one is actually above the other, they will continue to appear so regardless of the angle from which viewed.

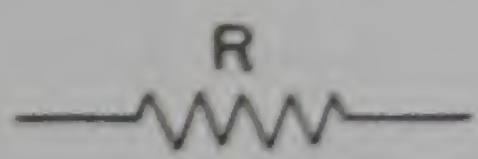
Parallax may be put to use in reading instruments like electric meters in which a pointer indicates the reading by means of a dial. Since the pointer is a certain distance above the dial, the reading obtained depends on the angle of observation. The correct reading will be the one obtained when the line of sight passes through the pointer perpendicular to the plane of the dial. Some dials are equipped with mirrors. The perpendicular line is then easy to find: the line of sight will pass through both the pointer and its image, that is, the pointer appears to be directly above its image, and covers it.

In optical instruments parallax may be used to determine when two images are in the same plane, or when a set of cross hairs is accurately placed in the plane of an image. A motion of the head back and forth should not produce any relative motion of the two images.

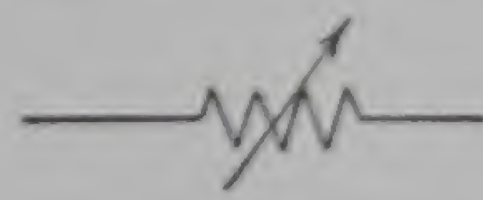
G. Electrical Circuit Elements

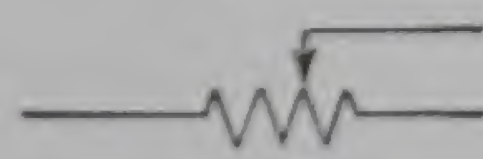
1. *Standard symbols* for the various elements encountered in electric circuits are given in the table below:

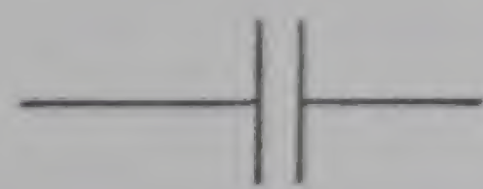
Connecting wire (negligible resistance) . . . . ————

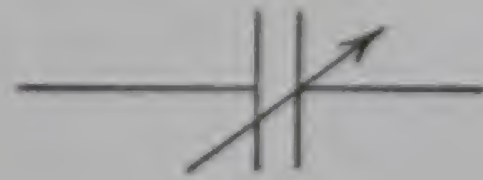
Resistance (fixed) . . . . . 

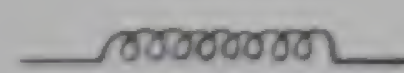
(NOTE: Symbol "Ω" stands for "ohms.")

Variable resistance (two terminals) such as a dial box . . . 

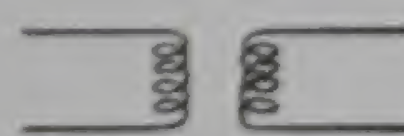
Rheostat or potentiometer (three terminals) . . . . 

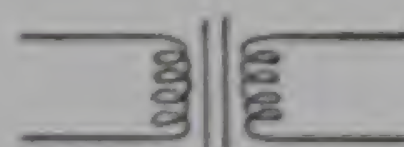
Capacitance (fixed) . . . . 

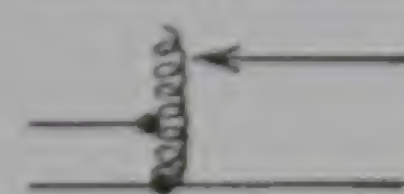
Capacitance (variable) . . . . 

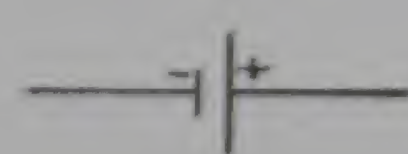
Inductance . . . . 

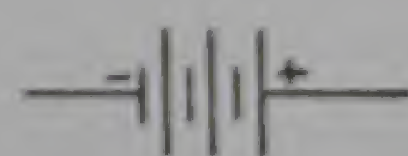
Inductance (iron core) or choke . . . . 

Transformer (or mutual inductance) . . . . 

Transformer (iron core) . . . . 

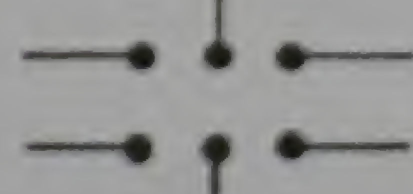
Autotransformer . . . . 

Voltaic cell . . . . 

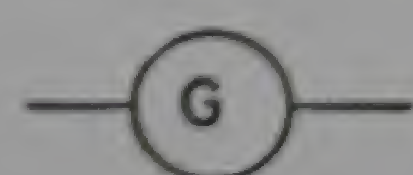
Battery (two or more cells in series) . . . . 

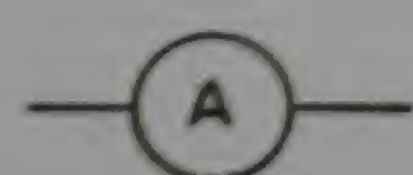
Fuse . . . . 

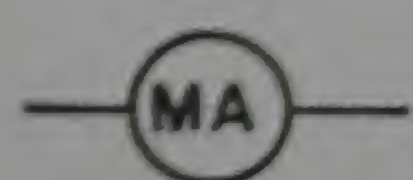
Switch [single-pole single-throw (SPST)] . . . . 

Switch [double-pole double-throw (DPDT)] . . . . 

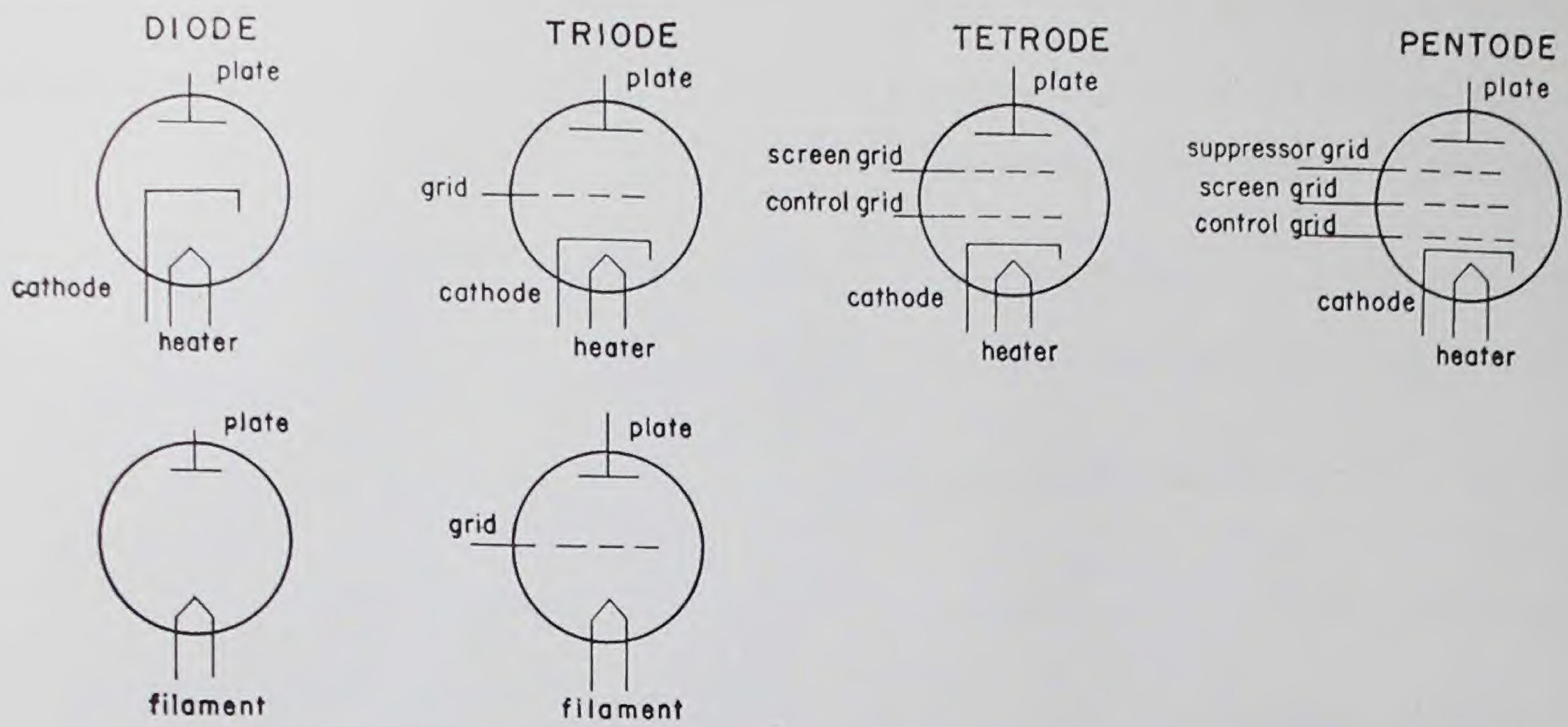
Tap key (SPST momentary contact) . . . . 

Galvanometer . . . . 

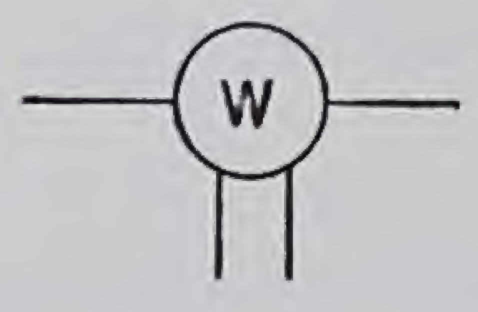
Ammeter . . . . 

Milliammeter . . . . 

Vacuum tubes:



Voltmeter . . . . 

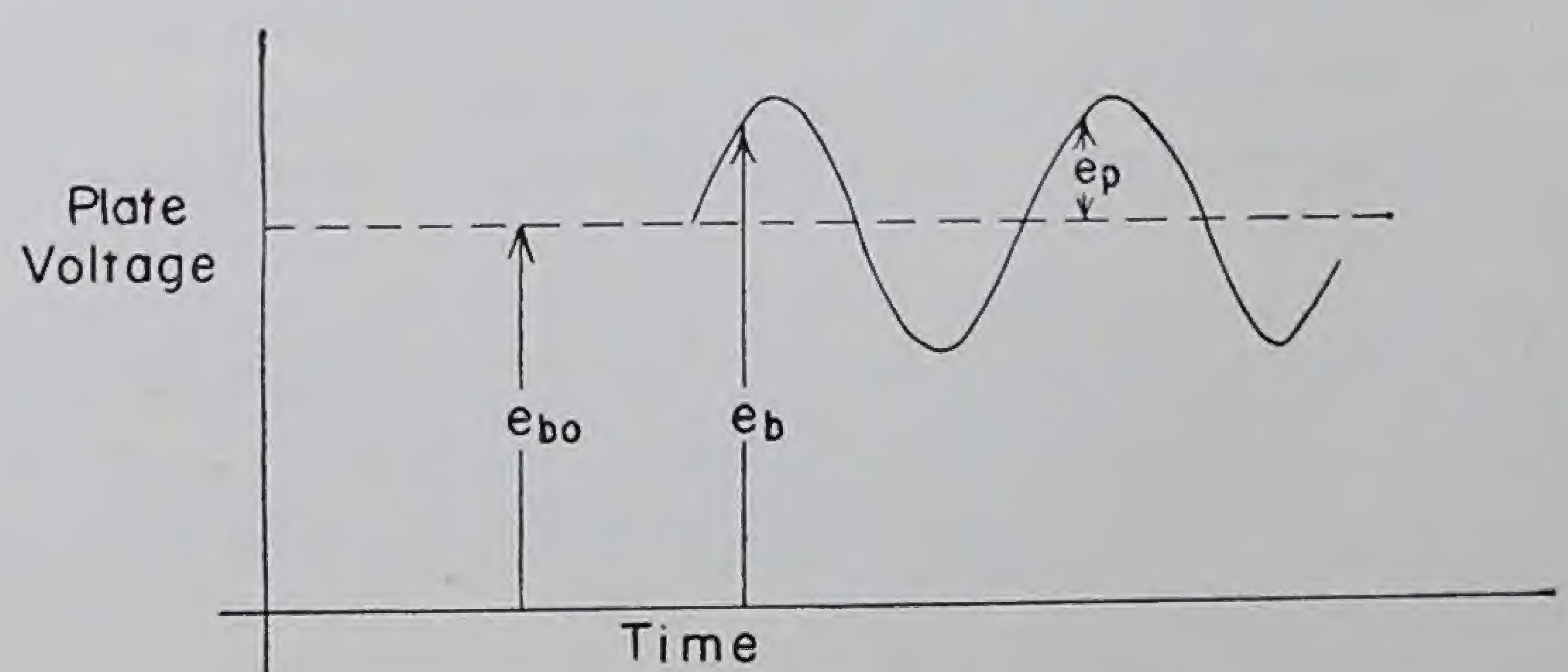
Wattmeter . . . . 

Symbols used in vacuum-tube circuits are given in the following explanation and table.

a. In plate circuits the subscripts $_b$ and $_p$ are used; in grid circuits the proper subscripts are $_c$ and $_g$.

b. Subscripts $_b$ and $_c$ indicate total instantaneous values of varying quantities. Subscripts $_p$ and $_g$ indicate instantaneous values of their alternating components. See the following table and figure:

Plate supply voltage	V_p
Filament or heater voltage	V_f
Instantaneous total grid voltage	e_c
Instantaneous total plate voltage	e_b
Instantaneous total plate current	i_b
Quiescent (d-c) value of plate voltage	e_{bo}
Quiescent (d-c) value of plate current	i_{bo}
Average (bias) value of grid voltage	e_{co}
Instantaneous value of alternating component of grid voltage	e_g
Instantaneous value of alternating component of plate voltage	e_p
Instantaneous value of alternating component of plate current	i_p



$e_b = e_{bo} + e_p$ (similarly: $i_b = i_{bo} + i_p$, $e_c = e_{co} + e_g$)

Amplification factor of tube $\left(= -\frac{\partial e_b}{\partial e_c}\right)$ . . . . μ

Plate resistance of tube $\left(= \frac{\partial e_b}{\partial i_b}\right)$ . . . . r_p

Control-grid, plate transconductance $\left(= \frac{\partial i_b}{\partial e_c}\right)$ . . . . g_m

Plate resistor (load resistor) . . . . R_L

Net d-c resistance of external plate circuit . . . . R_b

Net impedance of external plate circuit . . . . z_b

2. *Resistance Boxes.* A resistance box is a two-terminal variable resistance so designed that any desired known noninductive resistance may be introduced into the circuit of which it is a part. In the dial-type box, several dials are arranged in decades so that internal switches select the desired resistance. For example, one dial may select any number of thousands of ohms from one to nine. The next any number of hundreds of ohms from one to nine; the next any number of tens of ohms, and the fourth any number of ohms from one to nine. Thus by manipulating all four dials, any value from 1 to 9999 ohms may be set to the nearest ohm.

In the plug-type box, resistance is inserted by *removing* a plug which, when in place, short-circuits the resistance. The plugs must, therefore, be kept clean, and when removed from the box, placed on a clean sheet of paper. They are inserted with a clockwise motion and with only slight pressure. When a slight resistance to turning is felt a good contact has been obtained.

In either type, each coil is constructed so that it can dissipate $\frac{1}{4}$ watt safely. This rated capacity of 0.25 watt must not be exceeded since overheating a coil may change its resistance permanently.

$$P = I^2R \qquad \text{so} \qquad I = \sqrt{\frac{0.25}{R}}.$$

A 10-ohm coil can carry 0.16 amp; a 100-ohm coil, 0.050 amp; a 1000-ohm coil, 0.016 amp.

Most resistance boxes contain coils of manganin resistance wire, the resistivity of which does not change appreciably with small changes of temperature. The accuracy of such a resistance standard may be taken as $\pm 0.25\%$.

3. *Rheostats.* A rheostat usually consists of a solenoid of bare resistance wire wound on an insulating cylinder. A sliding contact introduces more or less of the resistance into the circuit, and thus controls the current of the circuit. Rheostats are able to dissipate a certain amount of power in the form of heat, and thus, like resistance boxes, have maximum current values. Those in use in the laboratory may be mounted on wooden bases, at the end of which is stamped the total resistance and the maximum allowable current. The rheostat will be too hot to touch at currents considerably under this maximum value.

Potentiometers are rheostats connected so that the entire resistance is in the circuit containing the main current. The slider then is able to tap off any fraction of the voltage drop across the resistance. See also Experiment 37. In small potentiometers the solenoid is often bent into a toroidal shape (doughnut) so that the slider may be mounted on a shaft, tapping off a voltage proportional to the angle of rotation if the winding is uniform.

4. *Condensers.* The simplest form of a condenser is a pair of metal plates facing each other. The capacitance of such a condenser varies directly with the area of the plates and inversely with the distance separating them. It also varies directly with the dielectric constant of the material separating the plates. The relation may be written in the form

$$C \propto \frac{KA}{d}, \tag{1}$$

where K = dielectric constant (= 1 for air),
A = area of each plate, and
d = distance separating the plates.

The capacitance of such a pair of plates of any convenient size is very small, and is totally inadequate for

most electrical purposes. One way of increasing the capacitance is to form a stack of plates connecting every other plate together, and having a sheet of dielectric between each pair of adjacent plates (Fig. A-10). Another way is to make the sheets of dielectric thinner, decreasing d. This cannot be carried too far, however, since if the dielectric becomes too thin, it becomes mechanically weak; it also may break down when even reasonable voltages are applied between the plates.

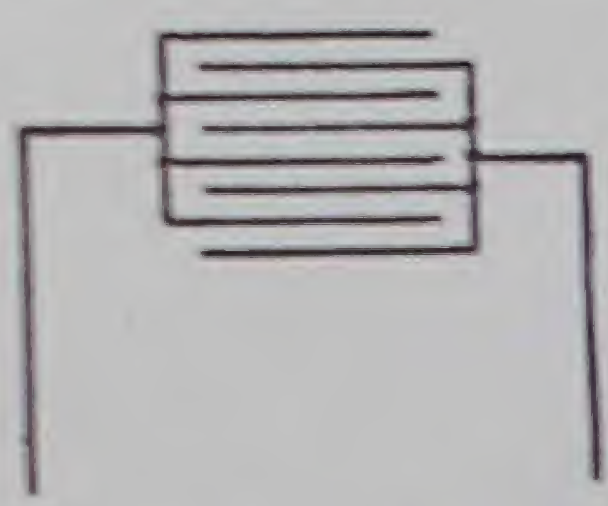


Fig. A-10.

For some purposes, air is used as the dielectric. An example of this is the tuning condenser of a radio, the capacitance of which is smoothly varied by interleafing one set of plates to any arbitrary extent with a fixed set of plates. A dielectric with useful properties of high resistance to voltage breakdown and stable value of K is mica. A stack of plates such as described in the preceding paragraph using mica as the dielectric is the usual form of a *standard condenser*. This type is still too bulky for most purposes, so the rigid plates are replaced by a long ribbon of aluminum foil, and the stiff mica by a strip of oil-impregnated paper. Using two strips of foil and two strips of paper and then rolling the four strips tightly into a cylinder forms a tubular paper condenser, in which each portion of one foil is adjacent to *two* equal portions of the other foil, one above it and one below it. In the laboratory, such a paper condenser has been rolled into a somewhat flat form, and the whole is encased in a metal container.

One other type which may be mentioned is the electrolytic condenser. In this type the metal of one plate is coated by a thin film, only a few molecules thick, of its own oxide, and the other "plate" of the condenser is an electrolyte which makes intimate contact with the oxide film. Thus the distance d between the "plates" is exceedingly small, and filled with a material of high dielectric constant; a very high capacitance is therefore possible in a limited volume. Such a condenser is polarized. The oxide film will stand considerable voltage in one direction; however, placing even a small voltage of incorrect polarity on the plates will destroy the coating and render the condenser useless.

5. *Autotransformer*. The voltage drop across an ordinary inductance is uniform from turn to turn of the coil if the coil winding itself is uniform and if the coil has an iron core. A sliding contact will "tap off" a fraction of the total voltage drop proportional to the number of turns it is from a reference turn, and thus performs the same function as the sliding contact on a potentiometer. An autotransformer often uses as a reference the first turn of the winding. In addition, the voltage source may be connected at points short of the ends of the winding. By transformer action, the sliding contact may tap off a voltage greater than that of the voltage source. A typical autotransformer connection is shown in Fig. A-11.

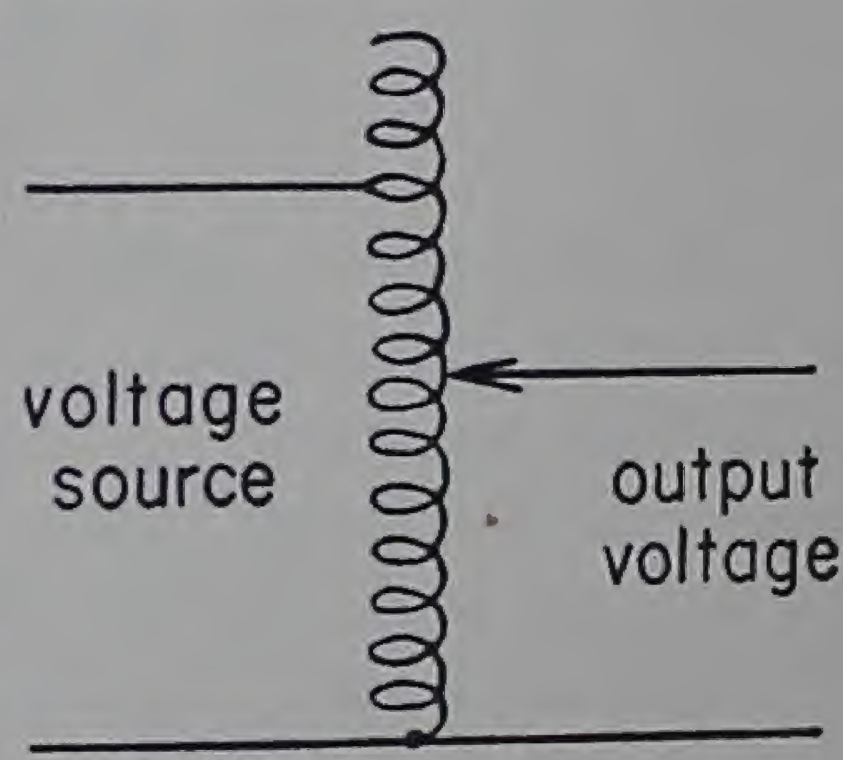


Fig. A-11.

Since this instrument is a transformer in every sense of the word (although it differs from the ordinary transformer in that it has only one winding) it will, of course, operate only on alternating current. Connecting it to a d-c source whose emf is higher than a very small fraction of the rated a-c voltage will cause excessive current, burning out the winding.

6. *Voltaic Cells*. These cells furnish, through chemical action, a source of emf and of current. The theory of these cells is covered in lectures and need not be described here. The so-called *dry cell* is used in the laboratory where only small currents (0.1 amp or less) are needed for short periods of time. For larger current demands, *storage cells* are used. These can furnish currents up to about 15 amp for short periods, or 1 or 2 amp for relatively long periods. A third and very important type of cell is the *standard cell*. This type is constructed so as to have a very constant emf. However, it must not be used to deliver any appreciable current—no more than a few microamperes. The standard cells in the laboratory, consequently, may be equipped with internal series resistances to limit the current to a safe value. However, the current that can still be drawn by short circuiting the terminals or by having low resistances across the terminals for any length of time exceeding a few seconds can affect the cell to the extent that its emf changes greatly, and several days of nonuse are necessary to restore it to normal. It should not be called upon to furnish even enough current to operate a voltmeter (see Note H, Section 2); *never call upon it to do more than deflect a table galvanometer slightly for a few seconds.*

7. *Switches.* A SPST switch is used to interrupt or to complete a single branch of a circuit. A double-pole single-throw (DPST) switch simultaneously performs the same operation in two branches of the circuit. The DPDT switch may be used to select either of two circuit elements for inclusion in the main circuit. It can also be used as a *reversing switch*, changing the direction of the current in any portion of a circuit. The alternate corner contacts of the switch are connected electrically as shown in Fig. A-12, so that with the blades to the right, the current through the motor is as shown by the arrow. With the blades to the left, the flow is opposite to that indicated.

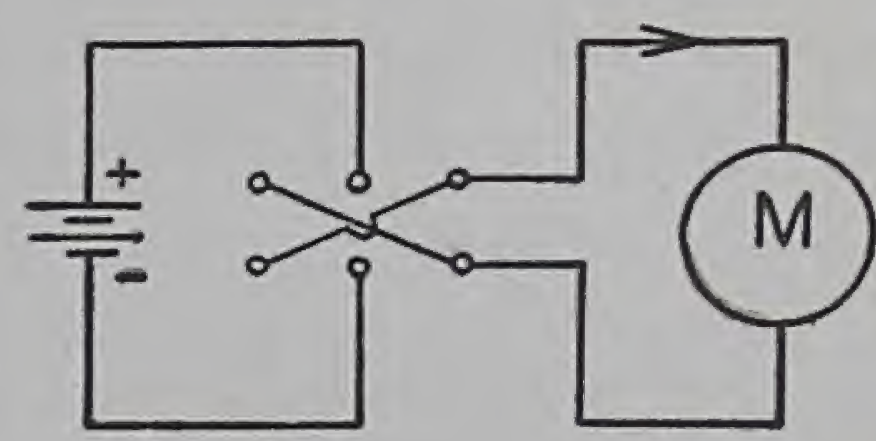


Fig. A-12.

H. Galvanometers

1. *General Notes.* A galvanometer is an instrument designed to detect small electric currents. The small portable type used in the general physics laboratory will deflect one (small) division with a current of approximately 0.00002 amp (20 μa). The wall galvanometer, or reflecting galvanometer, is considerably more sensitive, and one type in common use will deflect one division (1 mm) with a current of about 0.02 μa, and thus on this basis is 1000 times as sensitive as the portable or table type. An idea of the magnitude of this current is obtained with the realization that an ordinary 100-watt lamp bulb uses about 1 amp—one hundred

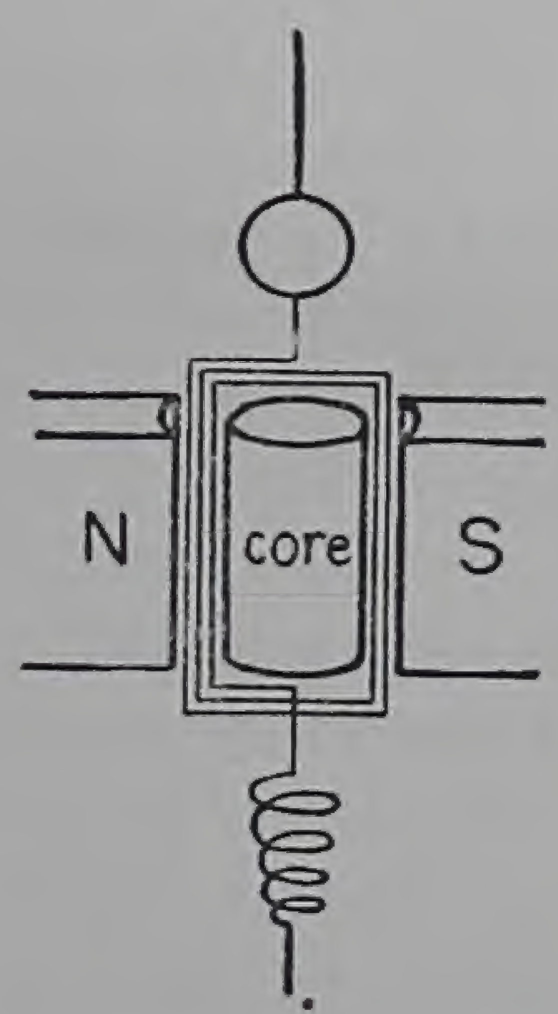


Fig. A-13a.

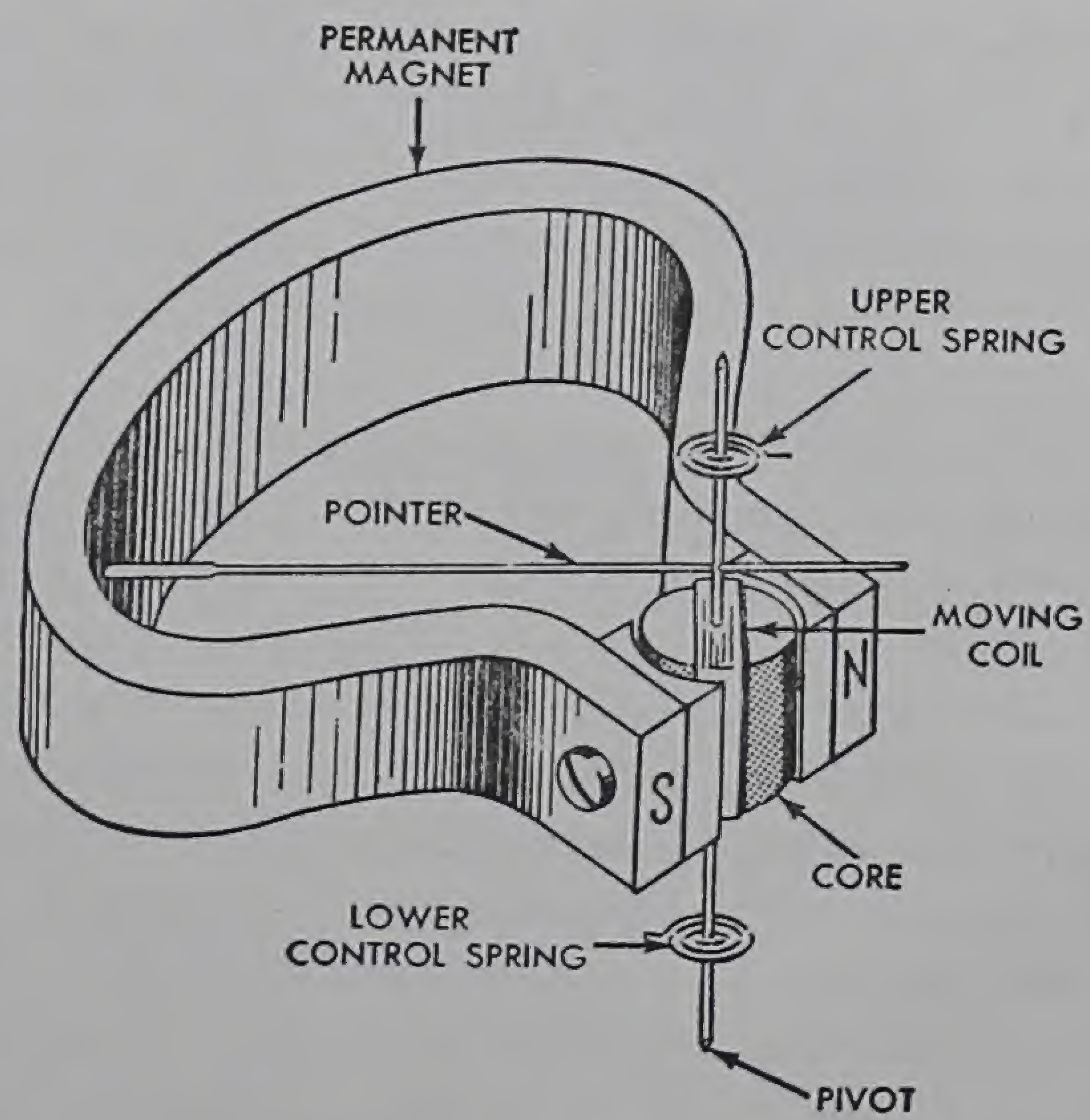


Fig. A-13b.

million times the current detectable on the wall galvanometer. Other galvanometers may be designed for different uses ranging to sensitivities 1000 times greater than that of the wall galvanometer. The amount of current necessary to deflect the galvanometer one scale division is called the *current sensitivity.* Other types of sensitivity may also be defined, such as voltage sensitivity or charge sensitivity.

Most galvanometers consist essentially of a coil of wire suspended in a stationary magnetic field (the d'Arsonval movement). See Fig. A-13. The interaction of this field with the field produced by a current in the wire causes the coil to turn in the stationary field. Since the field produced by the current in the wire is proportional to the current, the stationary field is designed to have a constant value regardless of the angular position of the coil in order that the deflecting torque be proportional to the current. A cylinder of soft iron (Core of Fig. A-13) is mounted at the center of the coil, and the pole pieces are often curved, to produce a radial field (Fig. A-14) which for relatively large deflections of the coil is constant in value. The coil is mounted in such a way that as it turns it twists a spring or suspension; it will turn, therefore, until the torque caused by the current will just be balanced by the restoring torque of the spring or suspension. The coil may be pivoted in jeweled bearings, or it may be suspended by a fine wire or ribbon which also serves to conduct the current to the coil. BY THE VERY NATURE OF THESE SENSITIVE DETECTORS, THEIR

CONSTRUCTION IS DELICATE, AND CARE MUST BE TAKEN TO LIMIT THE CURRENT IN THEM TO AN AMOUNT NOT SIGNIFICANTLY EXCEEDING THAT NEEDED FOR FULL SCALE DEFLECTION. An excess current can injure the instrument in one or both of two ways; the wire of the coil or the suspensions may be melted ("burned out") or the meter may deflect with great violence, injuring the moving parts.

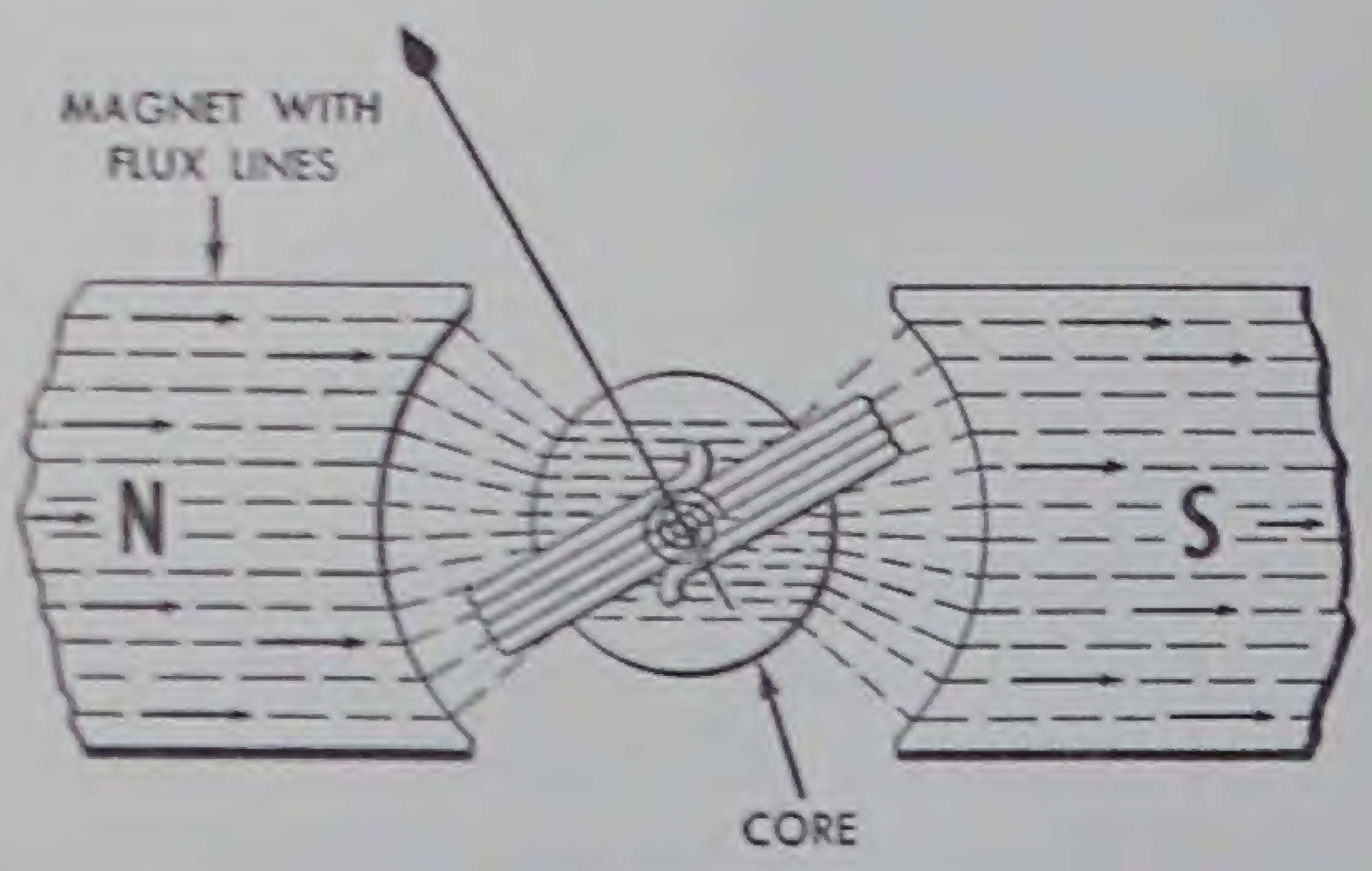


Fig. A-14a.

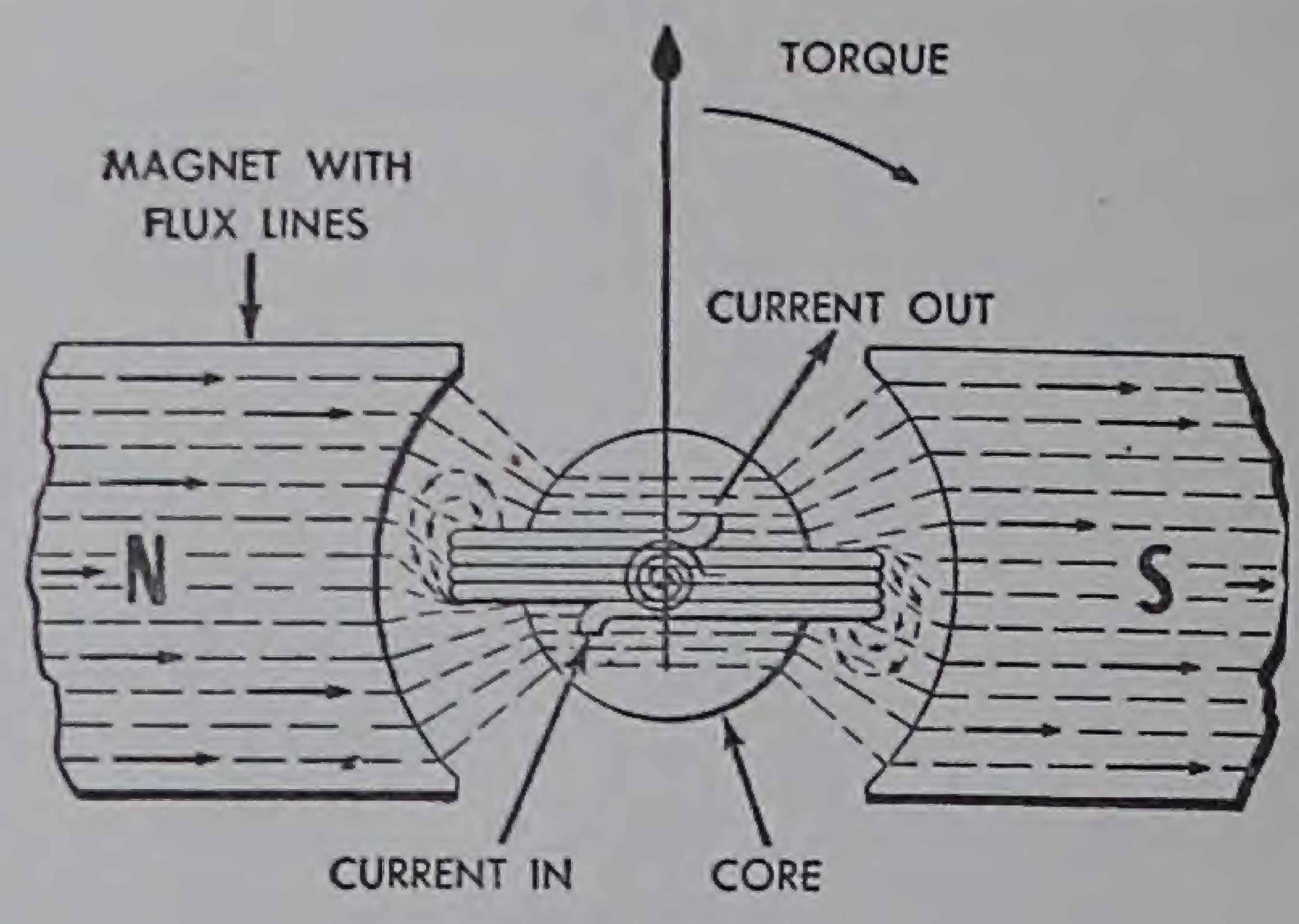


Fig. A-14b.

2. *The Table Galvanometer.* The coil of the portable galvanometer is mounted to pivot in jeweled bearings. Current is led into and out of the coil through springs coiled at the bearings. These springs also serve to return the galvanometer to zero when the current ceases. A pointer is attached to the coil, and indicates its position by means of a dial (Fig. A-13b). The zero of this type of galvanometer is usually in the center of the scale so that currents in either direction may be measured. The scale, it should be noted, is not usually calibrated in terms of any standard units but is divided uniformly so that the scale reading is *proportional* to the deflecting current.

Some table galvanometers are equipped with internal shunts so that only a portion of the total current passes through the movable coil, thus reducing the sensitivity of the meter. A switch selects either the reduced ("low") sensitivity or the full ("high") sensitivity of which the meter is capable. In circuits using galvanometers as null-indicators (that is, to indicate when the current in the galvanometer branch of the circuit has been reduced to zero) the initial readings should always be taken using the low sensitivity to guard against probable overloads. When the circuit has been adjusted to the desired null condition as nearly as possible using the low sensitivity, the high sensitivity may be used for the final adjustment.

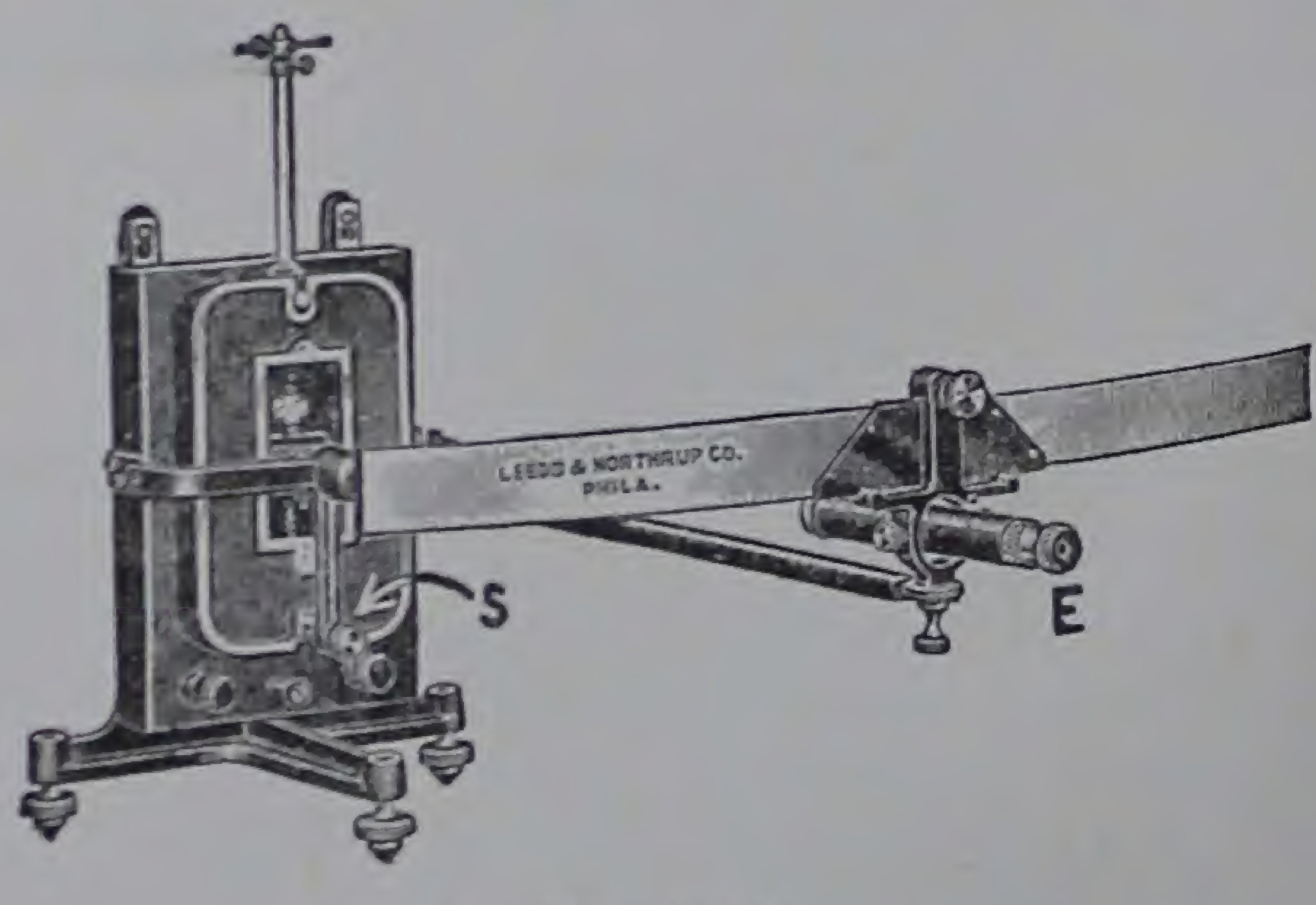


Fig. A-15.

3. *The Reflecting Galvanometer.* The moving element of this type of galvanometer consists of a rectangular coil wound on a light frame and suspended by a fine wire or ribbon of gold or of phosphor bronze which also furnishes the restoring torque to the coil. Moreover the suspension serves as one current lead to the coil, while the other lead consists of a very light wire helix attached to the lower end of the coil. A small mirror is fastened to the top of the coil, and turns with it, reflecting an image of a scale into a telescope mounted with the scale before the mirror. See Fig. A-13a and Fig. A-15. This use of reflection corresponds in effect to attaching to the coil a weightless pointer whose length is twice the distance between the mirror

and the scale. There are usually slight errors in the position of the zero, and in addition, the field is not everywhere truly radial. The resulting errors can usually be minimized by taking the deflection to each side of zero and using the average value.

Adjustment. Complete adjustment of the reflecting galvanometer is beyond the scope of the courses for which this manual is designed. Leveling the instrument and adjusting the suspension should not be attempted. IT IS OF UTMOST IMPORTANCE, THEREFORE, NOT TO MOVE THE GALVANOMETER IN ANY MANNER, SINCE THIS WILL PROBABLY THROW IT OUT OF LEVEL AND PREVENT IT FROM FUNCTIONING PROPERLY. If one of these adjustments is at fault, the assistance of the instructor should be obtained.

The telescope and scale are usually clamped to a rod which is attached adjustably to the galvanometer. The telescope may have to be adjusted to suit the individual eye. To do this, the eyepiece E should be moved *in* from a position *too far out* until the cross hairs of the telescope appear in sharp focus. Then the telescope is aimed directly at the mirror. Unless it is badly out of adjustment, the scale should now appear, and it is now possible to slide the second tube in the third tube from a position too far out until the image of the scale is seen most distinctly. If the scale is too high or too low in the field when the mirror is in the center of the field, raise or lower the scale arm by means of the screw S. The cross hairs should be in the plane of the image, that is, free from parallax. See Note F of this Appendix. If parallax is present, the eyepiece should be readjusted, necessitating a readjustment of the focusing. The scale may be moved with respect to the supporting rod to adjust the position of the zero mark.

A maladjustment which does not respond to the above treatment requires the attention of the instructor. The following notes are important in avoiding eyestrain and obtaining a clear image: observations should always be taken with both eyes open; if necessary, cover one eye with the hand. The *scale* should be well illuminated. It should be noted that light from *behind* the observer may be reflected from the glass face of the galvanometer, obscuring the scale image.

Damping. When the coil turns in the magnetic field, it acts as a generator. If the terminals are short-cuited, a current proportional to the angular velocity will be generated, setting up a magnetic field which reacts with the stationary field in such a way as to oppose the motion. Thus instead of allowing the coil to oscillate after each reading, it is possible to bring it to rest at the zero point by use of a damping key which short-circuits the coil.

4. *The Ballistic Galvanometer.* For many purposes the reflecting galvanometer as described in the preceding section may be used as a ballistic galvanometer, that is, to *measure a charge* passed rapidly through it. Accuracy here depends on the moving coil having a relatively large moment of inertia, so that all the charge has passed through the coil before it has been able to move appreciably. The coil receives an impulsive torque from the passage of the charge and the resulting swing or throw is proportional to the amount of the charge:

$$Q = KD \tag{2}$$

where D is the deflection in millimeters, and Q is the charge in coulombs. K is called the ballistic constant or *coulomb sensitivity* (coulombs per millimeter) of the galvanometer.

5. *The Ayrton Shunt.* Referring to Note J, Section 2, on The Ammeter, it is seen that by connecting a simple shunt across a galvanometer the instrument can be made to measure currents larger than its full-scale current. The shunt and the galvanometer form two branches of a divided circuit, the current in only one branch of which is measured. This current being known, the total current in the external circuit can be computed. For a galvanometer with any shunt, a portion of the total current I passes through the galvanometer. If this portion is denoted by I_g, the current passing through the shunt S is $I - I_g$. The potential drop across the galvanometer being equal to that across the shunt, we may write

$$I_gR_g = (I - I_g)S.$$

Thus for a measured current I_g, the total current is given by

$$I = I_g \frac{R_g + S}{S}. \tag{3}$$

The factor $(R_g + S)/S$ by which the current measured by the galvanometer must be multiplied to obtain the total current in the main circuit is called the *multiplying factor* of the shunt. In order to have this factor be a convenient number such as a power of ten, the value of S must be carefully adjusted in its relation to the resistance of the galvanometer as shown in Note J, Section 2, on the ammeter. The multiplying factor of this shunt will be a convenient number only for the galvanometer for which it was designed.

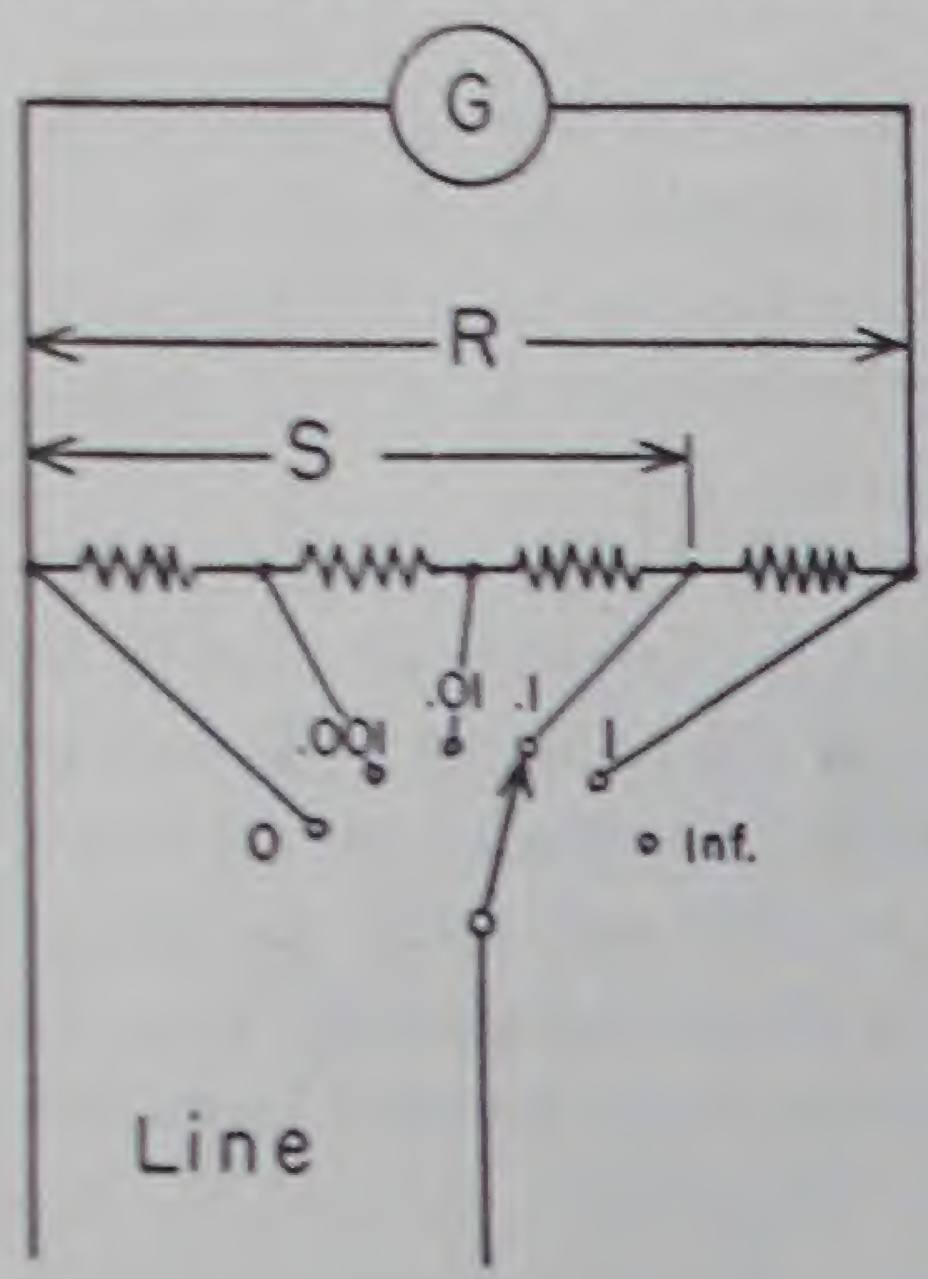


Fig. A-16.

Further, such a shunt will be useful only for current-measurement work, since in ballistic work, different shunts will damp the galvanometer different amounts, and thus not allow deflections proportional to the nominal multiplying factor. The Ayrton or universal shunt overcomes both of the aforementioned difficulties. In Fig. A-16 is shown the arrangement of the Ayrton shunt. A resistance R is connected permanently across the galvanometer, and one end of this combination is attached to the external circuit. The other lead of the external circuit is connected at will to points on R which are usually distant R, $0.1R$, $0.01R$, and $0.001R$ from the fixed external circuit lead. This distance corresponds to the shunt S of the simple shunt circuit previously discussed. The remainder of the resistance, $R - S$, is effectively in series with the galvanometer resistance, R_g, so far as the external circuit is concerned. With such an arrangement, a current I_g in the galvanometer indicates an external total current

$$I = I_g \frac{(R_g + R - S) + S}{S}$$

$$= I_g \frac{R_g + R}{S}, \tag{4}$$

since R_g in Eq. (3) is now effectively $R_g + R - S$. In this form of shunt, therefore, the multiplying factor varies inversely as S since the numerator remains constant even with varying S. For this reason the Ayrton shunt may be used with any galvanometer; *i.e.*, the multiplying factors will have the same ratios with respect to maximum sensitivity (when $S = R$) for any galvanometer; the numbers 1, 0.1, 0.01, and 0.001 on the shunt box indicate the *relative* values of I_g for the same current I in the main line.

It is to be noted that the figure 1 does not indicate that 100% of the external current is activating the galvanometer. In this position the galvanometer is shunted by R. For an R of 10,000 ohms connected to a galvanometer whose R_g = 100 ohms, the position 1 means that 0.99 of the current in the main circuit is in the galvanometer, or that the multiplying factor is 1.01. Position 0.1 gives a multiplying factor of 0.101, or one-tenth the sensitivity, and so forth. If the galvanometer is to be used with its Ayrton shunt a good deal, it is advisable to avoid using such multiplication factors by calibrating the galvanometer *with the Ayrton shunt in place* and with the setting unity. In this way the multiplying factors for different shunt settings with the new sensitivity are simply those shown on the shunt box. In any event, if R is much greater than the galvanometer resistance, most of the galvanometer sensitivity is realized. This is usually the case. If R is too small for the galvanometer employed, not only is the maximum sensitivity decreased, but the moving coil of the galvanometer may be too heavily damped, and therefore sluggish in action.

In addition to the advantage of definite current division, the Ayrton shunt has the capability of accurately dividing charge in open-circuit ballistic work. This is particularly useful when capacities of condensers are being compared by means of the ballistic galvanometer. The damping of the galvanometer is independent of the setting of the shunt, since the resistance across it is at all times equal to R. This, however, is not true in the case of a low-resistance line, which damps the galvanometer differently for different settings of the shunt.

It is possible to prepare an Ayrton shunt from a resistance box of the traveling plug type. The resistance R may be chosen to equal the critical damping value for the galvanometer, rendering the instrument "dead beat" for all settings of the shunt, even with an external circuit of very high resistance.

It should be noted that in the case of a low-resistance external circuit, the total external current will depend on the setting of the shunt, and thus the shunt is not applicable to such circuits without a knowledge of the resistance of the circuit involved.

3

The resistances in an Ayrton shunt of 10,000 ohms total resistance are given in the following table:

Setting of Shunt	*Value of S, Ohms*
0	0
0.001	10
0.01	100
0.1	1000
1	10,000
infinity	infinity

J. Meters

The basic structure of many meters is the d'Arsonval galvanometer, the most common types having the coil supported in jeweled bearings. A galvanometer may be converted into a voltmeter, an ammeter, or an ohmmeter of any range greater than a certain minimum if two factors are known: the *current sensitivity* (Note H, Section 1) and the *resistance of the coil*. In addition, for convenience in use, the number of divisions on the scale of the galvanometer should be taken into account. The computation will be shown in detail in the sections below.

1. *The Voltmeter*. The resistance of the coil of a galvanometer being constant, the deflection will be proportional to the voltage across the coil. Thus the galvanometer in its basic form may be considered to be a voltmeter of a very low range. However, it is, in general, not a convenient range, since the factor relating the voltage to the deflection is not ordinarily a simple integer.

Let k = current sensitivity,
R_g = coil resistance,
N = number of divisions between zero and full scale on galvanometer dial,
I_f = full-scale current (amount necessary to deflect galvanometer N divisions),
E_f = full-scale voltage (amount necessary to deflect galvanometer N divisions),
R = a resistance in series with the galvanometer,
R_v = total resistance of the voltmeter $(R + R_g)$, and
V = voltage at which it is desired that the *volt*meter read full scale.

See Fig. A-17.

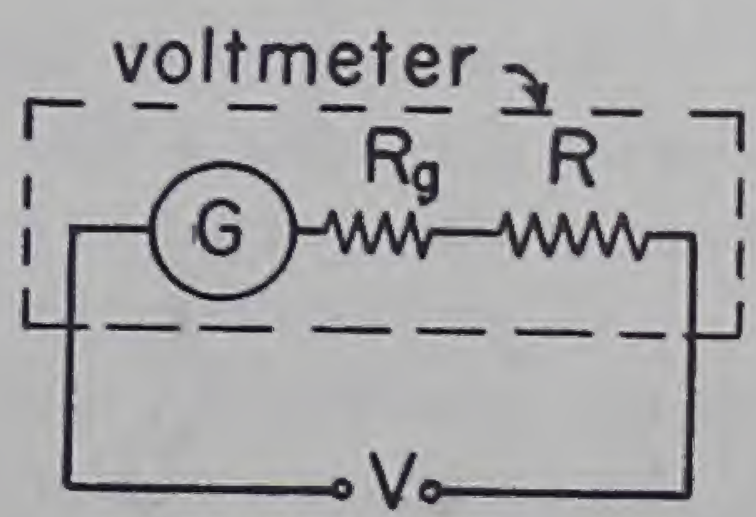


Fig. A-17.

Now the current necessary for full-scale deflection is the amount necessary for a one-division deflection times the number of divisions:

$$I_f = kN. \tag{5}$$

If it is desired that a voltage V cause the voltmeter to deflect to full scale, this voltage must produce a current I_f. Then

$$V = I_f(R_v) = I_f(R + R_g), \tag{6}$$

so that the size of the series resistance that it is necessary to add in order to convert the galvanometer into a voltmeter with a range of V volts is

$$R = \frac{V}{I_f} - R_g. \tag{6a}$$

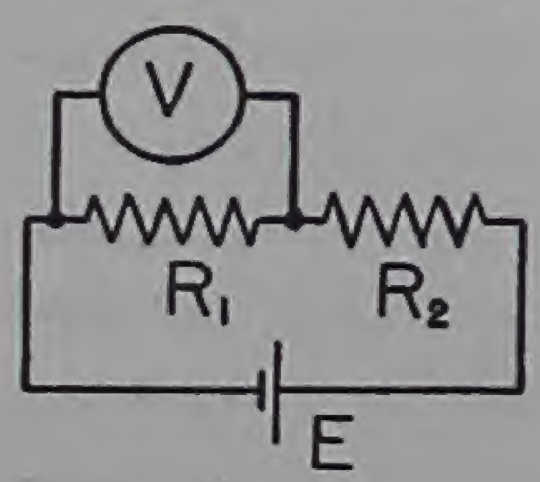


Fig. A-18.

A voltmeter is supposed to measure the potential difference across any part of an electrical circuit without itself changing the circuit. Ideally this means that the voltmeter resistance R_v should be infinite, for consider the measurement of the voltage drop across a resistance, as shown in Fig. A-18.

When the voltmeter is connected across R_1, the net resistance of the combination of R_1 and the voltmeter is less than R_1:

$$R_{\text{net}} = \frac{R_1 R_v}{R_1 + R_v}.$$

The total resistance of the circuit is now reduced by an amount $R_1 - R_{\text{net}}$ so that the current is greater, causing a larger IR drop across R_2, and, since the applied emf is constant, a smaller drop across R_1 than

before the voltmeter was applied. Thus the voltmeter always reads a value lower than the true value. As the ratio of the voltmeter resistance to the circuit resistance increases, however, the error becomes smaller. If the voltmeter resistance is 100 times as large as the circuit resistance, the error is about 1%. For careful work, therefore, the voltmeter resistance must be taken into account. For the highest quality voltmeters, the galvanometer current sensitivity should be very small. (Remember, of course, that the smaller the current sensitivity, the more sensitive is the galvanometer.)

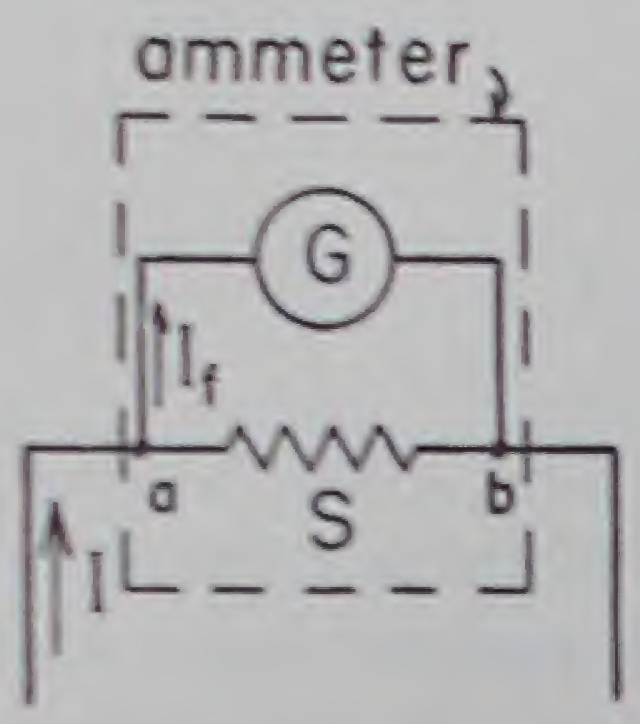


Fig. A-19.

Since the voltmeter is simply a modified galvanometer, the warning at the end of Section 1 of Note H applies. *Do not apply more voltage than the full scale amount.*

2. *The Ammeter.* The galvanometer may be used as an ammeter whose range is $I_f = kN$. If larger currents are to be measured, say a maximum current of I, all but the amount I_f must be shunted through a parallel path around the galvanometer, as shown in Fig. A-19. The easiest way to calculate the correct value of S for any particular current range is to consider the following facts: when there is a current in the circuit, point a is at some definite potential, V_a; point b is at the potential V_b. The *difference* $(V_a - V_b)$ must equal E_f when the external current equals I, for by our assumption, the current I in the external circuit is to cause full scale deflection of the galvanometer. However, the potential difference E_f also exists across the shunt resistance S. The current in this resistance is $I - I_f$. Thus the value of S is given by

$$S = \frac{E_f}{I - I_f} = \frac{I_f}{I - I_f} R_g. \tag{7}$$

An ammeter is supposed to measure the current in any circuit without itself changing the circuit. Ideally this means that the ammeter resistance R_a should be zero; for consider the measurement of the current in a circuit like the one of Fig. A-20.

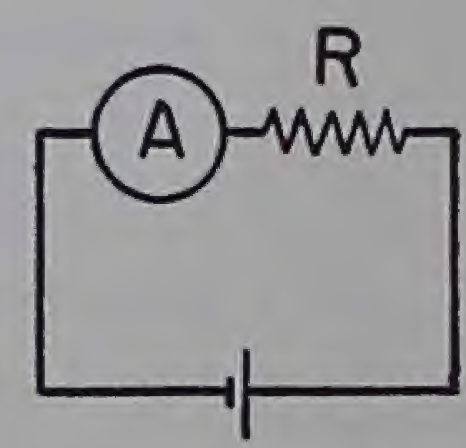


Fig. A-20.

When the ammeter is connected in the circuit, the total resistance of the circuit increases by the amount R_a, and thus the current falls below that when the ammeter is not present. If the resistance of the ammeter is 1/100 as much as that of the rest of the circuit, the error is 1%. For careful work, therefore, and in cases where the circuits have low resistances, the ammeter resistance must be taken into account. For the highest quality ammeters, the galvanometer should have a low k, which means that R_a will be small.

WARNING: An ammeter must never be placed across a voltage source; it must only be used in *series* with a load which does not draw more than the full scale current.

3. *The Ohmmeter.* The common type of ohmmeter consists of a galvanometer in series with a battery and a resistor, R_o, as shown in Fig. A-21. It is clear from the sketch that the amount of current in the ohmmeter when connected across an unknown resistance, R_x, depends on the values of the resistances R_g, R_o, and R_x. If R_x is an open circuit, that is if $R_x = \infty$, then the current is zero. The rest position of the pointer on the dial may be marked "∞." It is also clear that if R_x is a short circuit, that is if $R_x = 0$, then the current is limited only by $R_g + R_o$. Thus R_o must be of such a size that the current with $R_x = 0$ is just the full scale current, I_f. Suppose further that $R_x = R_o + R_g$. The current is now half the amount it was for $R_x = 0$. Thus the point on the dial corresponding to this value of R_x is halfway between the zero and infinity marks. The scale is obviously not linear.

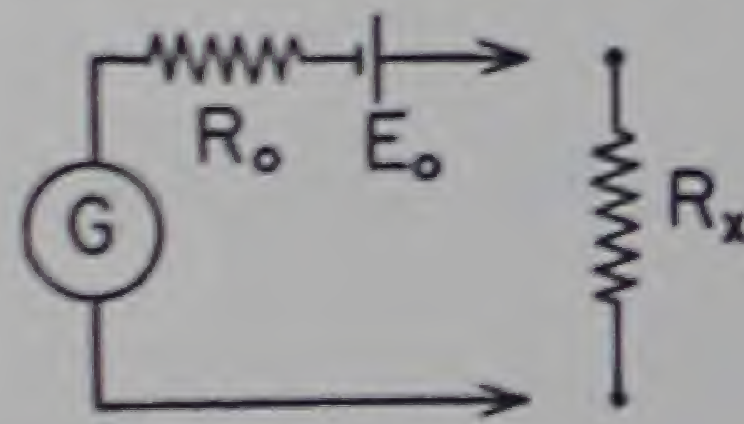


Fig. A-21.

Different scales are obtained by charging both R_o and E_o.

4. *The Wattmeter.* This instrument is designed to measure the power consumed in an electric circuit element. It is constructed on the electrodynamometer principle and has two windings, a "current" winding and a "voltage" winding. See Fig. A-22. The former winding is placed in series with the circuit element to be measured, and therefore the current in this winding is the same in phase and magnitude as that in the measured element. The voltage winding is placed "across" the measured element, and since the resistance of this winding is high, the current through it is in the same phase and is proportional to the voltage across the measured element. The deflection of the wattmeter is accomplished in much the same way as in the

ordinary d'Arsonval movement: the current winding furnishes the stationary field for the movable coil which is itself the voltage winding; the torque is proportional to the product of the currents in the windings, that is, to the product of E and I in the measured element. Further, since the currents in the two windings must be in phase for maximum torque, and since when the phases differ by 90° a maximum current in one winding corresponds to zero current in the other, giving zero torque, it may be shown that the torque will also be proportional to the cosine of the phase angle. The net effect is now seen to be the a-c power equation, $P = EI \cos \theta$, translated into mechanical motion. The factor "$\cos \theta$" is called the *power factor*.

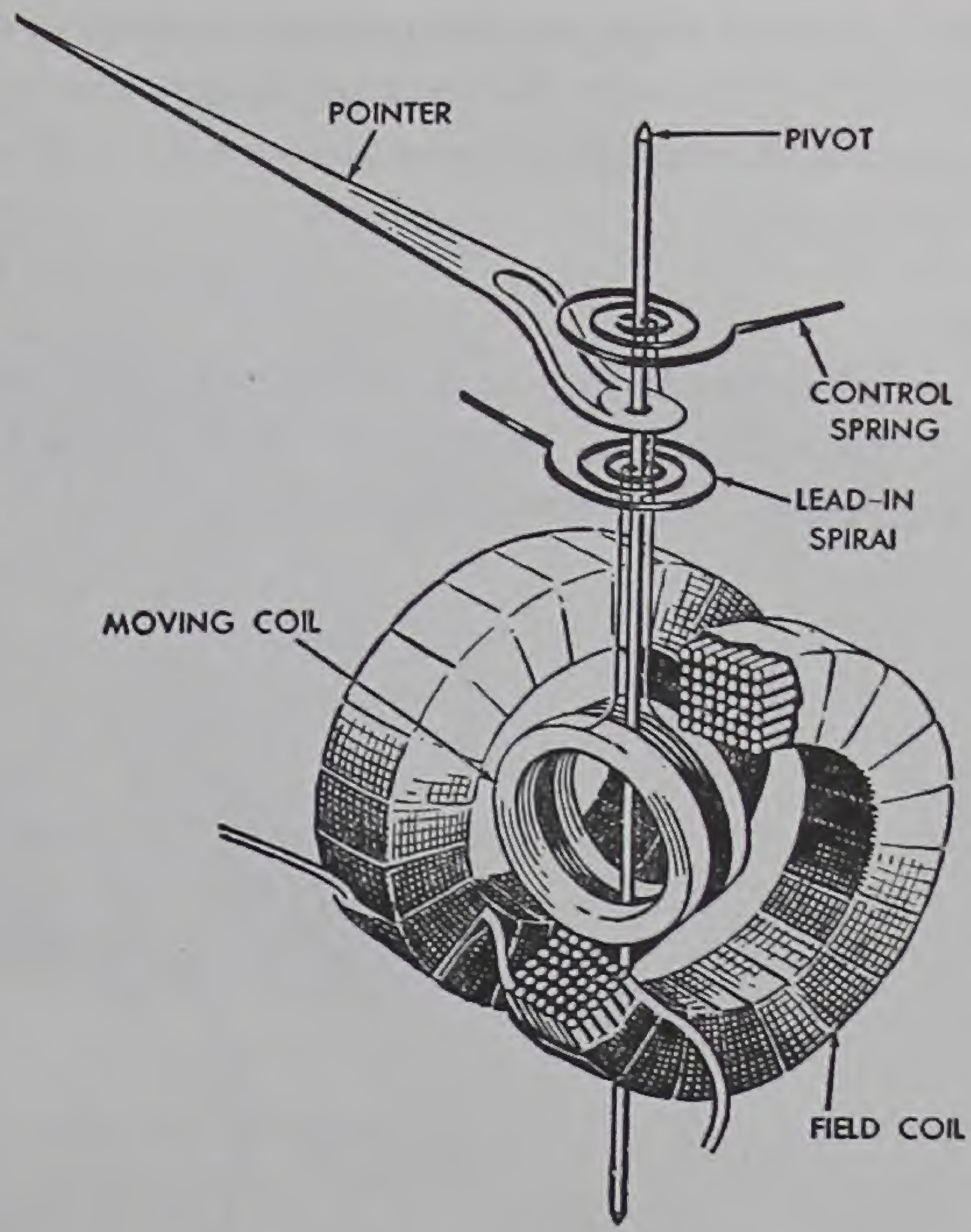


Fig. A-22.

The wattmeter may also be used to measure power in d-c circuits. In this case the power factor, $\cos \theta$, is always unity.

Note that the wattmeter in its simplest form has four terminals, two current terminals and two voltage terminals. One terminal of each pair is marked "$\pm$." These must both be connected *toward the same end* (high potential or low potential) of the circuit.

K. Magnetic Flux Standards

A flux standard is an instrument designed to deliver a very brief voltage pulse produced in a coil by having it cut a known number of flux linkages. The emf thus produced is given by the equation (Faraday's law of electromagnetic induction),

$$e = -N \frac{d\phi}{dt} \times 10^{-8} \qquad \text{volt,} \tag{8}$$

where N is the number of turns in the coil cutting the flux, and ϕ is the number of lines of flux linking the coil at time t.

The rate of change of the linkage is $\frac{d}{dt}(N\phi) = N\frac{d\phi}{dt}$ since N is constant. There are two different types of flux standard used in the laboratory.

1. *The Hibbert Standard.* This standard consists essentially of a coil which may be dropped through a radial magnetic field of known magnetic flux produced by a permanent magnet formed by the shell and core of the standard. The coil slides on a shaft which passes through the center of the field. At the top of the shaft is a catch which holds the coil out of the field, and which, when released, allows the coil to fall under the action of gravity into the field. In its initial position, there is no magnetic flux from the Hibbert standard linking the coil. In its final position, the entire flux of the standard links the coil. See the sketch, Fig. A-23, which shows the cross section of the Hibbert standard.

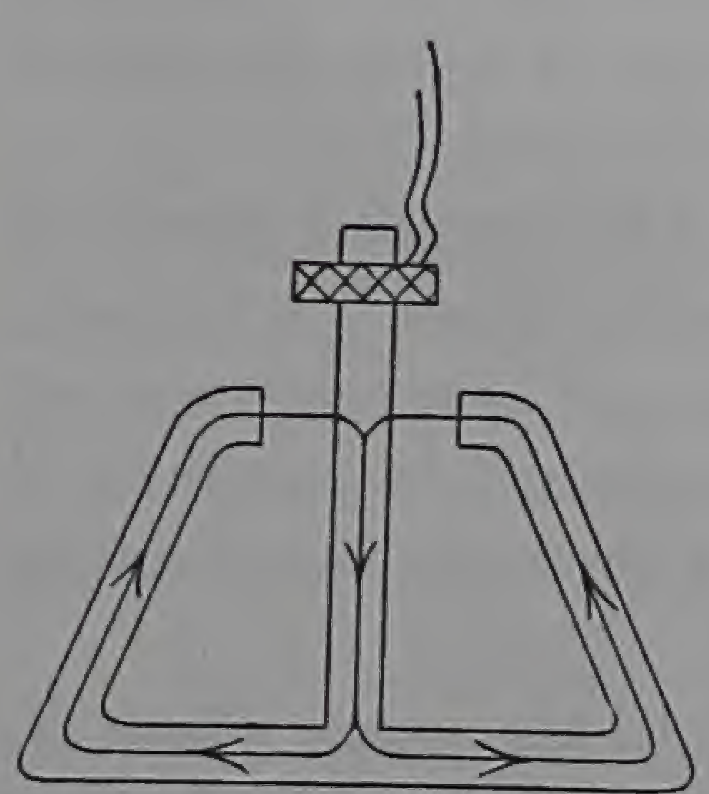


Fig. A-23.

2. *The Cenco Standard.* This standard is constructed with a d'Arsonval movement (see Note H, Section 1). A rather strong spring returns the coil to the zero deflection position if the coil is deflected by hand and released. The coil, rotating in the radial field, cuts magnetic flux in its return to zero.

As in the Hibbert standard, the number of flux *linkages* cut is the product of the number of *lines* and the number of *turns* on the coil. In this instrument, only the product is given. A pointer is attached to the coil in the same manner as in an ordinary voltmeter or ammeter, and indicates on the scale the net change in flux linkage that occurs upon release of the coil. Because of differences among individual instruments, each is separately calibrated at several points along the dial, and each point is marked. The marks give significant figures only, and the multiplying factor is printed elsewhere on the dial.

The selection mechanism of the desired number of flux linkages, and the subsequent release of the coil, is as follows: a large knurled knob just above the coil is turned counter clockwise to the zero position. At that point it engages the pointer, which is already at zero. The knob may now be turned clockwise to move the pointer to the desired setting. To release the pointer and coil, the protruding button on the knob is pressed, disengaging the pointer and coil which then snap back to zero.

L. The Spectrometer

The spectrometer is essentially an instrument for the measurement of angles of deviation of light rays due to reflection, refraction, and diffraction. Its essential features are represented diagrammatically in Fig. A-24. A circular horizontal plate K, the edge of which is graduated in degrees, is supported upon a

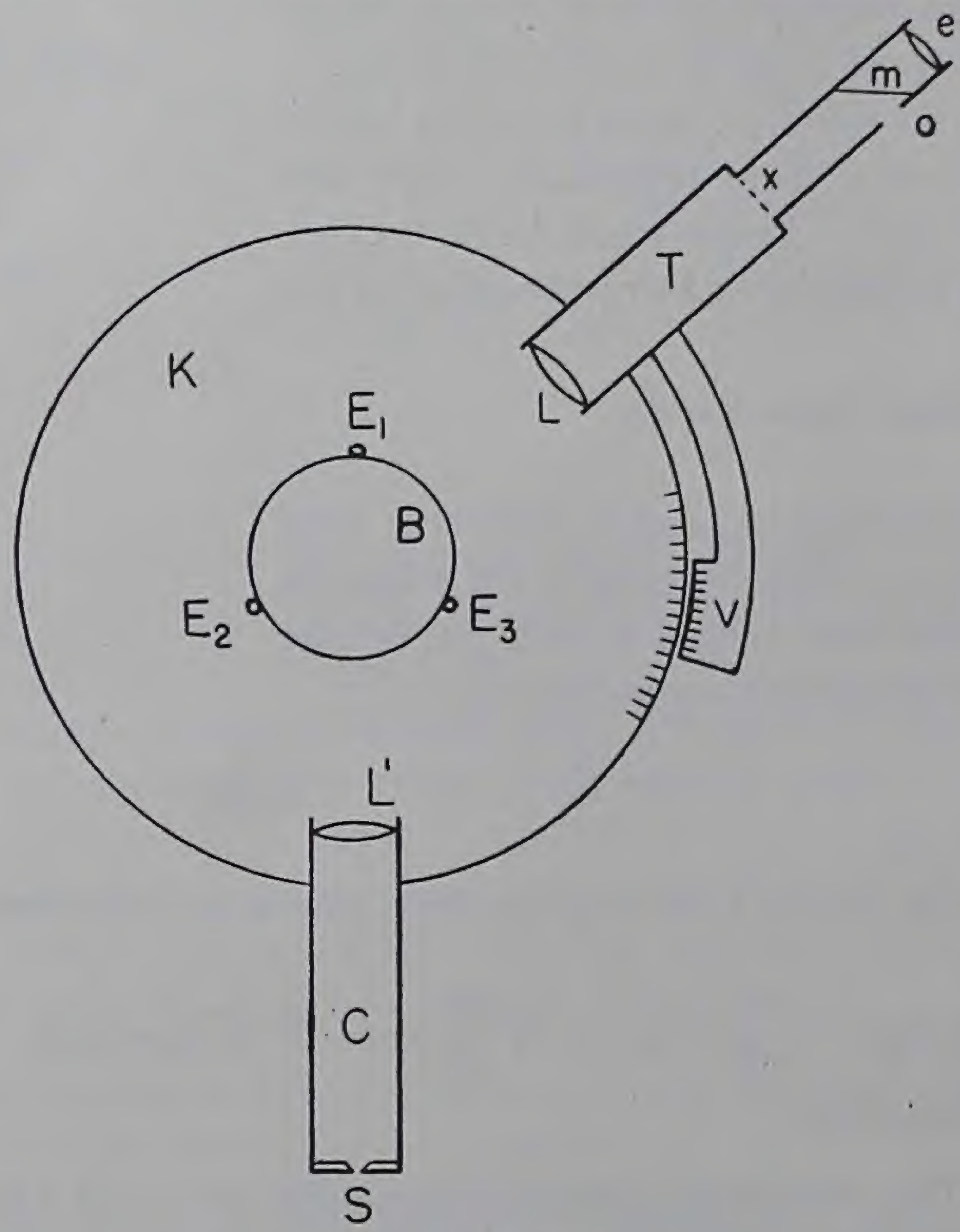


Fig. A-24.

vertical mounting which carries also a telescope T and a collimator C. The latter consists merely of a tube carrying a slit S of adjustable width and so mounted that it may be placed in the principal focal plane of the lens L'. This arrangement makes it possible to regard S as an infinitely distant source of light, for waves (rays) which originate at S become plane waves (parallel rays) after passing through the lens L'.

The telescope T is mounted so as to rotate about the vertical axis of the plate K. The angular position of the telescope with reference to the graduations on the plate is read by means of a vernier scale (or scales), V, attached to the telescope. A small piece of plane glass m is inserted in the eyepiece e of the telescope so as to make an angle of 45° with the axis of the telescope. The purpose of this arrangement is to make it possible to illuminate the cross hairs at x by projecting a beam of light into the eyepiece through the circular opening at O. Such an eyepiece is called a Gauss eyepiece.

A second smaller circular plate B, the spectrometer table, is mounted at the center of the large plate K. It may be leveled by means of leveling screws E_1, E_2, E_3. On this table is placed the optical device which is being examined, *e.g.*, prism, diffraction grating, etc. This table may be rotated about its vertical axis independently of the rotation of the telescope. It may also be adjusted for height.

An examination of the spectrometer will reveal that there is a large number of screws and clamps on it which are to be used in adjusting the instrument. For example, there are leveling screws and clamps on both the telescope and the collimator in addition to focusing devices. There are clamps for the adjustment

of the spectrometer table either as to height or as to rotation. The telescope and verniers attached to it may be clamped in position except for slow motion afforded by a tangent screw. The same is true for the collimator. Hence it is necessary for the student to examine carefully the functions of these various screws and clamps before trying to use the spectrometer. See Fig. A-25 for a photograph of a Gaertner spectrometer.

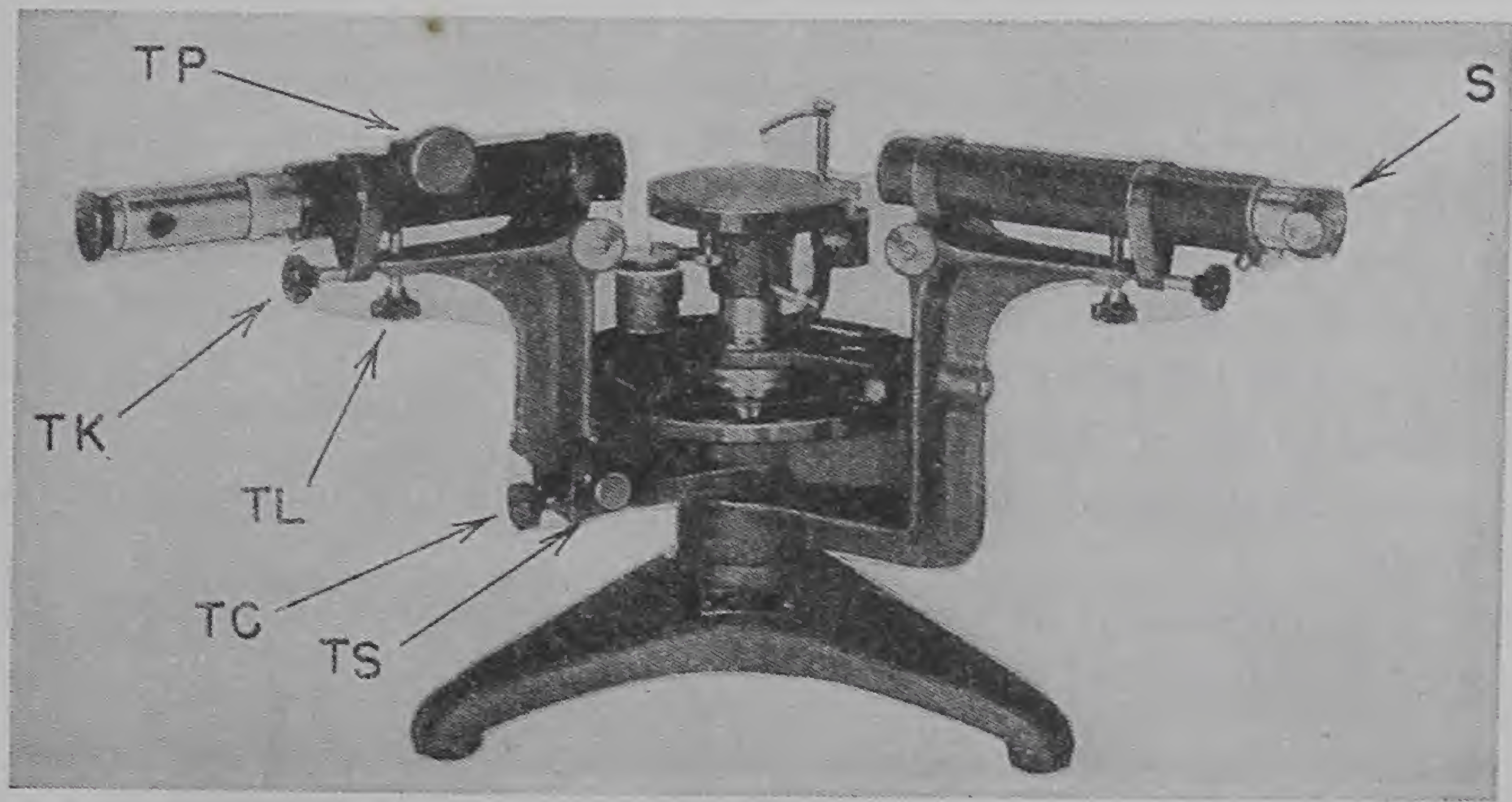


Fig. A-25.

TK = leveling clamp of telescope, *TL* = leveling screw of telescope, *TC* = telescope clamp, *TS* = tangent screw for slow motion of telescope, *TP* = rack and pinion for focusing telescope. Corresponding clamps and leveling screws exist for the collimator. *S* = slit of collimator.

Adjustment of Spectrometer. The complete adjustment of the spectrometer is a long and arduous task for the novice. For highly accurate work it is necessary to have the spectrometer in complete adjustment. This is usually done in advanced work in experimental optics but it is hardly advisable in beginning courses in optics. It will serve our purpose to make an approximate adjustment of the spectrometer only. In making this approximate adjustment, the following steps should be taken in the order given.

1. Focus the eyepiece on the cross hairs of the telescope. This may be done by directing the telescope toward some bright object, drawing out the *eyepiece tube* of the telescope its full amount, then slowly pushing it in until the cross hairs are *first* distinctly observed. Focusing in this manner places the cross hairs in the focal plane of the eyepiece. Light rays from the cross hairs which enter the eye are, therefore, parallel rays. This is the condition of most comfortable vision for most people.

2. Focus the telescope for parallel rays. In most experiments with the spectrometer it is necessary that rays coming from the collimator be parallel and hence the telescope must be adjusted for parallel rays. This may be done by focusing the telescope on a distant object. Direct the telescope toward an open window at some distant object. By use of the rack and pinion focusing device on the telescope (which moves both eyepiece and cross hairs) rack out the tube as far as it will go, then rack it in slowly until a clear image of the distant object *first* comes into distinct view. Focusing in this manner places the real image of the distant object (produced by the objective lens of the telescope) in the focal plane of the eyepiece and, therefore, in coincidence with the cross hairs. Under these conditions there should be no parallax between the image and the cross hairs. See Appendix II, Note F. If parallax exists, the telescope should be refocused.

3. Alignment. Align the telescope and collimator with the center of the spectrometer table. *In the Gaertner spectrometers this adjustment has already been made by the manufacturer and should not be disturbed.* In other types this is done by unclamping the telescope or the collimator from its supporting arm, sighting along its barrel, turning it until it points to the center of the table, then reclamping it.

4. Level the spectrometer. Place a level on the plate K so that it is parallel to a line through two of the spectrometer feet, and level by adjusting the foot screws. Then place the level at right angles to its first position, and level by adjusting the third foot screw.

5. Level the spectrometer table. Place a level on the table so that it is parallel to a line through two

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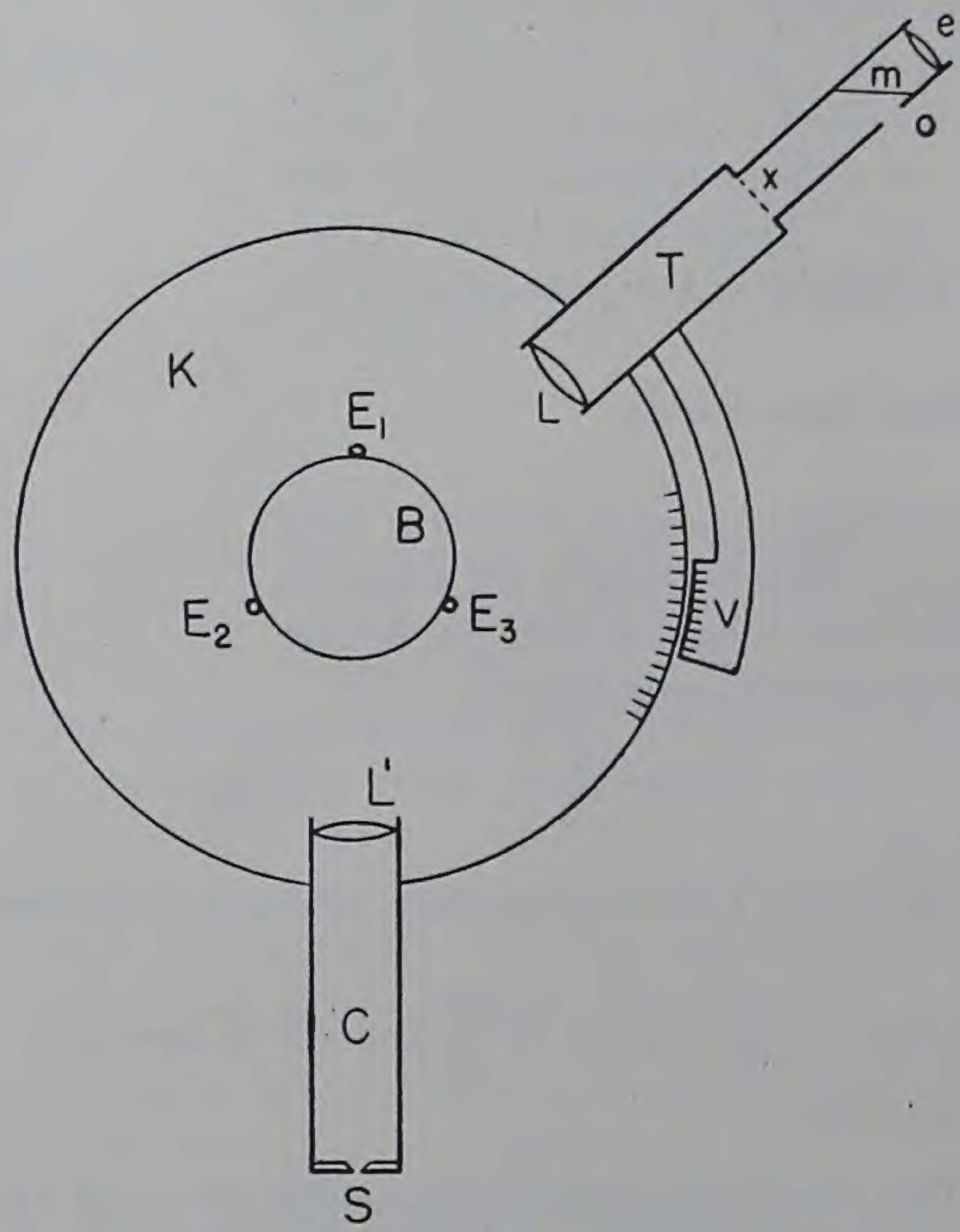

Fig. A-24.

vertical mounting which carries also a telescope T and a collimator C. The latter consists merely of a tube carrying a slit S of adjustable width and so mounted that it may be placed in the principal focal plane of the lens L'. This arrangement makes it possible to regard S as an infinitely distant source of light, for waves (rays) which originate at S become plane waves (parallel rays) after passing through the lens L'.

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3. In using a flame as a source of illumination for the collimator, be sure to keep the flame sufficiently far from the slit to avoid overheating the slit mechanism.

4. It is frequently advisable to cover the prism table, telescope, and collimator with a black cloth to eliminate extraneous light.

M. The Analytical Balance

1. *General Discussion.* a. The analytical balance is used to determine accurately the mass of an object. Since it is one of the most delicate (and expensive) pieces of apparatus used in the laboratory, and is easily damaged or put out of adjustment, great care must be exercised in its use. The student should be thoroughly familiar with the functions of its various parts and the precautions to be used in handling them before attempting to use the balance.

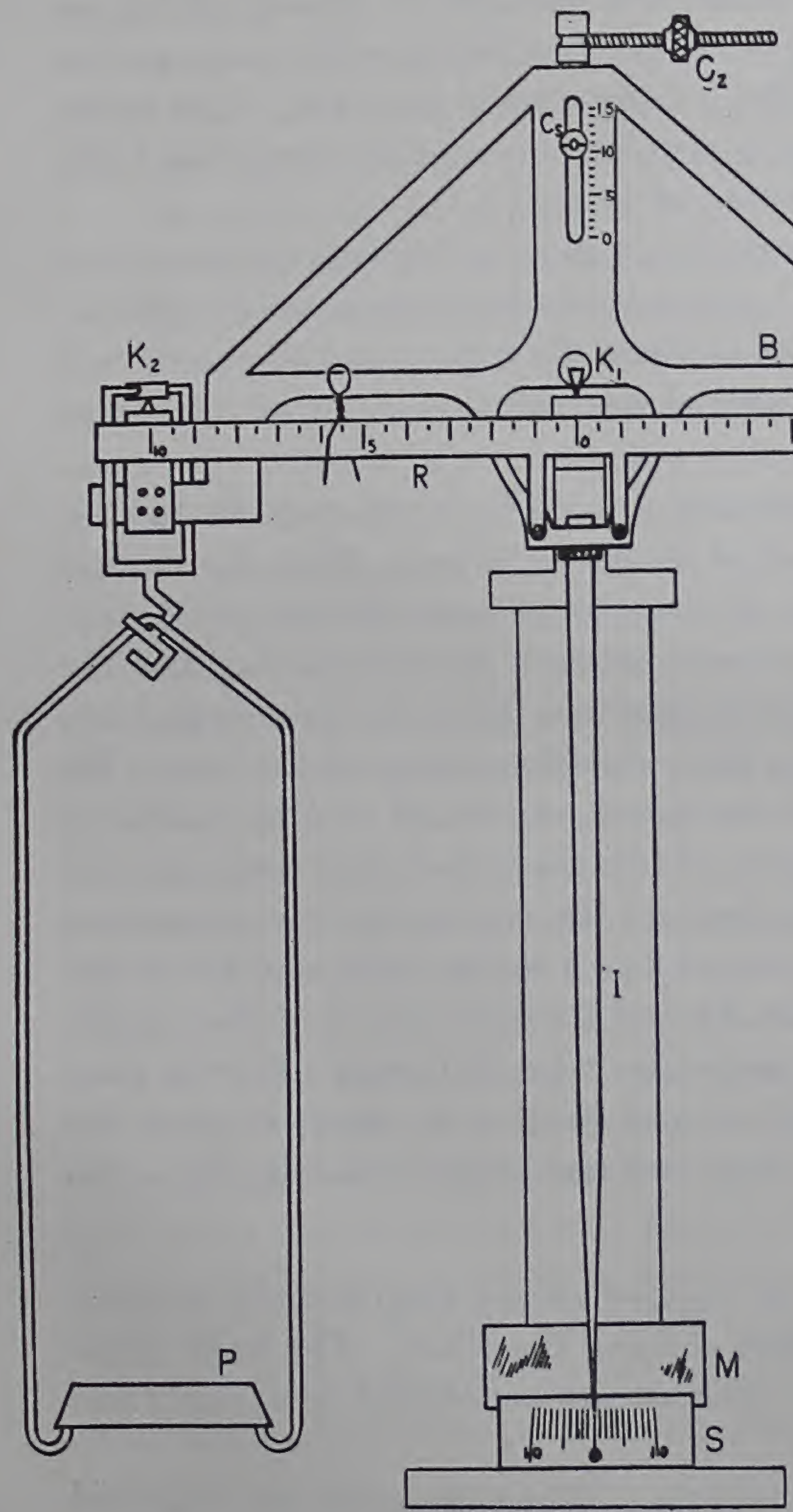


Fig. A-26a.

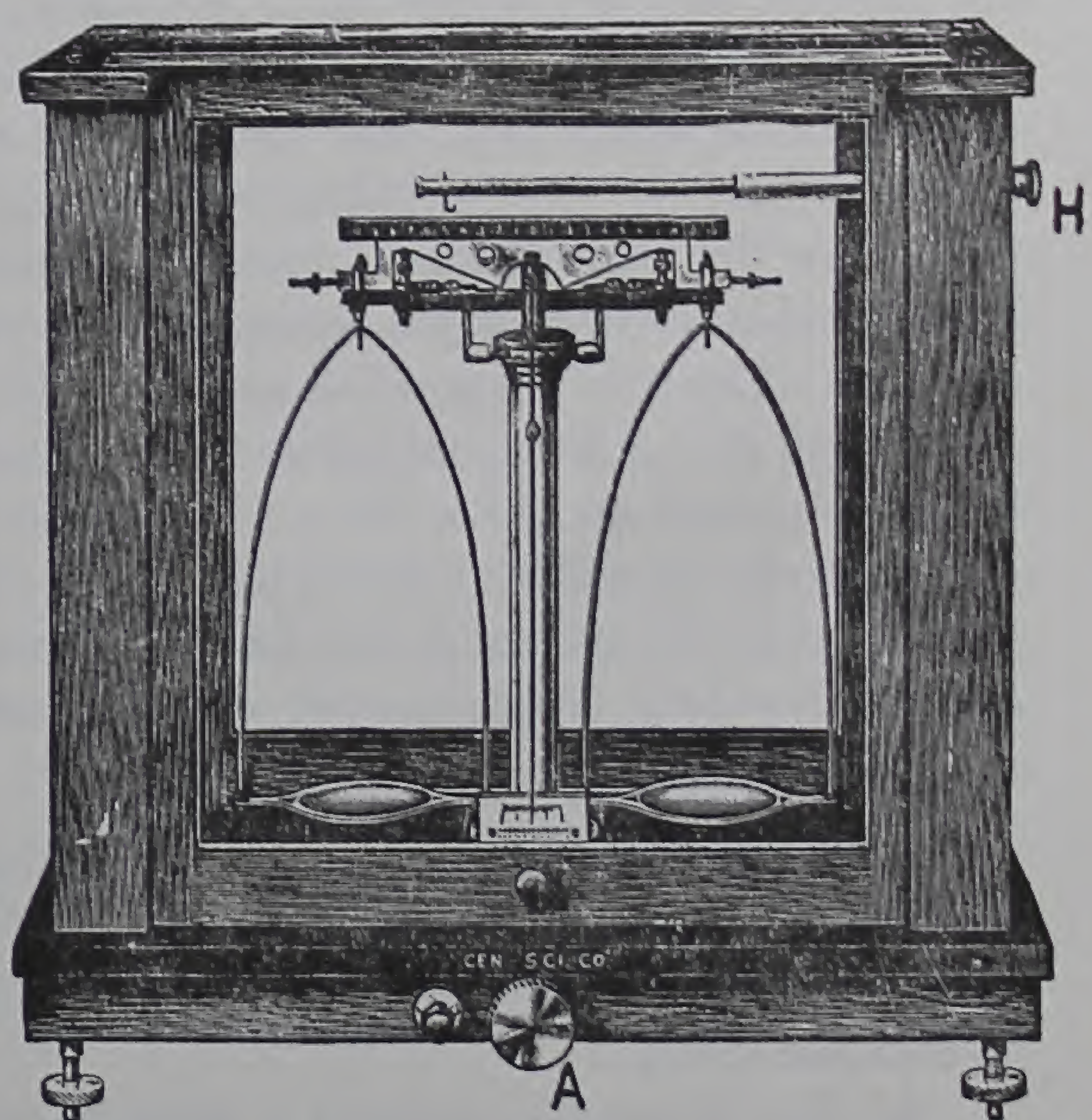


Fig. A-26b.

b. The balance consists of a beam, B, pivoting by means of an agate knife-edge, K_1, on an agate surface; two pans, P, hung on the beam at equal distances from K_1 by means of secondary knife-edges, K_2; a long pointer, I, attached to the beam, and indicating the position of the beam by means of a short scale, S, behind which a mirror, M, is placed; and a scale, R, attached to the beam, on which a small rider may be placed for fine adjustments in balance. In addition, there is an arm and hook arrangement, H, by means of which the rider may be lifted and moved without opening the case, and an arrestment, A, which lifts the beam and pans off their knife-edges at all times except during an actual balancing operation. Two adjustments are provided: C_s, by moving vertically, changes the center of mass of the beam vertically with respect to the knife-edge K_1, and thus changes the sensitivity of the balance; C_z, by moving horizontally, moves the center of

mass of the beam in the same direction, allowing adjustment of the unloaded rest position to coincide with the zero point of the scale, S. *In general neither of these adjustments is to be attempted by the student.*

c. An object is "weighed" by placing it on one pan (usually the left) and placing members of a special set of "weights," called analytical weights, in the other pan until balance is obtained. It might be noted here that this process of weighing is essentially equivalent to comparing the mass of the body to the mass of the "weights." It is a valid process regardless of the value of g, the acceleration due to gravity, provided this acceleration is the same at both pans of the balance.

The set of analytical weights usually consists of a group made of brass ranging from 1 to 100 gm and another group made of aluminum or of platinum ranging from 10 up to 500 mg. The previously mentioned rider provides adjustments from 0.1 up to 10 mg. This last adjustment may be made in one of two different ways, depending on the marking of the scale, R. If it is divided into 20 divisions, with zero in the center, a 10-mg rider is provided. Moving the rider to the tenth division either way amounts to placing 10 mg on the corresponding pan. Moving it 5 divisions amounts to placing 5 mg on the corresponding pan, and so forth. If the scale R is divided into 10 divisions, with zero to the left, a 5-mg rider is provided. The scales are balanced for the rider at the zero position. Moving the rider 10 divisions amounts to removing 5 mg from the left pan and adding 5 mg to the right pan, a total transaction of 10 mg.

d. In order to determine the mass of a body, the *unloaded* (normal) *rest point* of the balance must first be found. This is the position of the pointer I on the scale S when the *unloaded* balance ceases to vibrate; in general it will not be the center point of the scale. The balance between the object and the analytical weights has been obtained when the pointer comes to rest at this normal rest point (and not at the center point).

In practice, the scales swing for so long a time that it is impractical to wait for them to come to rest. The ultimate rest point of the balance may be found by the *method of swings* as follows: After the motion has become regular, record three *successive* extremes of the travel of the pointer past the center of scale (two on one side and one on the other). The rest point will lie halfway between the one reading and the average of the other two. For more careful work, record five successive extremes (three on one side and two on the other); the rest point lies halfway between the average of the three and the average of the two. We may show the logic of this procedure by postulating some natural rest point, say left 3, and by assuming that the balance loses equal amounts of amplitude each swing because of friction and air-damping, say 1/2 division. Then with an initial swing to the left of say 5 divisions beyond the rest point, the succeeding swings will go 4.5 divisions to the right of the rest point, 4.0 to the left of it, 3.5 to the right, and 3.0 to the left, giving readings of L8, R1.5, L7, R0.5, and L6. The averages are L7 and R1.0; the mean of these is L3, the originally postulated rest point. It is to be noted that this procedure when followed with the pans empty gives the *unloaded rest point*. When followed with the object and weights in place, it gives the *loaded rest point*. When enough weights have been added so that these two rest points coincide, the scales are balanced, and the two masses are equal.

2. *Precautions.* a. The arrestment of the balance must always be engaged except when actually measuring swings. It must be engaged when changing weights, even when shifting the rider. The knife edges are sharp and brittle, and any shock may ruin them permanently. The arrestment should be engaged and disengaged *gently*.

b. The balance is enclosed in glass to avoid the effects of air currents. When the scales are balanced to the point where swings may be recorded, the front window must be closed. After the window is closed, wait a few seconds for any air currents to die out before releasing the arrestment.

c. In reading the extremes of the swings, use the mirror behind the scale S to avoid parallax. (See Note F.) The reading is correct when the pointer and its image in the mirror coincide, *i.e.*, when the pointer exactly covers its image.

d. The analytical weights are very precisely calibrated. They are to be used only with the analytical balance. They must be *handled with the forceps* and never with the fingers, because oil and moisture from the hands will change their weights. This applies equally to the largest and the smallest weights. Each weight has a particular place in its box. When finished with a weight, *return it to its proper place in the box.* In order to avoid error in totaling the group of weights used in weighing a load, record each one individually in a column as they are being returned to their box, record the contribution of the rider, and add later.

e. Since there are two sizes of rider available, as noted in Section 1c above, be sure the balance to be used is equipped with the proper one. This may be checked by placing the rider in one pan and a 10-mg weight in the other and seeing if the scales balance. If they are far out of balance, the rider is of the 5-mg size.

3. *Weighing Procedure.* 1. With the case closed, insert the arrestment knob, and *gently* lower the beam onto its knife-edges. If it swings freely, find the unloaded rest point accurately, as indicated above, using swings of about 8 or 10 divisions total amplitude. The size of the swing can be controlled by the care with which the beam is lowered onto its knife-edges. If the balance does not swing freely, it is out of adjustment, and the instructor should be called. The student should not attempt to adjust it himself. Record the unloaded rest point and engage the arrestment.

b. Open the case and place the object to be "weighed" on the left-hand pan. On the right-hand pan place a single large weight of about the same mass as the object. *Gently* lower the arrestment. It will immediately become clear whether the mass of this weight is larger or smaller than that of the object, since the beam will definitely lean toward one side or the other as soon as the arrestment permits. Reengage the arrestment, and replace the weight by one nearer the mass of the object. Always use as the largest weight the first one which fails to overbalance the object.

Reengage the arrestment and add the next smaller weight to the pan. Gently release the arrestment to determine whether this addition is too large or too small. If too small, leave the weight on the pan, and add the next smaller weight after reengaging the arrestment.

Continue in this fashion until (with the rider in its zero position) an added 10-mg weight in the right-hand pan overbalances the object being weighed. Remove the 10-mg weight, and finish the balancing operation with the rider, using the method of swings and taking only three successive extremes until a good balance is obtained, *i.e.*, until the loaded rest point very nearly equals the unloaded rest point. For ordinary work the mass is determined to the nearest milligram, since in order for the tenths of a milligram to be significant, special corrections for the buoyant force of the air, etc., are usually necessary.

c. For work which demands an 0.1-mg accuracy, the *sensitivity* of the balance should be obtained. Determine the unloaded rest point as above, using the method of swings with five successive extremes. Repeat. Then, by means of the rider, add the equivalent of 1 mg to the right-hand pan, and determine the new rest point. *The shift in the rest point for the addition of* 1 *mg is known as the sensitivity of the balance.* Knowing the sensitivity, it is no longer necessary to bring the loaded rest point exactly into coincidence with the unloaded rest point when determining the mass of a body. Having determined the distance of the loaded from the unloaded rest point, the number of milligrams which *would be* necessary to reduce this distance to zero may easily be calculated.

d. When finding the *difference* between two very nearly equal weights, the effect of small errors in the analytical weights may be radically reduced by using the *same* large weights to balance both loads. In this way only errors in the small weights making up the difference will enter into the final computations. A still better way to proceed is to balance the heavier object first by placing sufficient weights in the right pan. Then, *leaving* these weights in the *right* pan of the balance, replace the heavier object by the lighter object in the left pan. Finally, add small weights to the *left* pan until balance is again restored. The total of the weights *added* to the *left* pan will equal the *difference* in the weights of the two objects. In this manner any error due to inequality of the arms of the balance will be eliminated.

N. The Cathode-ray Oscilloscope

1. *Description.* A very versatile and useful instrument in many fields is the cathode-ray oscilloscope. By its use, nearly any kind of electrical data may be presented visually. It is the purpose of this note to describe its operation, its principal functions, and some of its common uses. The heart of the cathode-ray oscilloscope is the cathode-ray tube. This is an elongated vacuum tube with a source of electrons at one end, and a phosphorescent screen at the other. The electrons are emitted from the "electron gun," accelerated and focused into a narrow beam, and sent down the length of the tube onto the phosphorescent screen at the far end. In the absence of any means of deflecting the electrons, the beam strikes only one spot on the screen which emits a visible glow where it is struck. If the beam is deflected, it leaves a phosphorescent path on the screen which dies out rapidly or slowly, depending on the type of screen. Since the beam pos-

sesses very little inertia, it is capable of responding extremely rapidly to deflection forces. In the elec static type of cathode-ray tube, these forces are applied by means of plates placed in the tube near the bea path. The plates are arranged facing each other in pairs; usually one pair is arranged with one plate ab the beam's path and the other below, and the other pair is set perpendicular to the first pair, one on each of the beam's path. Placing a potential difference across a pair of opposite plates causes the beam to deflected toward the more positive plate. The course of the beam is thus altered, and it strikes the fac the tube in a different place. If the potential on the plates is varied, the beam swings accordingly, trac out a corresponding path on the face of the tube.

In the cathode-ray oscilloscope, such a tube is installed along with various electronic circuits wh apply the voltages and signals necessary for its operation. Besides the power supply which furnishes proper voltages to operate the electron gun and the accelerating and focusing electrodes, there are usu three principal circuits. First, a vacuum-tube amplifier which drives the vertical deflection plates, causes a measurable vertical beam deflection in response to a small input signal. Second, another vacuu tube amplifier which has a similar function with respect to the horizontal deflection plates. Third, a sw circuit which drives the beam horizontally across the scope and enables wave forms to be plotted as fun cti of time. Each deflection amplifier is provided with one or more gain controls which can be adjusted to ca any given signal above a certain minimum size to occupy as much of the scope face as desired. The sw circuit applies a "saw-tooth" shaped wave form to the horizontal amplifier which puts it on the horizo deflection plates instead of an external signal. The saw tooth sweeps the beam across the tube face (usu from left to right) at a constant speed, and then quickly returns it to the left edge to repeat the sweep ag The repetition frequency of the sweep can be adjusted to present different phenomena as functions of t to the best advantage.

2. *Uses.* *a. Wave forms as functions of time.* Very slowly changing electrical voltages can be obser by use of a voltmeter which shows the actual voltage at any instant. If the variation is faster, either mechanism of the meter will fail to follow it because of its inertia, or the eye will be unable to recognize fu the movements of the needle. The cathode-ray oscilloscope overcomes these difficulties by using as a mov element an electron beam which has so little inertia that it can respond to deflection forces varying milli of times per second. As recording element, it uses a phosphorescent screen which can retain a record of passage of the electron beam over its surface for as long as necessary. For phenomena which occur o once, and happen rapidly, a screen with a relatively long persistence is useful. The record is thus preser for a long enough time for the eye to register it. If the phenomenon is repetitive, or if a camera is to rec the passage of a singly occurring wave form, then a screen of shorter persistence is desirable. For ordina cathode-ray oscilloscopes used primarily for visual observation, a screen with medium persistence is us Patterns are retained for an appreciable fraction of a second on the common green screen (phosphor 5).

By using a saw-tooth sweep on the horizontal plates and presenting the changing voltage on the verti plates, a plot of the voltage wave form as a function of time is obtained. Since it is possible to sweep electron beam across the scope face in less than a microsecond, wave forms lasting less time than this can presented in detail. Ordinary oscilloscopes, however, usually can only present data occurring at frequenc of a few tens of thousands per second or under. The amplifiers driving the deflection plates are the princi offenders in this behavior since they rarely are designed to have good high-frequency response. The ho zontal amplifier is usually poorer than the vertical amplifier in this respect by a factor of ten or more.

b. Frequency comparison. It is often desirable to present two simultaneously varying electrical volta as functions of each other. In such a case, one voltage is fed into the vertical amplifier and one into horizontal amplifier. If the two voltages are sinusoidal in form, and of some fixed rational frequency ra the resulting pattern on the oscilloscope takes the form of a Lissajous figure.

One application of such a presentation is to adjust the frequency of some sine wave source to be acc rately a multiple or submultiple of a known frequency. The unknown is fed into the vertical amplifier a the known sine wave is fed into the horizontal amplifier. The unknown frequency is adjusted until Lissajous pattern characteristic of the desired frequency is obtained. Another application is the comparis of the frequency of any repetitive wave form with the output of a signal generator. Putting one signal in each amplifier of the oscilloscope, the signal-generator frequency is adjusted until the resulting pattern sho

a one-to-one relationship between the unknown and the signal from the generator. The frequency of the generator is then equal to that of the unknown phenomenon.

c. Phase comparison. Still another application is the determination of the phase difference between two sine waves of the same frequency. Putting one into each input of the scope results in an ellipse on the scope face. If the two waves are in phase or 180° out of phase, the ellipse degenerates into a straight line. At any other phase relationship, the ellipse is nondegenerate; at 90° its major and minor axes are parallel to the axes of the scope. (When the amplitudes of the two waves are equal, the 90° ellipse becomes a circle.) To find the phase relationship expressed by any ellipse, consider the following:

$$e_1 = E_1 \sin \omega t,$$
$$e_2 = E_2 \sin (\omega t + \phi),$$

where ϕ is the phase angle between e_1 and e_2. We can write e_2 as

$$e_2 = E_2 (\sin \omega t \cos \phi + \cos \omega t \sin \phi).$$

Now when $e_1 = 0$, then necessarily $\sin \omega t = 0$, since E_1, the amplitude of e_1, is a finite quantity. Then under these conditions,

$$e_2 = E_2 \sin \phi.$$

Thus, if one draws a line through the center of the ellipse, parallel to the e_2 axis, the distance from the center of the ellipse to a place where the ellipse cuts this line, s, is proportional to $E_2 \sin \phi$. See Fig. A-27. The distance from the center of the ellipse to the extreme deflection in the e_2 direction, S, is proportional to E_2, the amplitude. The ratio of these two distances, s/S, is therefore equal to $\sin \phi$. In practice, it is simpler to measure the double distances in each case: the distance between the points where the ellipse cuts the axis and the outside dimension of the ellipse in the e_2 direction. A little thought will confirm the fact that measuring the distances along the e_1 axis will yield the same angle. To avoid confusion, the phase angle determined as above is to be taken simply as the difference in phase between the two voltages. If the major axis is in the first and third quadrants, the phase angle is less than 90°; if it lies in the second and fourth quadrants, the phase angle is between 90 and 180°. The shape and position of the ellipse does not indicate which voltage leads the other; this must be determined from other considerations.

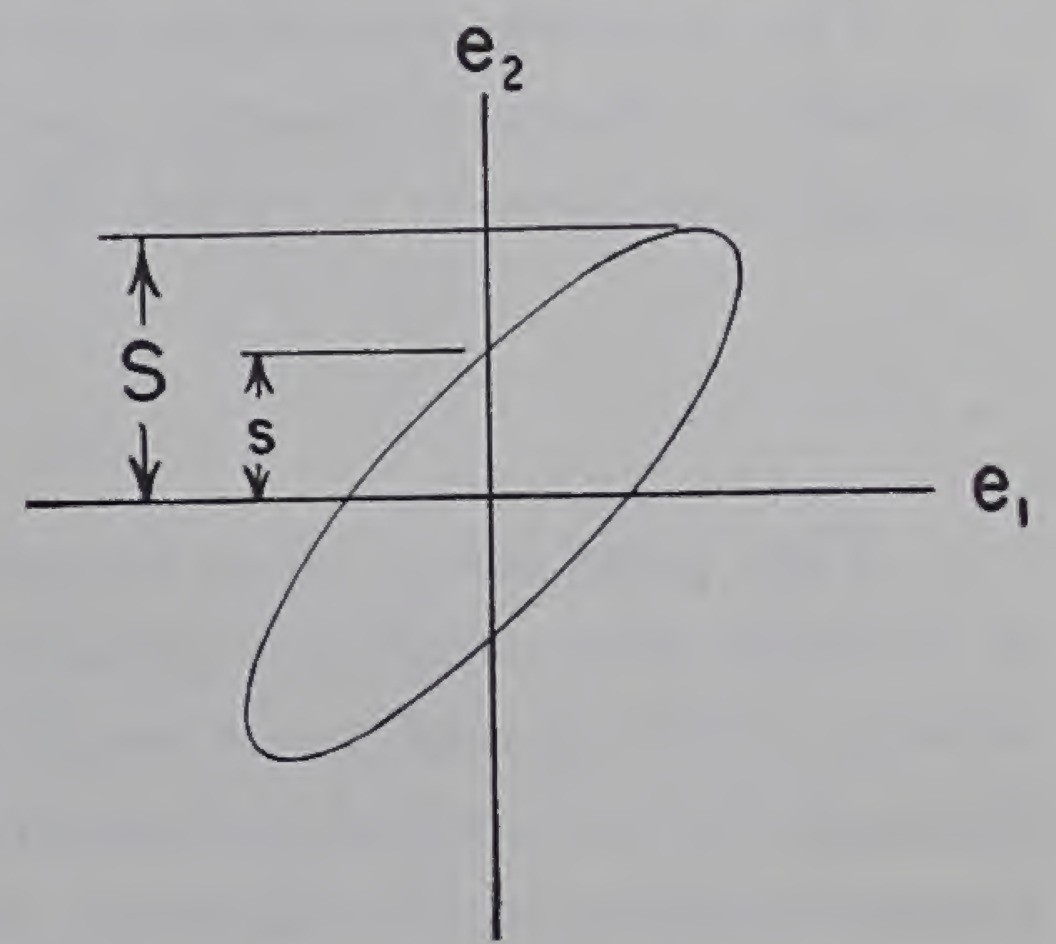


Fig. A-27.

d. Amplitude comparison. There are a myriad other uses for the cathode-ray oscilloscope, but only one more will be mentioned. It may be used to compare the amplitudes of different alternating voltages of any frequency within the frequency response characteristics of its vertical amplifier. It may be used most accurately to match the amplitudes of two voltages by putting the first one into the vertical amplifier and noting the amplitude of the resulting deflection; and then putting the other voltage into the vertical amplifier, leaving it set at the same gain, and adjusting the voltage to cause the same deflection amplitude. Within the accuracy of linearity of the vertical amplifier (that is, the limits within which the deflection on the scope face is accurately proportional to the amplitude of the input voltage to the deflection amplifier) the scope may be used to measure any alternating voltages by first calibrating it against a known voltage. The known voltage is fed to the vertical amplifier, and the gain of the amplifier is adjusted so that the beam is deflected by a convenient amount, say one scale division to the volt. The unknown voltage is then fed in with the same gain setting, and the resulting deflection measured. The amplitude of the unknown voltage is equal to the standard voltage times the ratio of its deflection to the standard deflection.

3. *Operation.* The controls of the cathode-ray oscilloscope include one to adjust the intensity of the trace and one to focus the beam. As mentioned earlier, each deflection amplifier has one or more gain controls, which often include a decade attenuator arrangement which allows the operator to cut incoming signals to one-tenth size before they reach the amplifier. The horizontal amplifier input has a switch which connects it to the sweep circuit or to the external signal jacks. This switch may be separate or it may be incorporated into the coarse frequency control switch of the sweep circuit. The sweep circuit has in addition a

fine or vernier frequency control, which provides a continuously variable frequency between the steps provided by the coarse control. The pattern may be centered, or moved from side to side or up and down by means of the horizontal and vertical centering controls, which insert a d-c "bias" onto the deflection plates without changing the character of the signal deflection voltages. Finally there are controls which enable the sweep to be synchronized with some other signal. This signal may be a repetitive phenomenon being fed to the vertical amplifier, in which case the "Synch. Selector" switch is set to "Internal," and the synchronizing signal comes from the vertical amplifier; the signal may be the line (60-cycle) frequency, in which case the selector is set to "Line"; or it may be some other external signal, in which case the selector is set to "External" and some synchronizing signal is fed to a special "Synch" jack on the panel of the oscilloscope. The synchronizing signal is fed into the sweep generator in such a way as to force it to alter its frequency to match the frequency of the signal. This can be done only if the natural frequency of the saw-tooth generator is set fairly near the frequency of the additional signal. The "Synch. Amplitude" control determines how much force is to be used to coerce the saw-tooth generator. Since a large force causes some distortion, it is usually best to use as little synchronizing signal amplitude as will do the job. When a repetitive phenomenon is being studied, the sweep is thus operating so that the wave form appears in the same place on the scope face each time it occurs. Thus wave forms which occur too rapidly to be ordinarily seen are made to appear repeatedly in the same place and become visible by virtue of such repetition. If the repetition is more rapid than about 60 per second, the wave form appears to be stationary, because of the eye's inability to discern such rapid changes.

The controls described above are the basic ones found on nearly all cathode-ray oscilloscopes, regardless of make. More elaborate instruments have, in addition, controls to operate other circuits which have special functions or perform the usual duties more precisely.

P. The FP400 Diode

This tube has a pure tungsten filament located axially in a cylindrical zirconium-coated nickel anode. It is built with sufficient precision so that it may be used in a variety of emission and space-charge experiments. The anode has a small hole centrally located so that a short portion of the filament may be viewed through it for optical-pyrometer and traveling-microscope measurements. The following tables show the characteristics of the filament and anode.

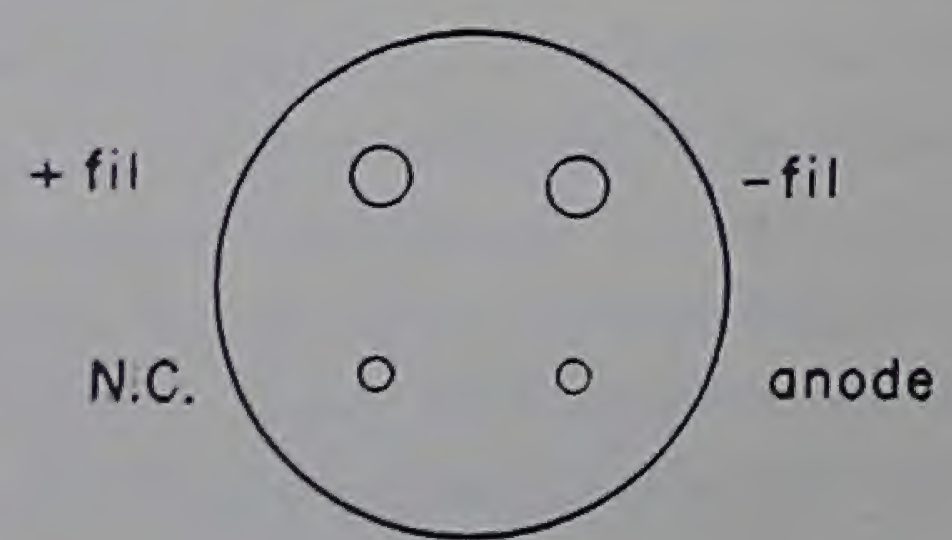
Filament:

Voltage	4.0 volts; max: 4.75 volts
Current	2.25 amp
Lead resistance	0.08 ohm (mostly in positive lead)
Length	1.25 in. Effective length, 1.0 in. (corrected for end effect)
Diameter (cold)	0.005 in.

Anode:

Voltage	125 volts max
Current	25 ma max
Power dissipation	15 watts max
Length	1.5 in.
Diameter (inside)	0.620 in.

The basing connections are shown in the figure below. The view is from the bottom of the tube.



Appendix III.

Tables

Table A

Natural Trigonometric Functions

sin

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9		
0°	.0000	.0017	.0035	.0052	.0070	.0087	.0105	.0122	.0140	.0157	.0175	89°
1°	.0175	.0192	.0209	.0227	.0244	.0262	.0279	.0297	.0314	.0332	.0349	88°
2°	.0349	.0366	.0384	.0401	.0419	.0436	.0454	.0471	.0488	.0506	.0523	87°
3°	.0523	.0541	.0558	.0576	.0593	.0610	.0628	.0645	.0663	.0680	.0698	86°
4°	.0698	.0715	.0732	.0750	.0767	.0785	.0802	.0819	.0837	.0854	.0872	85°
5°	.0872	.0889	.0906	.0924	.0941	.0958	.0976	.0993	.1011	.1028	.1045	84°
6°	.1045	.1063	.1080	.1097	.1115	.1132	.1149	.1167	.1184	.1201	.1219	83°
7°	.1219	.1236	.1253	.1271	.1288	.1305	.1323	.1340	.1357	.1374	.1392	82°
8°	.1392	.1409	.1426	.1444	.1461	.1478	.1495	.1513	.1530	.1547	.1564	81°
9°	.1564	.1582	.1599	.1616	.1633	.1650	.1668	.1685	.1702	.1719	.1736	80°
10°	.1736	.1754	.1771	.1788	.1805	.1822	.1840	.1857	.1874	.1891	.1908	79°
11°	.1908	.1925	.1942	.1959	.1977	.1994	.2011	.2028	.2045	.2062	.2079	78°
12°	.2079	.2096	.2113	.2130	.2147	.2164	.2181	.2198	.2115	.2233	.2250	77°
13°	.2250	.2267	.2284	.2300	.2317	.2334	.2351	.2368	.2385	.2402	.2419	76°
14°	.2419	.2436	.2453	.2470	.2487	.2504	.2521	.2538	.2554	.2571	.2588	75°
15°	.2588	.2605	.2622	.2639	.2656	.2672	.2689	.2706	.2723	.2740	.2756	74°
16°	.2756	.2773	.2790	.2807	.2823	.2840	.2857	.2874	.2890	.2907	.2924	73°
17°	.2924	.2940	.2957	.2974	.2990	.3007	.3024	.3040	.3057	.3074	.3090	72°
18°	.3090	.3107	.3123	.3140	.3156	.3173	.3190	.3206	.3223	.3239	.3256	71°
19°	.3256	.3272	.3289	.3305	.3322	.3338	.3355	.3371	.3387	.3404	.3420	70°
20°	.3420	.3437	.3453	.3469	.3486	.3502	.3518	.3535	.3551	.3567	.3584	69°
21°	.3584	.3600	.3616	.3633	.3649	.3665	.3681	.3697	.3714	.3730	.3746	68°
22°	.3746	.3762	.3778	.3795	.3811	.3827	.3843	.3859	.3875	.3891	.3907	67°
23°	.3907	.3923	.3939	.3955	.3971	.3987	.4003	.4019	.4035	.4051	.4067	66°
24°	.4067	.4083	.4099	.4115	.4131	.4147	.4163	.4179	.4195	.4210	.4226	65°
25°	.4226	.4242	.4258	.4274	.4289	.4305	.4321	.4337	.4352	.4368	.4384	64°
26°	.4384	.4399	.4415	.4431	.4446	.4462	.4478	.4493	.4509	.4524	.4540	63°
27°	.4540	.4555	.4571	.4586	.4602	.4617	.4633	.4648	.4664	.4679	.4695	62°
28°	.4695	.4710	.4726	.4741	.4756	.4772	.4787	.4802	.4818	.4833	.4848	61°
29°	.4848	.4863	.4879	.4894	.4909	.4924	.4939	.4955	.4970	.4985	.5000	60°
30°	.5000	.5015	.5030	.5045	.5060	.5075	.5090	.5105	.5120	.5135	.5150	59°
31°	.5150	.5165	.5180	.5195	.5210	.5225	.5240	.5255	.5270	.5284	.5299	58°
32°	.5299	.5314	.5329	.5344	.5358	.5373	.5388	.5402	.5417	.5432	.5446	57°
33°	.5446	.5461	.5476	.5490	.5505	.5519	.5534	.5548	.5563	.5577	.5592	56°
34°	.5592	.5606	.5621	.5635	.5650	.5664	.5678	.5693	.5707	.5721	.5736	55°
35°	.5736	.5750	.5764	.5779	.5793	.5807	.5821	.5835	.5850	.5864	.5878	54°
36°	.5878	.5892	.5906	.5920	.5934	.5948	.5962	.5976	.5990	.6004	.6018	53°
37°	.6018	.6032	.6046	.6060	.6074	.6088	.6101	.6115	.6129	.6143	.6157	52°
38°	.6157	.6170	.6184	.6198	.6211	.6225	.6239	.6252	.6266	.6280	.6293	51°
39°	.6293	.6307	.6320	.6334	.6347	.6361	.6374	.6388	.6401	.6414	.6428	50°
40°	.6428	.6441	.6455	.6468	.6481	.6494	.6508	.6521	.6534	.6547	.6561	49°
41°	.6561	.6574	.6587	.6600	.6613	.6626	.6639	.6652	.6665	.6678	.6691	48°
42°	.6691	.6704	.6717	.6730	.6743	.6756	.6769	.6782	.6794	.6807	.6820	47°
43°	.6820	.6833	.6845	.6858	.6871	.6884	.6896	.6909	.6921	.6934	.6947	46°
44°	.6947	.6959	.6972	.6984	.6997	.7009	.7022	.7034	.7046	.7059	.7071	45°
		.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	

cos

sin

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9		
45°	.7071	.7083	.7096	.7108	.7120	.7133	.7145	.7157	.7169	.7181	.7193	44°
46°	.7193	.7206	.7218	.7230	.7242	.7254	.7266	.7278	.7290	.7302	.7314	43°
47°	.7314	.7325	.7337	.7349	.7361	.7373	.7385	.7396	.7408	.7420	.7431	42°
48°	.7431	.7443	.7455	.7466	.7478	.7490	.7501	.7513	.7524	.7536	.7547	41°
49°	.7547	.7559	.7570	.7581	.7593	.7604	.7615	.7627	.7638	.7649	.7660	40°
50°	.7660	.7672	.7683	.7694	.7705	.7716	.7727	.7738	.7749	.7760	.7771	39°
51°	.7771	.7782	.7793	.7804	.7815	.7826	.7837	.7848	.7859	.7869	.7880	38°
52°	.7880	.7891	.7902	.7912	.7923	.7934	.7944	.7955	.7965	.7976	.7986	37°
53°	.7986	.7997	.8007	.8018	.8028	.8039	.8049	.8059	.8070	.8080	.8090	36°
54°	.8090	.8100	.8111	.8121	.8131	.8141	.8151	.8161	.8171	.8181	.8192	35°
55°	.8192	.8202	.8211	.8221	.8231	.8241	.8251	.8261	.8271	.8281	.8290	34°
56°	.8290	.8300	.8310	.8320	.8329	.8339	.8348	.8358	.8368	.8377	.8387	33°
57°	.8387	.8396	.8406	.8415	.8425	.8434	.8443	.8453	.8462	.8471	.8480	32°
58°	.8480	.8490	.8499	.8508	.8517	.8526	.8536	.8545	.8554	.8563	.8572	31°
59°	.8572	.8581	.8590	.8599	.8607	.8616	.8625	.8634	.8643	.8652	.8660	30°
60°	.8660	.8669	.8678	.8686	.8695	.8704	.8712	.8721	.8729	.8738	.8746	29°
61°	.8746	.8755	.8763	.8771	.8780	.8788	.8796	.8805	.8813	.8821	.8829	28°
62°	.8829	.8838	.8846	.8854	.8862	.8870	.8878	.8886	.8894	.8902	.8910	27°
63°	.8910	.8918	.8926	.8934	.8942	.8949	.8957	.8965	.8973	.8980	.8988	26°
64°	.8988	.8996	.9003	.9011	.9018	.9026	.9033	.9041	.9048	.9056	.9063	25°
65°	.9063	.9070	.9078	.9085	.9092	.9100	.9107	.9114	.9121	.9128	.9135	24°
66°	.9135	.9143	.9150	.9157	.9164	.9171	.9178	.9184	.9191	.9198	.9205	23°
67°	.9205	.9212	.9219	.9225	.9232	.9239	.9245	.9252	.9259	.9265	.9272	22°
68°	.9272	.9278	.9285	.9291	.9298	.9304	.9311	.9317	.9323	.9330	.9336	21°
69°	.9336	.9342	.9348	.9354	.9361	.9367	.9373	.9379	.9385	.9391	.9397	20°
70°	.9397	.9403	.9409	.9415	.9421	.9426	.9432	.9438	.9444	.9449	.9455	19°
71°	.9455	.9461	.9466	.9472	.9478	.9483	.9489	.9494	.9500	.9505	.9511	18°
72°	.9511	.9516	.9521	.9527	.9532	.9537	.9542	.9548	.9553	.9558	.9563	17°
73°	.9563	.9568	.9573	.9578	.9583	.9588	.9593	.9598	.9603	.9608	.9613	16°
74°	.9613	.9617	.9622	.9627	.9632	.9636	.9641	.9646	.9650	.9655	.9659	15°
75°	.9659	.9664	.9668	.9673	.9677	.9681	.9686	.9690	.9694	.9699	.9703	14°
76°	.9703	.9707	.9711	.9715	.9720	.9724	.9728	.9732	.9736	.9740	.9744	13°
77°	.9744	.9748	.9751	.9755	.9759	.9763	.9767	.9770	.9774	.9778	.9781	12°
78°	.9781	.9785	.9789	.9792	.9796	.9799	.9803	.9806	.9810	.9813	.9816	11°
79°	.9816	.9820	.9823	.9826	.9829	.9833	.9836	.9839	.9842	.9845	.9848	10°
80°	.9848	.9851	.9854	.9857	.9860	.9863	.9866	.9869	.9871	9874	.9877	9°
81°	.9877	.9880	.9882	.9885	.9888	.9890	.9893	.9895	.9898	.9900	.9903	8°
82°	.9903	.9905	.9907	.9910	.9912	.9914	.9917	.9919	.9921	.9923	.9925	7°
83°	.9925	.9928	.9930	.9932	.9934	.9936	.9938	.9940	.9942	.9943	.9945	6°
84°	.9945	.9947	.9949	.9951	.9952	.9954	.9956	.9957	.9959	.9960	.9962	5°
85°	.9962	.9963	.9965	.9966	.9968	.9969	.9971	.9972	.9973	.9974	.9976	4°
86°	.9976	.9977	.9978	.9979	.9980	.9981	.9982	.9983	.9984	.9985	.9986	3°
87°	.9986	.9987	.9988	.9989	.9990	.9990	.9991	.9992	.9993	.9993	.9994	2°
88°	.9994	.9995	.9995	.9996	.9996	.9997	.9997	.9997	.9998	.9998	.9998	1°
89°	.9998	.9999	.9999	.9999	.9999	1.000	1.000	1.000	1.000	1.000	1.000	0°
		.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	

cos

tan

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9		
0°	.0000	.0017	.0035	.0052	.0070	.0087	.0105	.0122	.0140	.0157	.0175	89°
1°	.0175	.0192	.0209	.0227	.0244	.0262	.0279	.0297	.0314	.0332	.0349	88°
2°	.0349	.0367	.0384	.0402	.0419	.0437	.0454	.0472	.0489	.0507	.0524	87°
3°	.0524	.0542	.0559	.0577	.0594	.0612	.0629	.0647	.0664	.0682	.0699	86°
4°	.0699	.0717	.0734	.0752	.0769	.0787	.0805	.0822	.0840	.0857	.0875	85°
5°	.0875	.0892	.0910	.0928	.0945	.0963	.0981	.0998	.1016	.1033	.1051	84°
6°	.1051	.1069	.1086	.1104	.1122	.1139	.1157	.1175	.1192	.1210	.1228	83°
7°	.1228	.1246	.1263	.1281	.1299	.1317	.1334	.1352	.1370	.1388	.1405	82°
8°	.1405	.1423	.1441	.1459	.1477	.1495	.1512	.1530	.1548	.1566	.1584	81°
9°	.1584	.1602	.1620	.1638	.1655	.1673	.1691	.1709	.1727	.1745	.1763	80°
10°	.1763	.1781	.1799	.1817	.1835	.1853	.1871	.1890	.1908	.1926	.1944	79°
11°	.1944	.1962	.1980	.1998	.2016	.2035	.2053	.2071	.2089	.2107	.2126	78°
12°	.2126	.2144	.2162	.2180	.2199	.2217	.2235	.2254	.2272	.2290	.2309	77°
13°	.2309	.2327	.2345	.2364	.2382	.2401	.2419	.2438	.2456	.2475	.2493	76°
14°	.2493	.2512	.2530	.2549	.2568	.2586	.2605	.2623	.2642	.2661	.2679	75°
15°	.2679	.2698	.2717	.2736	.2754	.2773	.2792	.2811	.2830	.2849	.2867	74°
16°	.2867	.2886	.2905	.2924	.2943	.2962	.2981	.3000	.3019	.3038	.3057	73°
17°	.3057	.3076	.3096	.3115	.3134	.3153	.3172	.3191	.3211	.3230	.3249	72°
18°	.3249	.3269	.3288	.3307	.3327.	.3346	.3365	.3385	.3404	.3424	.3443	71°
19°	.3443	.3463	.3482	.3502	.3522	.3541	.3561	.3581	.3600	.3620	.3640	70°
20°	.3640	.3659	.3679	.3699	.3719	.3739	.3759	.3779	.3799	.3819	.3839	69°
21°	.3839	.3859	.3879	.3899	.3919	.3939	.3959	.3979	.4000	.4020	.4040	68°
22°	.4040	.4061	.4081	.4101	.4122	.4142	.4163	.4183	.4204	.4224	.4245	67°
23°	.4245	.4265	.4286	.4307	.4327	.4348	.4369	.4390	.4411	.4431	.4452	66°
24°	.4452	.4473	.4494	.4515	.4536	.4557	.4578	.4599	.4621	.4642	.4663	65°
25°	.4663	.4684	.4706	.4727	.4748	.4770	.4791	.4813	.4834	.4856	.4877	64°
26°	.4877	.4899	.4921	.4942	.4964	.4986	.5008	.5029	.5051	.5073	.5095	63°
27°	.5095	.5117	.5139	.5161	.5184	.5206	.5228	.5250	.5272	.5295	.5317	62°
28°	.5317	.5340	.5362	.5384	.5407	.5430	.5452	.5475	.5498	.5520	.5543	61°
29°	.5543	.5566	.5589	.5612	.5635	.5658	.5681	.5704	.5727	.5750	.5774	60°
30°	.5774	.5797	.5820	.5844	.5867	.5890	.5914	.5938	.5961	.5985	.6009	59°
31°	.6009	.6032	.6056	.6080	.6104	.6128	.6152	.6176	.6200	.6224	.6249	58°
32°	.6249	.6273	.6297	.6322	.6346	.6371	.6395	.6420	.6445	.6469	.6494	57°
33°	.6494	.6519	.6544	.6569	.6594	.6619	.6644	.6669	.6694	.6720	.6745	56°
34°	.6745	.6771	.6796	.6822	.6847	.6873	.6899	.6924	.6950	.6976	.7002	55°
35°	.7002	.7028	.7054	.7080	.7107	.7133	.7159	.7186	.7212	.7239	.7265	54°
36°	.7265	.7292	.7319	.7346	.7373	.7400	.7427	.7454	.7481	.7508	.7536	53°
37°	.7536	.7563	.7590	.7618	.7646	.7673	.7701	.7729	.7757	.7785	.7813	52°
38°	.7813	.7841	.7869	.7898	.7926	.7954	.7983	.8012	.8040	.8069	.8098	51°
39°	.8098	.8127	.8156	.8185	.8214	.8243	.8273	.8302	.8332	.8361	.8391	50°
40°	.8391	.8421	.8451	.8481	.8511	.8541	.8571	.8601	.8632	.8662	.8693	49°
41°	.8693	.8724	.8754	.8785	.8816	.8847	.8878	.8910	.8941	.8972	.9004	48°
42°	.9004	.9036	.9067	.9099	.9131	.9163	.9195	.9228	.9260	.9293	.9325	47°
43°	.9325	.9358	.9391	.9424	.9457	.9490	.9523	.9556	.9590	.9623	.9657	46°
44°	.9657	.9691	.9725	.9759	.9793	.9827	.9861	.9896	.9930	.9965	1.000	45°
		.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	

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	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9		
45°	1.000	1.003	1.007	1.011	1.014	1.018	1.021	1.025	1.028	1.032	1.036	44°
46°	1.036	1.039	1.043	1.046	1.050	1.054	1.057	1.061	1.065	1.069	1.072	43°
47°	1.072	1.076	1.080	1.084	1.087	1.091	1.095	1.099	1.103	1.107	1.111	42°
48°	1.111	1.115	1.118	1.122	1.126	1.130	1.134	1.138	1.142	1.146	1.150	41°
49°	1.150	1.154	1.159	1.163	1.167	1.171	1.175	1.179	1.183	1.188	1.192	40°
50°	1.192	1.196	1.200	1.205	1.209	1.213	1.217	1.222	1.226	1.230	1.235	39°
51°	1.235	1.239	1.244	1.248	1.253	1.257	1.262	1.266	1.271	1.275	1.280	38°
52°	1.280	1.285	1.289	1.294	1.299	1.303	1.308	1.313	1.317	1.322	1.327	37°
53°	1.327	1.332	1.337	1.342	1.347	1.351	1.356	1.361	1.366	1.371	1.376	36°
54°	1.376	1.381	1.387	1.392	1.397	1.402	1.407	1.412	1.418	1.423	1.428	35°
55°	1.428	1.433	1.439	1.444	1.450	1.455	1.460	1.466	1.471	1.477	1.483	34°
56°	1.483	1.488	1.494	1.499	1.505	1.511	1.517	1.522	1.528	1.534	1.540	33°
57°	1.540	1.546	1.552	1.558	1.564	1.570	1.576	1.582	1.588	1.594	1.600	32°
58°	1.600	1.607	1.613	1.619	1.625	1.632	1.638	1.645	1.651	1.658	1.664	31°
59°	1.664	1.671	1.678	1.684	1.691	1.698	1.704	1.711	1.718	1.725	1.732	30°
60°	1.732	1.739	1.746	1.753	1.760	1.767	1.775	1.782	1.789	1.797	1.804	29°
61°	1.804	1.811	1.819	1.827	1.834	1.842	1.849	1.857	1.865	1.873	1.881	28°
62°	1.881	1.889	1.897	1.905	1.913	1.921	1.929	1.937	1.946	1.954	1.963	27°
63°	1.963	1.971	1.980	1.988	1.997	2.006	2.014	2.023	2.032	2.041	2.050	26°
64°	2.050	2.059	2.069	2.078	2.087	2.097	2.106	2.116	2.125	2.135	2.145	25°
65°	2.145	2.154	2.164	2.174	2.184	2.194	2.204	2.215	2.225	2.236	2.246	24°
66°	2.246	2.257	2.267	2.278	2.289	2.300	2.311	2.322	2.333	2.344	2.356	23°
67°	2.356	2.367	2.379	2.391	2.402	2.414	2.426	2.438	2.450	2.463	2.475	22°
68°	2.475	2.488	2.500	2.513	2.526	2.539	2.552	2.565	2.578	2.592	2.605	21°
69°	2.605	2.619	2.633	2.646	2.660	2.675	2.689	2.703	2.718	2.733	2.747	20°
70°	2.747	2.762	2.778	2.793	2.808	2.824	2.840	2.856	2.872	2.888	2.904	19°
71°	2.904	2.921	2.937	2.954	2.971	2.989	3.006	3.024	3.042	3.060	3.078	18°
72°	3.078	3.096	3.115	3.133	3.152	3.172	3.191	3.211	3.230	3.251	3.271	17°
73°	3.271	3.291	3.312	3.333	3.354	3.376	3.398	3.420	3.442	3.465	3.487	16°
74°	3.487	3.511	3.534	3.558	3.582	3.606	3.630	3.655	3.681	3.706	3.732	15°
75°	3 732	3.758	3.785	3.812	3.839	3.867	3.895	3.923	3.952	3.981	4.011	14°
76°	4.011	4.041	4.071	4.102	4.134	4.165	4.198	4.230	4.264	4.297	4.331	13°
77°	4.331	4.366	4.402	4.437	4.474	4.511	4.548	4.586	4.625	4.665	4.705	12°
78°	4.705	4.745	4.787	4.829	4.872	4.915	4.959	5.005	5.050	5.097	5.145	11°
79°	5.145	5.193	5.242	5.292	5.343	5.396	5.449	5.503	5.558	5.614	5.671	10°
80°	5.671	5.730	5.789	5.850	5.912	5.976	6.041	6.107	6.174	6.243	6.314	9°
81°	6.314	6.386	6.460	6.535	6.612	6.691	6.772	6.855	6.940	7.026	7.115	8°
82°	7.115	7.207	7.300	7.396	7.495	7.596	7.700	7.806	7.916	8.028	8.144	7°
83°	8.144	8.264	8.386	8.513	8.643	8.777	8.915	9.058	9.205	9.357	9.514	6°
84°	9.514	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20	11.43	5°
85°	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95	14.30	4°
86°	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46	19.08	3°
87°	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27	28.64	2°
88°	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08	57.29	1°
89°	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0	∞	0°
		.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	

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Table B

Table of Logarithms to Base 10

N	0	1	2	3	4	5	6	7	8	9	P. P.				
											1	2	3	4	5
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	5	7	9
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	6
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4

NOTE: $\log_e N = \log_e 10 \log_{10} N = 2.3026 \log_{10} N$

$\log_{10} e^x = x \log_{10} e = 0.43429\, x$

N	0	1	2	3	4	5	6	7	8	9	P. P. 1	2	3	4	5
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	3	3
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2

Table C

Densities

(Grams per cubic centimeter)

Gases (0°C, 76 cm Hg)

Air (dry)	0.001293	Hydrogen	0.00008988
Carbon dioxide	0.001965	Oxygen	0.001429
Helium	0.0001784		

Liquids (20°C)

Alcohol, ethyl	0.789	Mercury	13.546
Carbon tetrachloride	1.60	Water	0.998
Ether, ethyl	0.715		

Solids

Aluminum	2.70	Silver	10.5
Brass (70% Cu; 30% Zn)	8.44	Tungsten	18.8
Copper	8.87	Wood:	
Glass (common)	2.4 to 2.6	maple	0.6 to 0.9
Gold	19.3	oak	0.6 to 0.9
Iron	7.87	pine	0.4 to 0.7
Platinum	21.5	cork	0.2 to 0.3

Table D

Saturated Aqueous Vapor Pressure

Temperature, °C	*Vapor pressure,* mm Hg	*Temperature,* °C	*Vapor pressure,* mm Hg
−10	2.0	97.0	682.0
− 9	2.1	97.2	687.0
− 8	2.3	97.4	692.0
− 7	2.6	97.6	697.1
− 6	2.8	97.8	702.2
− 5	3.0	98.0	707.3
− 4	3.3	98.2	712.4
− 3	3.6	98.4	717.5
− 2	3.9	98.6	722.7
− 1	4.2	98.8	728.0
0	4.6	99.0	733.3
1	4.9	99.2	738.5
2	5.3	99.4	743.8
3	5.7	99.6	749.1
4	6.1	99.8	754.5
5	6.5	100.0	760.0
6	7.0	100.2	765.5
7	7.5	100.4	771.0
8	8.0	100.6	776.5
9	8.6	100.8	782.0
10	9.2		
11	9.8		
12	10.5		
13	11.2		
14	12.0		
15	12.8		
16	13.6		
17	14.5		
18	15.5		
19	16.5		
20	17.6		
21	18.7		
22	19.8		
23	21.1		
24	22.4		
25	23.8		
26	25.2		
27	26.8		
28	28.4		
29	30.1		
30	31.9		
31	33.7		
32	35.7		
33	37.8		
34	40.0		
35	42.2		
36	44.6		
37	47.1		
38	49.8		
39	52.5		
40	55.4		

Table E

Coefficients of Linear Expansion; Specific Heats

Substance	Coefficient	Specific heat
Aluminum	$22.2 \times 10^{-6}/°C$	0.210
Brass (70% Cu; 30% Zn)	18.8	0.089
Copper	16.2	0.092
Iron, steel	11.7	0.104
Glass	8.0	0.19
Lead	29.4	0.031
Pyrex	3.3	0.20

Table F

Barometer Correction

(Brass scale correct at 0°C. The correction is to be subtracted from the observed height.)

Temperature °C	Correction in mm Hg if observed height is		
	720 mm	740 mm	760 mm
15	1.8	1.8	1.9
16	1.9	1.9	2.0
17	2.0	2.0	2.1
18	2.1	2.2	2.2
19	2.2	2.3	2.3
20	2.3	2.4	2.5
21	2.5	2.5	2.6
22	2.6	2.6	2.7
23	2.7	2.8	2.8
24	2.8	2.9	3.0
25	2.9	3.0	3.1
26	3.0	3.1	3.2
27	3.2	3.2	3.3
28	3.3	3.4	3.5
29	3.4	3.5	3.6
30	3.5	3.6	3.7
31	3.6	3.7	3.8
32	3.7	3.8	4.0
33	3.9	4.0	4.1
34	4.0	4.1	4.2
35	4.1	4.2	4.3

Table G

Indexes of Refraction

Substance	Sodium D line (5890 A)	Mercury green line (5461 A)
Water (20°C)	1.3330	1.3345
Glass, crown	1.5170	1.5191
Glass, flint	1.6499	1.6546
Alcohol, amyl	1.41	
Alcohol, ethyl	1.367	
Carbon tetrachloride	1.464	
Lucite	1.51	
Turpentine	1.47	

Table H

Principal Spectrum Lines of Certain Elements

e lengths shown in angstrom units. Letters indicate following colors: red, ge, yellow, green, blue, violet. Wave lengths of brightest lines are in ace type.

ury arc:	4047	v
	4078	v
	4358	v
	4916	bg
	4960	g
	5461	**g**
	5770	**y**
	5791	**y**
	6152	o
	6232	o
Lithium flame:	4132	v
	4602	b
	6104	o
	6708	**r**
Potassium flame:	**4044**	v
	4047	v
	5802	y
	7668	**r**
	7702	**r**
Sodium flame:	**5890**	**y** (D_2 line)
	5896	**y**
Strontium flame:	**4607**	**b**
	6387	o

Table J

Coefficients of Kinetic Friction

Surfaces	*Coefficient*
Metals on hardwood, dry	0.5 to 0.6
Metals on hardwood, wet	0.24 to 0.26
Metals on metals, dry	0.15 to 0.20
Metals on metals, wet	0.3
Wood on wood, dry	0.25 to 0.5
Steel on agate, dry	0.20
Steel on agate, oiled	0.107
Smooth surfaces, oiled	0.05 to 0.08
Rolling friction	0.004 to 0.006

Table K

Thermal emf's for Copper-Constantan Thermocouple, Standard

(One junction at 0°C)

Junc temp, °C	Emf, mv	Diff, mv	Junc temp, °C	Emf, mv	Diff, mv
−200	5.54	0.16	0	0.00	0.39
−190	5.38	0.18	10	0.39	0.40
−180	5.20	0.18	20	0.79	0.40
−170	5.02	0.20	30	1.19	0.42
−160	4.82	0.22	40	1.61	0.42
−150	4.60	0.22	50	2.03	0.44
−140	4.38	0.24	60	2.47	0.44
−130	4.14	0.25	70	2.91	0.45
−120	3.89	0.27	80	3.36	0.45
−110	3.62	0.27	90	3.81	0.47
−100	3.35	0.29	100	4.28	0.47
− 90	3.06	0.29	110	4.75	0.48
− 80	2.77	0.31	120	5.23	0.48
− 70	2.46	0.32	130	5.71	0.49
− 60	2.14	0.33	140	6.20	0.50
− 50	1.81	0.34	150	6.70	0.51
− 40	1.47	0.36	160	7.21	0.51
− 30	1.11	0.36	170	7.72	0.51
− 20	0.75	0.37	180	8.23	0.53
− 10	0.38	0.38	190	8.76	0.53
− 0	0.00		200	9.29	0.53
			210	9.82	0.54
			220	10.36	0.55
			230	10.91	0.55
			240	11.46	0.55
			250	12.01	0.56
			260	12.57	0.57
			270	13.14	0.57
			280	13.71	0.57
			290	14.28	0.58
			300	14.86	

Table L

Miscellaneous Constants and Numbers

Name	Symbol	Value
Absolute zero	0°K	−273.16°C
Alpha particle, rest mass	m_α	6.598×10^{-24} gm
Alpha particle, number emitted per second from a 1-millicurie source	..	3.71×10^{7}/sec
Angstrom unit	A	10^{-8} cm
Avogadro's number	N_o	6.023×10^{23}/mol
Boltzman's constant	k	1.3805×10^{-16} erg/deg
e (base of natural logs)	e	2.71828
$\log_{10} e$	..	0.43429
Earth's magnetic field, horizontal component (Minneapolis)	H	0.17 oersted
Earth's magnetic field, vertical component (Minneapolis)	V	0.59 oersted
Electron, charge	e	1.6008×10^{-19} coulomb
Electron, rest mass	m_o	9.1066×10^{-24} gm
Electron, charge/mass ratio	e/m	1.759×10^{7} abs emu/gm
Electron volt; one ev is equivalent to	ev	1.59×10^{-12} erg
		1.16×10^{4} deg Abs
		1.07×10^{-9} mass unit
		1.77×10^{-33} gm
		12,336 A
Faraday	F	96,489 coulombs/equivalent
Gas constant	R	8.314×10^{7} erg/mol deg
Horsepower	hp	550 ft-lb/sec
Inch	in.	2.540 cm
Latent heat of fusion of H_2O	L_f	79.6 cal/gm
Latent heat of vaporization of H_2O	L_v	539 cal/gm
Mechanical equivalent of heat	J	4.185 joules/cal
Pi	π	3.14159
$\log_{10} \pi$	..	0.49715
Planck's constant	h	6.624×10^{-27} erg sec
Pound	lb	453.59 gm
Pressure coefficient, perfect gas	α_p	0.00367/°C
Specific heat ratio, $\frac{c_p}{c_v}$ (air)	γ	1.402
Standard conditions	P_o, T_o	760 mm Hg, 0°C
Surface tension of H_2O (at 20°C)	T	73 dynes/cm
Velocity of light (vacuum)	c	2.9989×10^{10} cm/sec
Velocity of sound (dry air at 0°C)	v_a	3.3136×10^{4} cm/sec

Table M

Elastic Constants

(dynes per square centimeter)

Substance	Elasticity (Young's modulus)	Rigidity (shear modulus)	Volume elasticity (bulk modulus)
Aluminum	6.8 to 7.1×10^{11}	2.4 to 2.6×10^{11}	7.5×10^{11}
Brass	9 to 11	3.5 to 4.1	11
Copper	11 to 13	4.1 to 4.7	13
Glass	4 to 6	1.6 to 2.4	4
Mercury	....	....	0.26
Steel	20.0 to 20.5	7.3 to 8.3	16
Water	....	....	0.20

Table N

Work Functions

(adapted from Smithsonian Tables)

Substance	*Work function (w_o) Electron Volts*	*Substance*	*Work function (w_o) Electron Volts*
Aluminum	2.70	Platinum	6.27
Copper	2.75	Silver	3.55
Iron	2.80	Tantalum	4.07
Lead	3.80	Tungsten	4.52
Molybdenum	4.44	Zinc	2.65
Nickel	2.70	Zirconium	4.13

Table P

Acceleration Due to Gravity

The acceleration due to gravity at various latitudes and altitudes above sea level may be obtained by interpolation from the table below, or by use of the following approximate formula:

$$g = 978.04 + 5.17 \sin^2 \lambda - 0.000092A, \qquad (\text{cm/sec}^2)$$

where λ is the latitude in degrees and A is the altitude above sea level in feet.

Latitude	*g (sea level)*	*Altitude*	*Subtract*
20°	978.64 cm/sec²	500 ft	0.05 cm/sec²
25	978.96	1000	0.09
30	979.33	1500	0.14
35	979.74	2000	0.18
40	980.17	2500	0.23
45	980.62	3000	0.28
50	981.07	3500	0.32
55	981.51	4000	0.37
		4500	0.41
		5000	0.46

Table Q

Standardizing Factors for Wratten Filters and Tungsten Filament at 3000°K

Color	Filter No.	λ	S_λ
Infrared	88*	8700 A	5.8
Red	29*	7000	1.7
Red	26	6800	1.6
Orange-red	71A*	6500	0.078
Green	53*	5400	0.075
Green	61	5300	0.16
Green	58	5200	0.17
Green	60	5200	0.23
Green	67*	5100	0.20
Green	69	5100	0.28
Blue	49	4500	0.028
Blue	49A*	4500	0.067
Violet	39*	3800	0.10

* Preferred.

$$S_\lambda = \frac{J_\lambda f_\lambda \, \Delta\lambda}{1000}$$

where J_λ = relative intensity of tungsten filament ($J_{6000} = 1.0$),
f_λ = transmission coefficient of filter at wave length λ,
$\Delta\lambda$ = effective band width of filter in angstrom units.

Table R

Temperature of a Tungsten Filament*

The absolute temperature of a long uniform tungsten filament is related to the current in the filament, i_f, and to the diameter of the filament, d, as shown in the following table. (End effects neglected.)

T°K	$\frac{i_f}{d^{3/2}}$ amp/cm$^{3/2}$	Differences
500	47.6	27.6
600	75.2	33.2
700	108	40
800	148	45
900	193	51
1000	244	57
1100	301	62
1200	363	68
1300	431	73
1400	504	77
1500	581	81
1600	662	85
1700	747	89
1800	836	91
1900	927	95
2000	1022	97
2100	1119	98
2200	1217	102
2300	1319	103
2400	1422	104
2500	1526	106
2600	1632	109
2700	1741	108
2800	1849	112
2900	1961	111
3000	2072	115
3100	2187	114
3200	2301	117
3300	2418	119
3400	2537	120
3500	2657	120
3600	2777	

* Adapted from *The Characteristics of Tungsten Filaments as Functions of Temperature*, by Howard A. Jones and Irving Lang General Electric Review 30, pp. 310–319. June 1927.

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